FAST TRACKING RLS ADAPTATION ALGORITHMS OF THE SECOND-ORDER VOLTERRA DIGITAL FILTERS

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Abstract

Two new fast tracking exponentially weighted conventional recursive least-squares (RLS) algorithms of adaptation of the adaptive Volterra filters (AVF) for time-varying systems are presented. The new algorithms are based on a modification of a priciple of variable forgetting factor with unity zone [1]. Their additional computational complexity due to the forgetting factor adaptation is negligible compared to that of conventional RLS algorithms. The performance properties of the proposed algorithms are verified via computer experiments.

Keywords:

Volterra filters, adaptive filters, RLS algorithms, forgetting factor adaptation

1. Introduction

Nonlinear Volterra digital filters belong to a group of nonlinear estimators, and they are based on the approximation of nonlinear functionals by a truncated discrete Volterra series [2]. Under the condition that they are used for non-Gaussian signal processing or if their application is connected with nonlinear system modelling, they give better signal processing than some linear filters, and can be applied with advantage to noise cancelling, echo cancelling, prediction of signals, speech intelligibility enhancement, etc.

An important requirement of recursive estimators for adaptive signal processing lies in their ability to track nonstationary environment parameter changes. From this point of view, the famous standard RLS algorithm which is known to have the optimal properties in stationary environments is unsuitable for nonstationary environments.

In the field of adaptive Volterra filtering the only one attempt has been directed to the development of a modified version of the RLS algorithm to include tracking capability in time-varying environments. In 1992, Gu,Y.H. [3] presented recursive least-squares algorithms of adaptation of the AVF with a sliding-window (RLS-SW). The RLS-SW AVFs described in [3] have good convergence properties. The sliding-window application results in also the good tracking capability in time-varying environments. The disadvantage of the RLS-SW algorithms are their approximatively two times higher computational complexity than that of the RLS adaptation algorithm of the AVF [4, 5].

In this paper, two modifications of the RLS adaptation algorithms of the second-order AVF using the variable forgetting factor are proposed. The application as well as a conveniet form of modification of the variable forgetting factor principle with unity zone described in [1] is used as a basis for the new algorithm derivation. Because of working with a forgetting factor adaptation depending on input signal stationarity, the algorithms presented here have very good tracking capability. The advantage of the proposed algorithms is also their substantially smaller computational complexity than that of the RLS-SW algorithms.

2. The conventional RLS second-order Volterra filters

Let $d(k)$ and $x(k)$ represent the desired response signal and the input signal respectively, to the AVF. The problem is then to find an exponentially weighted, conventional RLS solution for the coefficients of the AVF that minimizes cost function

$$J(n) = \sum_{k=0}^{n} \lambda^{n-k}[d(k) - d(n,k)]^2 \tag{1}$$

at each time instant n, where $d(n,k)$ is the second-order Volterra filter response to the input signal $x(k)$ given by

$$d(n,k) = \sum_{i=0}^{N-1} h_i(n)x(k-i) + \sum_{i=0}^{N-1}\sum_{j=i}^{N-1} h_{i,j}(n)x(k-i)x(k-j) \tag{2}$$

where $h_i(n)$ and $h_{i,j}(n)$ are linear and quadratic coefficients, respectively of the AVF and $N-1$ represents the number of delay elements used by the AVF. We will assume without loss of generality that the quadratic coefficients are symmetric in their indices (i.e.,

$h_{i,j}(n) = h_{j,i}(n)$ for all i, j). A parameter λ is a forgetting (or data weighting) factor of $0 < \lambda \leq 1$ that controls the rate at which the adaptive filter tracks time-varying parameters. For simplicity of representation, the following notation will be used. The input vector $\mathbf{X}(k)$ at time k, which has $M = N(N+3)/2$ elements, is defined as

$$\mathbf{X}(k) = [x(k), x^2(k), x(k)x(k-1), ..., x(k)x(k-N+1), x(k-1)$$
$$x^2(k-1), x(k-1)x(k-2), ..., x(k-N+1), x^2(k\_N+1)]^T \quad (3)$$

where $[.]^T$ denotes the transpose of [. Also, the coefficient vector $\mathbf{H}(n)$ is defined as

$$\mathbf{H}(n) = [h_0(n), h_{0,0}(n), h_{0,1}(n), ..., h_{0,N-1}(n), h_1(n),$$
$$h_{1,1}(n), h_{1,2}(n), ..., h_{N-1}(n), h_{N-1,N-1}(n)]^T \quad (4)$$

Then, the exponentially weighted least-squares problem under consideration is to find, at each time moment n, the optimal coefficient vector $\mathbf{H}(n)$ that would minimize the cost function

$$J(n) = \sum_{k=0}^{n} \lambda^{n-k}[d(k) - \mathbf{H}^T(n)\mathbf{X}(k)]^2 \quad (5)$$

It is easy to show that the optimal solution to the problem is given by

$$\mathbf{H}(n) = \mathbf{R}^{-1}(n)\mathbf{P}(n) \quad (6)$$

where

$$\mathbf{R}(n) = \sum_{k=0}^{n} \lambda^{n-k}\mathbf{X}(k)\mathbf{X}^T(k) \quad (7)$$

is the least-squares autocorrelation matrix of the input vector $\mathbf{X}(n)$ and

$$P(n) = \sum_{k=0}^{n} \lambda^{n-k}\mathbf{X}(k)d(k) \quad (8)$$

is the least-squares crosscorrelation of $\mathbf{X}(n)$ and $d(n)$.

Direct evaluation of this solution requires $O(N^6)$ multiplications at each time instant. Using the matrix inversion lemma [6], this complexity can be reduced to $O(N^4)$ multiplications per iteration. The matrix inversion lemma application will result in the RLS algorithm of adaptation of the second-order AVF given in the Table 1. In this table, $a(n)$ is the *a priori* estimation error and $\mathbf{k}(n$ is well-known Kalman gain vector. The detailed derivation and description of the RLS AVF which is similar to that for the RLS adaptive linear filter (ALF) can be found e.g. in [5].

3. New RLS adaptation algorithms of the second-order AVF with time-variable forgetting factor

From the tracking capability analysis of the RLS algorithm given in [1,6,7] it follows that its above mentioned imperfection can be cancelled by using forgetting factor adaptation depending on the input signal stationarity. In the stationary case, we can estimate the environment parameters with a forgetting factor $\lambda = 1$.

Initialization:
$\mathbf{X}(0) = \mathbf{0}_{Mx1}$
$\mathbf{H}(0) = \mathbf{0}_{Mx1}$
$\mathbf{R}(0) = \delta\mathbf{I}_{MxM}$
where
M - the number of elements of the vectors H(n) and X(n)
$\mathbf{0}_{Mx1}$ - M-by-1 null matrix
$\mathbf{I}_{MxM}$ - M-by-M identity matrix
δ - a small positive constant
Iteration at time instant $n = 1, 2, 3, ..., \infty$
$\mathbf{k}(n) = \dfrac{\lambda^{-1}\mathbf{R}^{-1}(n-1)\mathbf{X}(n)}{1 + \lambda^{-1}\mathbf{X}^T(n)\mathbf{R}^{-1}(n-1)\mathbf{X}(n)}$
$\mathbf{R}^{-1}(n) = \lambda^{-1}\mathbf{R}^{-1}(n-1) - \lambda^{-1}\mathbf{k}(n)\mathbf{X}^T(n)\mathbf{R}^{-1}(n-1)$
$a(n) = d(n) - \mathbf{H}^T(n-1)\mathbf{X}(n)$
$\mathbf{H}(n) = \mathbf{H}(n-1) + \mathbf{k}(n)a(n)$

Tab.1
The RLS adaptive second-order Volterra filter

In the nonstationary case, we require to be small enough to estimate quickly the local trend of a nonstationary signal by using a finite number of recent available data. Therefore must be varied adequately to give a good tracking adaptability and low parameter error variance. If the forgetting factor is kept small when the parameters are changed abruptly, and is increased to unity appropriately so that the estimated parameter vector converges to the true value, then the algorithm has good tracking capabilities during the transient stage and fewer misadjustment errors of parameters in the steady-state.

Very interesting algorithm of the forgetting factor adaptation for the conventional RLS adaptation algorithm of the ALF with the above described properties and negligible additional computational complexity was presented in [1]. In this paper, D.J.Park et al. proposed the following strategy for choosing the variable forgetting factor:

$$\lambda(n+1) = \lambda_{min} + (1-\lambda_{min})2^{L(n)} \quad (9)$$

$$L(n) = -NINT[pa^2(n)] \quad (10)$$

NINT is defined as the nearest integer to [.] , p is a design parameter which controls the width of a unity zone and λ_{min} is a constant representing a minimum value of the forgetting factor. In expression (9), the minimum value of the forgetting factor is obtained when $a(n)$ goes to infinity and when $a(n)$ decreases to zero, the forgetting factor goes to unity at an exponential rate. The time interval in which the forgetting factor becomes unity due to small errors is called a unity zone. Then, the rate of which the forgetting factor goes to unity as well as the width of the unity zone is controlled by the sensitivity gain p. The main advantage of the forgetting factor adaptation by (9) and (10) is that after convergence or after a transient process when $\lambda(n)$ is of $<\lambda_{min}, 1>$ the forgetting factor can be quickly switched to unity in order to reduce steady-state errors.

The forgetting factor adaptation in accordance with expressions (9) and (10) can be without invoking any further assumption straight-lined extended to the AVF. This modification of the RLS algorithm of adaptation of the AVF which is given in the Tab.1 supplemented by the expressions (9) and (10) we will call the RLS adaptation algorithm of the AVF with the variable forgetting factor (RLS-A).

The RLS-A AVF will work satisfactorily also in a nonstationary environment if *a priori* estimation error $a(n)$ is from a certain constant dynamic range for which a corresponding optimal value of parameter p is selected. If the dynamic range of $a(n)$ is increased (decreased) and contemporary if the parameter p is selected by the previous recommendation then the forgetting factor sensitivity to $a(n)$ will be increased (decreased). This effect will result in misadjustment increasing irf steady-state (tracking capability decreasing in a transient stage). Such a behaviour of the RLS-A algorithm which can be unacceptable in some applications have been confirmed by a number of simulations.

This imperfection of the RLS-A algorithm can be suppressed if the forgetting factor adaptation is made in according with the expression (9) where the function $L(n)$ is defined as follows

$$L(n) = -NINT[pa^2(n)/\sigma_a^2(n)] \quad (11)$$

where

$$\sigma_a^2(n) = \lambda' \sigma_a^2(n-1) + a^2(n) \quad (12)$$

$\sigma_a^2(n)$ in the equation (11) is the energy of *a priori* estimation error of the AVF computed by means of an exponential weighted function (12) where λ' is a constant forgetting factor. Then, the RLS algorithm of adaptation of the AVF given by the Tab.1 and supplemented by the expressions (9), (11) and (12) we will call the RLS adaptive algorithms of the AVF with normalized adaptation of the forgetting factor (RLS-AN). The meaning of p parameter in the expression (11) is the same as in (10). *A priori* error normalization by equation (11) enables for $\lambda(n)$ to be sensitive mainly to dynamism of $a(n)$ changes. The effect of the changes of *a priori* estimation error energy to misadjustment in steady-state will be strongly suppressed by using the proposed normalization.

The common property of the RLS-A and RLS-AN algorithms is their negligible small additional computational complexity due to the forgetting factor adaptation. Some other properties of the algorithms proposed in this paper will be demonstrated by the computer simulations in the next section.

4. Computer simulations

For the purpose of the comparison of the some properties of the RLS,RLS-SW, RLS-A and RLS-AN and adaptation algorithms of the AVF, the following computer experiments were performed.

The nonlinear plant identifier set up in Fig.1. was used in the experimental results presented here.

A white zero-mean and pseudorandom signal with gaussian distribution with variance $\sigma_x = 1$ was used as a reference signal of the AVF. The nonlinear system was represented by the time-varying nonlinear second-order Volterra system. Its memory span was four samples long

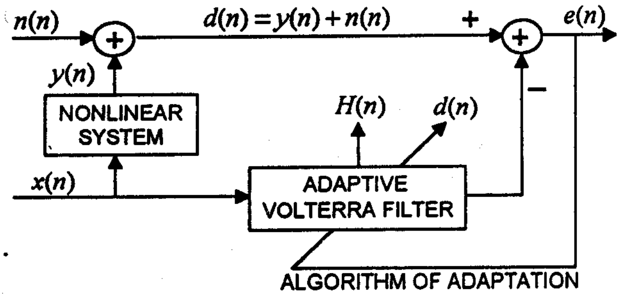

Fig.1
Nonlinear plant identification

(i.e.$N=4$). The values of the elements of its Volterra kernels were

$$\mathbf{H}^*(n) = [h_0(n), h_{0,0}(n), h_{0,1}(n), h_{0,2}(n), h_{0,3}(n),$$
$$h_1(n), h_{1,1}(n), h_{1,2}(n), h_{1,3}(n), h_2(n),$$
$$h_{2,0}(n), h_{2,1}(n), h_{2,3}(n), h_3(n), h_{3,3}(n)]^T =$$
$$= [-0.78, 0.54, 3.72, 1.86, -0.76, -1.48, -1.62, 0.76,$$
$$-0.12, 1.39, 1.41, -1.52, 0.04, -0.13]^T$$

which were changed to

$$\mathbf{H}^*(n) = [1.3, 0.04, 1.27, -1.86, 0.36, -0.5, -0.63,$$
$$-1.36, 1, 0.73, 2.41, 0.52, -0.38, -0.48]^T$$

at iteration time 2000. The input and the output of nonlinear plant were $x(n)$ and $y(n)$, respectively. A white zero-mean and pseudorandom signal with the gaussian distribution was used as a plant noise $n(n)$. The signal $n(n)$was uncorrelated with signal $x(n)$. The primary signal $d(n)$to the plant identifier was obtained as an aditive mixture of $y(n)$ and $n(n)$. Within our simulations, three diffrent experiments were made. In these experiments, the variance of the plant noise $n(n)$ were adjusted so that the signal to noise ratios (SNR) of plant noise and output signal of the nonlinear plant were subsequently 0dB, -10dB and -20dB.

The results presented here are asemble averages of 3 independent runs using 8000 samples each. The performance index used in our computer experiments is a normalized Euclid's norm of the vector of AVF coefficient estimation error defined as

$$N(n) = 10 * \log_{10} \frac{\|\mathbf{H}^*(n) - \mathbf{H}(n)\|^2}{\|\mathbf{H}^*(n)\|^2}; [dB$$

Besides index performance $N(n)$, comparisons between estimated filter parameter $h_1(n)$ and optimum filter parameter $h_1(n)$ will be also given to demonstrate performance properties of the proposed algorithms.

As an AVF was used the second-order AVF. Its memory span was four samples long. The RLS, RLS-SW, RLS-A and RLS-AN algorithms were used subsequently for the AVF adaptation. The parameters controlling the rate and the stability of the convergence of these algorithms were selected in the such a way as to reach the best performance from the viewpoint of tracking of time-varying nonlinear plant parameters as well as from the viewpoint of values of $N(n)$ in steady-state. In accordance with these conditions the following values of these parameters were choosen:

Algorithm:	SNR [dB]:	Parameters:
RLS	0, -10,-20	$\lambda = 0.998$
RLS-RW	0,-10,-20	$L = 200;$
RLS-A	0.00	$p = 0.0035$
RLS-A	-10.00	$p = 0.045$
RLS-A	-20.00	$p = 0.1$
RLS-AN	0, -10, -20	$p = 60, \lambda' = 0.999$

where L is the window length.

The results obtained are presented in the Fig.2-9. It can be seen from the Fig.2. and Fig.6. that the RLS algorithm do not work well in the case of the time-varying environment because it tracks the changes of the environment very slowly. Because of that imperfection, it cannot be probably applied with succes in the time-varying environment.

On the other hand, we can observe from the Fig.3-5 and Fig. 7-9 that the RLS-A, RLS-AN and RLS-SW algorithms have very good tracking capabilities in the time-varying environments. The rate of tracking is approximatively the same for all of these algorithms. The values of the index performance $N(n)$ of the algorithms proposed in this paper are a bit smaller than that of the RLS-SW algorithm. With regard to that fact, the quantities such as misadjustment and the estimation error variance of the optimal coefficients of the AVF will be also a bit smaller.

The dependance of the behaviour of the RLA-A and RLS-AN algorithms on the selection of the parameter p is demonstrated at the Fig.4 and Fig.5. This results indicate that *a priori* estimation error normalization by its energy can cancell the effect of the plant noise variance variability very effectively.

5. Conclusion

In this paper, two RLS algorithms of adaptation of the AVF called RLS-A and RLS-AN algorithms were introduce to improve the tracking capability of the conventional RLS algorithms of adaptation of the AVF. The proposed algorithms use the variable forgetting factor with unity zone as well as the modifications of this principle. They have very good tracking capabilities in the time-varying environment as well as good properties in steady-state. Their performance properties are similar to that of the RLS-SW algorithm. The extra computational requirement is not complicated compared with the computational complexity of the conventional RLS and FRLS algorithms. The computational complexity of the proposed algorithms is approximatively twice smaller for the RLS-A and RLS-AN algorithms than that of the RLS-SW algorithm. The disadvantage of the RLS-A algorithm is its dependence on plant noise variance variability. We believe that this imperfection will not represent a invincible obstacle for its many practical applications.

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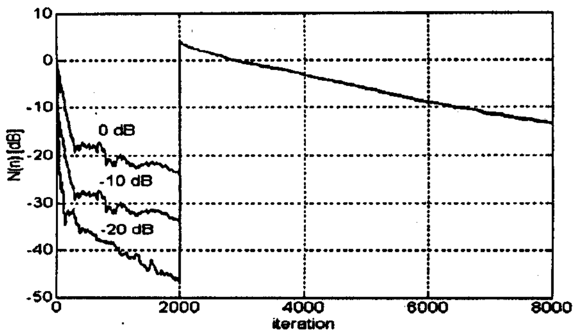

Fig.2
RLS algorithm. Index performance N(n) versus iteration.

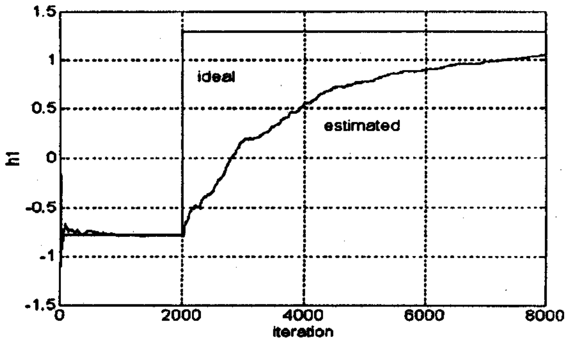

Fig.6
RLS algorithm. Coefficient h_1 (n) versus iteration. SNR= -20dB.

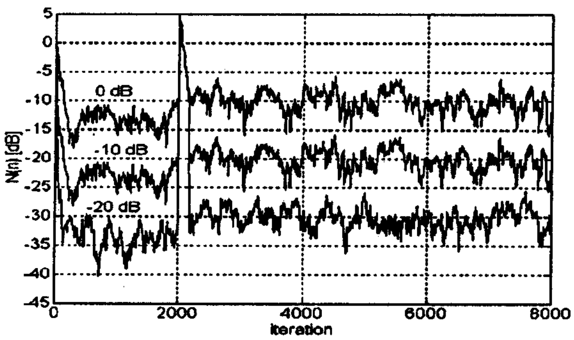

Fig.3
RLS-SW algorithm. Index performance N(n) versus iteration.

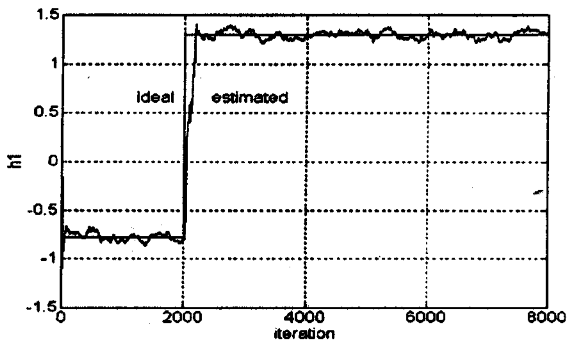

Fig.7
RLS-SW algorithm. Coefficient h_1 (n) versus iteration. SNR= -20dB.

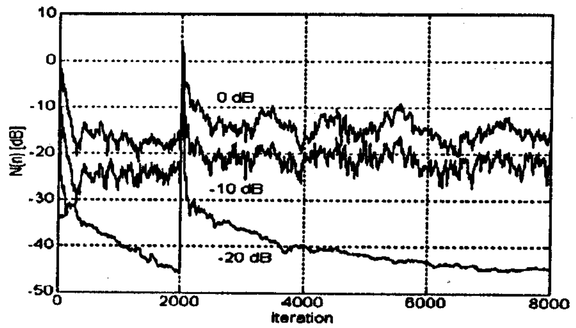

Fig.4
RLS-A algorithm. Index performance N(n) versus iteration.

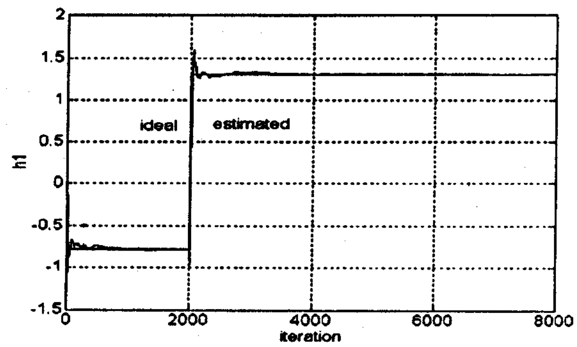

Fig.8
RLS-A algorithm. Coefficient h_1 (n) versus iteration. SNR= -20dB.

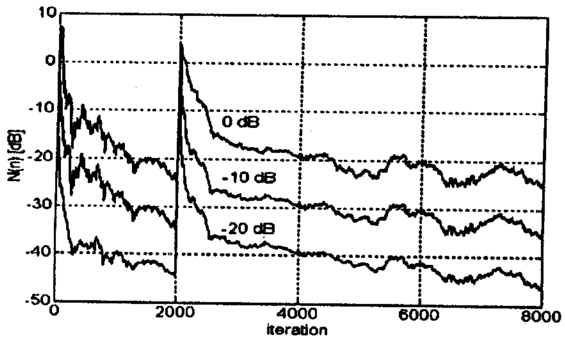

Fig.5
RLS-AN algorithm. Index performance N(n) versus iteration.

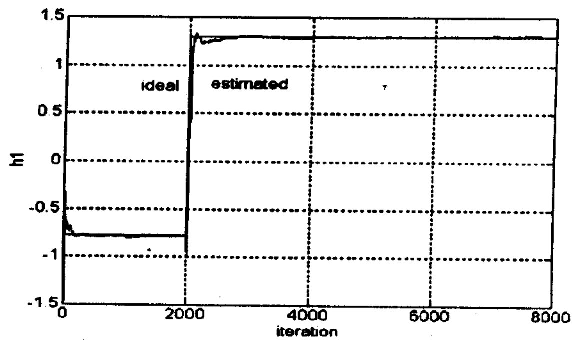

Fig.9
RLS-AN algorithm. Coefficient h_1 (n) versus iteration. SNR= -20dB.

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