TRANSFER FUNCTIONS AND CHAIN PARAMETERS OF SC EQUIVALENTS

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Abstract

When designing SC filters, the question of charge transfer functions and chain parameters becomes very interesting. A complete analysis of this question based on standard charge conservation equations and their solution by algebraic cofactors is presented here.

First, the analytic method used is reviewed (both a standard and a modified nodal voltage method are used), second, the voltage and charge transfer functions of a standard SC four-port model are discussed (including some special cases, such as SC circuits using full bilinear simulation of resistors); and, finally, a chain parameters analogy for SC circuit is derived (including special cases with non existing input/output nodes in one phase).

Further possible ways to establish chain parameters are presented, and the relation between trace parameters of a chain matrix and standard transfer functions is discussed. The entire solution is demonstrated with some simple examples, and possible methods of computer-aided evaluation of the transfer functions are briefly discussed.

Keywords:

Switched capacitor circuits, switched capacitor filters, computer aided analysis, two-port parameters, four-port parameters, Z-domain analysis, charge conservation equations, chain parameters

1. Introduction

In designing SC filters from analogue prototypes with imitance convertors and designing SC filters with OTA amplifiers in current mode (see also [10]), the use of charge transfer functions and chain parameter analogies seems to be very efficient. In the following article, first the analytic charge conservation method used will be reviewed and then the charge transfer functions and chain parameters of SC circuits will be derived.

2. Charge Conservation Equations

The SC filter design is carried out for idealized SC circuits for many practical reasons, using mostly slight modification of basic charge conservation equations. The method with the shifted Z transform using $z^{-1/2}$ factors used below. For more details, see [10,1]. The basic definition of circuit equations (for two-phases switching) is given in the matrix form as

$$\begin{bmatrix} \mathbf{Q}_E(z) \\ \mathbf{Q}_O(z) \end{bmatrix} = \begin{bmatrix} \mathbf{C}_{EE} & -\mathbf{C}_{EO}z^{-1/2} \\ -\mathbf{C}_{OE}z^{-1/2} & \mathbf{C}_{OO} \end{bmatrix} \begin{bmatrix} \mathbf{V}_E(z) \\ \mathbf{V}_O(z) \end{bmatrix} \quad (1)$$

where the matrices $\mathbf{C}_{EE}$, $\mathbf{C}_{EO}$, $\mathbf{C}_{OE}$, $\mathbf{C}_{OO}$ together form the basic capacitance matrix of an SC circuit, which can be rewritten, for example, as follows:

$$\begin{bmatrix} Q_{1E} \\ Q_{2E} \\ \vdots \\ Q_{1O} \\ Q_{2O} \\ \vdots \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1\bar{1}} & C_{1\bar{2}} & \cdots \\ C_{21} & C_{22} & \cdots & C_{2\bar{1}} & C_{2\bar{2}} & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \\ C_{\bar{1}1} & C_{\bar{1}2} & \cdots & C_{\bar{1}\bar{1}} & C_{\bar{1}\bar{2}} & \cdots \\ C_{\bar{2}1} & C_{\bar{2}2} & \cdots & C_{\bar{2}\bar{1}} & C_{\bar{2}\bar{2}} & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} V_{1E} \\ V_{2E} \\ \vdots \\ V_{1O} \\ V_{2O} \\ \vdots \end{bmatrix} . \quad (2)$$

In this way a described SC circuit can be seen as a four-port, when deriving the basic transfer functions, see Fig. 1.

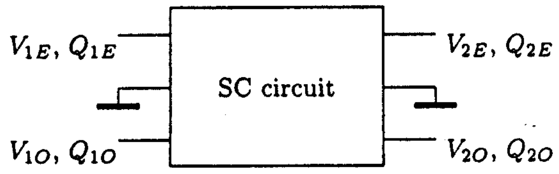

Fig.1
Four-port model

The basic solution of this four-port, using algebraic cofactors, can be written as follows:

$$\begin{bmatrix} V_{1E} \\ V_{1O} \\ V_{2E} \\ V_{2O} \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \Delta_{11} & \Delta_{1\bar{1}} & \Delta_{12} & \Delta_{1\bar{2}} \\ \Delta_{\bar{1}1} & \Delta_{\bar{1}\bar{1}} & \Delta_{\bar{1}2} & \Delta_{\bar{1}\bar{2}} \\ \Delta_{21} & \Delta_{2\bar{1}} & \Delta_{22} & \Delta_{2\bar{2}} \\ \Delta_{\bar{2}1} & \Delta_{\bar{2}\bar{1}} & \Delta_{\bar{2}2} & \Delta_{\bar{2}\bar{2}} \end{bmatrix} \begin{bmatrix} Q_{1E} \\ Q_{1O} \\ Q_{2E} \\ Q_{2O} \end{bmatrix} . \quad (3)$$

This is, in fact, an SC analogy of impedance parameters. The entire following solution is based on these basic equations.

3. Voltage Transfer Functions

Voltage transfer functions are basic functions describing an SC circuit; nevertheless, their interpretation hides a lot of obscurities, and the results of some analytic programs are quite confusing. Insight into the nature of this problem was given in [1,2]. In the following section, the basic definitions using algebraic cofactors will be reviewed. The obtained results are fully compatible with those obtained in [1,2,4,10].

3.1 Basic Functions

In defining these functions, we shall start from our basic schedule and from the superposition theorem given in [3] which is taken as a standard for SC circuit design. Then for the port voltage relations, we can put down:

$$\begin{bmatrix} V_{2E} \\ V_{2O} \end{bmatrix} = \begin{bmatrix} K_{EE} \, K_{EO} \\ K_{OE} \, K_{OO} \end{bmatrix} \begin{bmatrix} V_{1E} \\ V_{1O} \end{bmatrix} . \tag{4}$$

In this way, defined four-transfer functions are assumed as the basic function set of an SC circuit, which is used to establish any other necessary transfer functions using a more complicated sampling system. The correct definition of transfer functions is, therefore, of great importance. For example:

$$K_{EE} = \frac{V_{2E}}{V_{1E}} \bigg|_{V_{1O} = 0, Q_{2E} = Q_{2O} = 0} \tag{5}$$

The zero voltage condition in the opposite phase follows from previous equations; the zero output charge condition is a basic condition of the voltage transfer function exactly as it is in the p domain. For the above function K_{EE} we can then write, based on (3)

$$\begin{aligned} V_{2E} &= \tfrac{1}{\Delta}(\Delta_{12} Q_{1E} + \Delta_{\bar{1}2} Q_{1O}), \\ V_{1E} &= \tfrac{1}{\Delta}(\Delta_{11} Q_{1E} + \Delta_{\bar{1}1} Q_{1O}), \\ V_{1O} &= \tfrac{1}{\Delta}(\Delta_{1\bar{1}} Q_{1E} + \Delta_{\bar{1}\bar{1}} Q_{1O}) = 0 \; . \end{aligned} \tag{6}$$

First we calculate Q_{1O} from the last equation, and then, using the dopple cofactors rule (see also [11]):

$$\boxed{\Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21} = \Delta_{11,22}\Delta} \tag{7}$$

we get

$$\frac{V_{2E}}{V_{1E}} = \frac{\Delta_{\bar{1}\bar{1}}\Delta_{12} - \Delta_{1\bar{1}}\Delta_{\bar{1}2}}{\Delta_{11}\Delta_{\bar{1}\bar{1}} - \Delta_{\bar{1}1}\Delta_{1\bar{1}}} = \frac{\Delta_{12,\bar{1}\bar{1}}}{\Delta_{11,\bar{1}\bar{1}}} . \tag{8}$$

The remaining three transfer functions can be derived analogically. Here only the result voltage transfer matrix is presented:

$$\mathbf{K} = \begin{bmatrix} \dfrac{\Delta_{12,\bar{1}\bar{1}}}{\Delta_{11,\bar{1}\bar{1}}} & \dfrac{\Delta_{\bar{1}2,11}}{\Delta_{\bar{1}\bar{1},11}} \\ \dfrac{\Delta_{1\bar{2},\bar{1}\bar{1}}}{\Delta_{11,\bar{1}\bar{1}}} & \dfrac{\Delta_{\bar{1}\bar{2},11}}{\Delta_{\bar{1}\bar{1},11}} \end{bmatrix} . \tag{9}$$

In this way, obtained results are the same as in [1,2,4]. However, definition of voltage transfers using simple cofactors, which can be found in the literature [7], is valid only for simplified circuits with no input in the odd phase. The output situation has no influence on transfer function definitions

$$K_{EE} = \frac{\Delta_{12}}{\Delta_{11}} \qquad K_{EO} = \frac{\Delta_{1\bar{2}}}{\Delta_{11}} \; . \tag{10}$$

3.2 Bilinear Circuits

There is one exception to this, which is very important. This exception is bilinear circuit which contains only capacitors, amplifiers and bilinearly-simulated resistors. These circuits do not change their structure by switching, and therefore we can describe them using only one condensed function (K_{BB}), but with two sampling times in one period. Their consistency with the previous results can be shown, for example, as follows:

$$e^{pT} \overset{!}{=} e^{\frac{pT}{2}} \Rightarrow z^{-\frac{1}{2}} \overset{!}{=} z^{-1} . \tag{11}$$

1 Re-defining the frequence scale

2 Using the definitions from [4]

$$K_{BE} = K_{EE} + K_{EO} \, , K_{BO} = K_{OE} + K_{OO} \, , \qquad K_{BB} = \frac{K_{BE} + K_{BO}}{2} . \tag{12}$$

3 Then rewriting the K_{BB} function and reducing it to its original order.

Example for a bilinear integrator, see Fig 2.

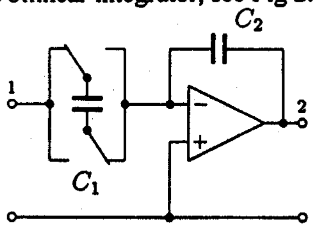

Fig. 2
Bilinear integrator

Using (9) we obtain

$$\begin{aligned} K_{EE} &= -\frac{C_1}{C_2}\frac{1+z^{-1}}{1-z^{-1}} & K_{EO} &= -\frac{C_1}{C_2}\frac{2z^{-\frac{1}{2}}}{1-z^{-1}} \\ K_{OE} &= -\frac{C_1}{C_2}\frac{2z^{-\frac{1}{2}}}{1-z^{-1}} & K_{OO} &= -\frac{C_1}{C_2}\frac{1+z^{-1}}{1-z^{-1}} . \end{aligned} \tag{13}$$

Adding the functions together we get

$$K_{BE} = K_{BO} = -\frac{C_1}{C_2}\frac{1+2z^{-\frac{1}{2}}+z^{-1}}{1-z^{-1}} . \tag{14}$$

Re-defining frequence scale gives as

$$K_{BB} = -\frac{C_1}{C_2}\frac{1+2z^{-1}+z^{-2}}{1-z^{-2}} . \tag{15}$$

And finally, using well-known algebraic formulae for K_{BB}, we get[1]

$$K_{BB} = -\frac{C_1}{C_2}\frac{1+z^{-1}}{1-z^{-1}} . \tag{16}$$

[1] It should be noted that only function K_{BB} can be compared with the transformed analogue functions via bilinear transform. The separation of phases causes difference in the denominator of higher order function and, therefore, shifts zeroes sustantially.

4. Charge Transfers

The charge transfer functions will be derived below. The basic set of equations (3) will be used again. The basic definition is the same as for the voltage functions (4):

$$\begin{bmatrix} Q_{2E} \\ Q_{2O} \end{bmatrix} = \begin{bmatrix} H_{EE}\; H_{EO} \\ H_{OE}\; H_{OO} \end{bmatrix} \begin{bmatrix} Q_{1E} \\ Q_{1O} \end{bmatrix} . \tag{17}$$

The initial conditions are the same as for the analogue current transfers. The results obtained are comparable with analogue prototypes (see examples below). For example, for H_{EE}

$$H_{EE} = \frac{Q_{2E}}{Q_{1E}} \bigg|\, Q_{1O} = 0, V_{2E} = V_{2O} = 0 . \tag{18}$$

4.1 Cofactor Definition

In evaluating these transfer functions, we use the last two equations from the basic set of an SC four-port, where the output voltages will be set to zero as well as the input voltage in the opposite phase

$$V_{2E} = \tfrac{1}{\Delta}(\Delta_{12}Q_{1E} + \Delta_{\bar{1}2}Q_{1O} + \Delta_{22}Q_{2E} + \Delta_{\bar{2}2}Q_{2O})$$

$$V_{2O} = \tfrac{1}{\Delta}(\Delta_{1\bar{2}}Q_{1E} + \Delta_{\bar{1}\bar{2}}Q_{1O} + \Delta_{2\bar{2}}Q_{2E} + \Delta_{\bar{2}\bar{2}}Q_{2O}) . \tag{19}$$

For example, for function H_{EE}:

$$\tfrac{1}{\Delta}(\Delta_{12}Q_{1E} + \Delta_{22}Q_{2E} + \Delta_{\bar{2}2}Q_{2O}) = 0$$
$$\tfrac{1}{\Delta}(\Delta_{1\bar{2}}Q_{1E} + \Delta_{2\bar{2}}Q_{2E} + \Delta_{\bar{2}\bar{2}}Q_{2O}) = 0 . \tag{20}$$

We evaluate Q_{2O} from the last equation and use it in the first one, where

$$\Delta_{12}Q_{1E}+\Delta_{22}Q_{2E}-\frac{\Delta_{\bar{2}2}}{\Delta_{\bar{2}\bar{2}}}(\Delta_{1\bar{2}}Q_{1E}+\Delta_{2\bar{2}}Q_{2E}) = 0 \tag{21}$$

and finally

$$\frac{Q_{2E}}{Q_{1E}} = \frac{\Delta_{12}\Delta_{\bar{2}\bar{2}} - \Delta_{1\bar{2}}\Delta_{\bar{2}2}}{\Delta_{22}\Delta_{\bar{2}\bar{2}} - \Delta_{2\bar{2}}\Delta_{\bar{2}2}} = -\frac{\Delta_{12,\bar{2}\bar{2}}}{\Delta_{22,\bar{2}\bar{2}}} . \tag{22}$$

The remaining transfer functions can be evaluated analogically. The whole charge transfer matrix then is

$$\mathbf{H} = \begin{bmatrix} -\dfrac{\Delta_{12,\bar{2}\bar{2}}}{\Delta_{22,\bar{2}\bar{2}}} & -\dfrac{\Delta_{\bar{1}2,\bar{2}\bar{2}}}{\Delta_{22,\bar{2}\bar{2}}} \\ -\dfrac{\Delta_{1\bar{2},22}}{\Delta_{\bar{2}\bar{2},22}} & -\dfrac{\Delta_{\bar{1}\bar{2},22}}{\Delta_{\bar{2}\bar{2},22}} \end{bmatrix} . \tag{23}$$

Exactly as for the voltages, reduced transfers can be derived using the simple cofactors. In this case, input node reduction is of no influence, which can be seen easily from the equation structures

$$H_{EE} = \frac{\Delta_{12}}{\Delta_{22}} \quad , \quad H_{OE} = \frac{\Delta_{\bar{1}2}}{\Delta_{22}} . \tag{24}$$

4.2 The Modified Voltage Method

Another possibility for evaluating charge transfer functions directly is the modified voltage method [7]. The non-regular elements are simulated by adding an extra equation to the basic set, which defines the current (charge) conditions of the investigated element. This causes expansion of the capacitance matrix by one or more columns and rows. For the switch, which is of the most importance now, we can write

$$V_{xw} = V_{yw} , \tag{25}$$

where index w is phase, and x, y are the switched nodes. The resulting "stamp matrix" of the switch is then

	V_x	V_y	$Q_{x,y}$
x			1
y			−1
	1	−1	0

(26)

For the charge transfer from x to y we can write now for the given phase (index a is the input node)

$$Q_{xw} = \frac{\Delta_{aw,xw}}{\Delta} Q_{aw} . \tag{27}$$

5. Chain Parameters

Based on the previous result we can now define the switched capacitor analogy of the chain parameters. The definition is given for the general case, where both phases are defined; the degenerated ones will be discussed later. Taking into account the phasing, we can now write (see also[6])

$$\begin{aligned} V_{1E} &= a_{11}V_{2E} + a_{1\bar{1}}V_{2O} + a_{12}Q_{2E} + a_{1\bar{2}}Q_{2O} \\ V_{1O} &= a_{\bar{1}1}V_{2E} + a_{\bar{1}\bar{1}}V_{2O} + a_{\bar{1}2}Q_{2E} + a_{\bar{1}\bar{2}}Q_{2O} \\ Q_{1E} &= a_{21}V_{2E} + a_{2\bar{1}}V_{2O} + a_{22}Q_{2E} + a_{2\bar{2}}Q_{2O} \\ Q_{1O} &= a_{\bar{2}1}V_{2E} + a_{\bar{2}\bar{1}}V_{2O} + a_{\bar{2}2}Q_{2E} + a_{\bar{2}\bar{2}}Q_{2O} \end{aligned} \tag{28}$$

which can be rewritten in matrix form

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{Q}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{a}_{11}\; \mathbf{a}_{12} \\ \mathbf{a}_{21}\; \mathbf{a}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_2 \\ \mathbf{Q}_2 \end{bmatrix} . \tag{29}$$

Both voltages and currents are vectors and the individual chain parameters are matrices 2 x 2. This fact is very important for following investigation.

$$\mathbf{V}_1 = [V_{1E}, V_{1O}]^T$$
$$\mathbf{a}_{11} = \begin{bmatrix} a_{11}\; a_{1\bar{1}} \\ a_{\bar{1}1}\; a_{\bar{1}\bar{1}} \end{bmatrix} \tag{30}$$

In this way, defined chain parameters can be evaluated practically via many different approaches, which will be briefly discussed below.

5.1 Basic Cofactor Definition

In evaluating the chain parameters of an SC circuit, we start again from the basic set of equations (3). These can be seen as the impedance parameters of the SC circuit in the matrix form, as was said in a previous section.[2] These parameters can be recalculated as the chain parameters now, exactly as in the analogue case. We have to only take into account the matrix nature of the individual chain parameters now.

$$\begin{bmatrix} V_{1E} \\ V_{1O} \\ \hline V_{2E} \\ V_{2O} \end{bmatrix} = \frac{1}{\Delta} \left[\begin{array}{cc|cc} \Delta_{11} & \Delta_{\bar{1}1} & \Delta_{21} & \Delta_{\bar{2}1} \\ \Delta_{1\bar{1}} & \Delta_{\bar{1}\bar{1}} & \Delta_{2\bar{1}} & \Delta_{\bar{2}\bar{1}} \\ \hline \Delta_{12} & \Delta_{\bar{1}2} & \Delta_{22} & \Delta_{\bar{2}2} \\ \Delta_{1\bar{2}} & \Delta_{\bar{1}\bar{2}} & \Delta_{2\bar{2}} & \Delta_{\bar{2}\bar{2}} \end{array}\right] \begin{bmatrix} Q_{1E} \\ Q_{1O} \\ \hline Q_{2E} \\ Q_{2O} \end{bmatrix} \tag{33}$$

The transformation formulae can be obtained easily

$$\begin{aligned} \mathbf{a}_{11} &= \mathbf{z}_{11}\mathbf{z}_{21}^{-1} \\ \mathbf{a}_{12} &= -\mathbf{z}_{12} + \mathbf{z}_{11}\mathbf{z}_{21}^{-1}\mathbf{z}_{22} \\ \mathbf{a}_{21} &= \mathbf{z}_{21}^{-1} \\ \mathbf{a}_{22} &= \mathbf{z}_{21}^{-1}\mathbf{z}_{22} \end{aligned} \tag{34}$$

and for individual sub-matrices we can write

$$\mathbf{a}_{11} = \begin{bmatrix} \frac{\Delta_{11,\bar{1}\bar{2}}}{\Delta_{12,\bar{1}\bar{2}}} & \frac{\Delta_{\bar{1}1,12}}{\Delta_{12,\bar{1}\bar{2}}} \\ \frac{\Delta_{1\bar{1},\bar{1}\bar{2}}}{\Delta_{12,\bar{1}\bar{2}}} & \frac{\Delta_{\bar{1}\bar{1},12}}{\Delta_{12,\bar{1}\bar{2}}} \end{bmatrix} \quad \mathbf{a}_{12} = \begin{bmatrix} \frac{\Delta_{11,\bar{1}2,22}}{\Delta_{12,\bar{1}\bar{2}}} & -\frac{\Delta_{11,\bar{1}2,\bar{2}2}}{\Delta_{12,\bar{1}\bar{2}}} \\ -\frac{\Delta_{1\bar{1},\bar{1}2,22}}{\Delta_{12,\bar{1}\bar{2}}} & \frac{\Delta_{1\bar{1},\bar{1}2,\bar{2}2}}{\Delta_{12,\bar{1}\bar{2}}} \end{bmatrix}$$

$$\mathbf{a}_{21} = \begin{bmatrix} \frac{\Delta_{\bar{1}\bar{2}}}{\Delta_{12,\bar{1}\bar{2}}} & \frac{-\Delta_{\bar{1}2}}{\Delta_{12,\bar{1}\bar{2}}} \\ \frac{-\Delta_{1\bar{2}}}{\Delta_{12,\bar{1}\bar{2}}} & \frac{\Delta_{12}}{\Delta_{12,\bar{1}\bar{2}}} \end{bmatrix} \quad \mathbf{a}_{22} = \begin{bmatrix} \frac{\Delta_{22,\bar{1}\bar{2}}}{\Delta_{12,\bar{1}\bar{2}}} & \frac{\Delta_{\bar{2}2,\bar{1}\bar{2}}}{\Delta_{12,\bar{1}\bar{2}}} \\ \frac{\Delta_{2\bar{2},12}}{\Delta_{12,\bar{1}\bar{2}}} & \frac{\Delta_{\bar{2}\bar{2},12}}{\Delta_{12,\bar{1}\bar{2}}} \end{bmatrix} \tag{35}$$

5.2 Re-calculation of Admitance Matrix

Exactly as in the previous sub-section, the chain parameters can be obtained, using the reduced admitance matrix and the dual transformation formulas from the impedance parameters (see also [6])

$$\begin{aligned} \mathbf{a}_{11} &= -\mathbf{y}_{21}^{-1}\mathbf{y}_{22} \\ \mathbf{a}_{12} &= \mathbf{y}_{12} + \mathbf{y}_{11}\mathbf{y}_{21}^{-1}\mathbf{y}_{22} \\ \mathbf{a}_{21} &= \mathbf{y}_{21}^{-1} \\ \mathbf{a}_{22} &= .\mathbf{y}_{11}\mathbf{y}_{21}^{-1} \end{aligned} \tag{36}$$

5.3 Direct Evaluating

If the admitance two-port parameters do not exist, which is the case in the investigated imitance convertor, the chain parameters can be obtained from the basic equations by direct evaluation. This example is given at the end of this article.

5.4 Relation to Previous Transfer Functions

For the trace elements of the chain matrix (that means a_{11} and a_{22}), we can write in the s domain the following direct relations[3]

$$\begin{aligned} a_{11} &= \tfrac{1}{K} \\ a_{22} &= \tfrac{1}{H} \end{aligned} \tag{37}$$

Now we try to derive analogical relations in the Z domain, which will be a bit more complex because of the switching process. Comparing the definition set (3) of the voltage transfer functions with their chain matrix counterpart a_{11} (supposing $Q_2 = 0$)

$$\begin{aligned} V_{1E} &= a_{11}V_{2E} + a_{1\bar{1}}V_{2L} \\ V_{1L} &= a_{\bar{1}1}V_{2E} + a_{\bar{1}\bar{1}}V_{2L} \end{aligned} \tag{38}$$

we see that (analogically for the charge transfers)

$$\begin{aligned} \mathbf{a}_{11} &= \mathbf{K}^{-1} \\ \mathbf{a}_{22} &= \mathbf{H}^{-1} \end{aligned} \tag{39}$$

The above defined relations are valid only in the case of a general SC circuit, that means one with input and output samples in both phases. In the case of the degeneration of one phase, the matrix structure is broken and the relations are simplified to the previous ones.

5.5 Special Cases

For a large family of switched capacitor circuits, especially when "Stray - insensitive" resistor simulation is used, some of the port nodes, mostly the input one, are not defined in one of the phases. Therefore, the matrix relation between the transfers is violated, and a special approach has to be used to evaluate the chain parameters. The use of a zero limited help capacitor was discussed in [6]; here we

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[2] Sometimes it can happen that the dopple cofactors from the definition do not exist. This case (again as with the convertor) can be explained as follows. We decompose the dopple cofactor into simple ones again using the basic formula:

$$\Delta\Delta_{11,22} = \Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21} \tag{31}$$

and suppose that the input and output nodes have the same column:

$$\Delta_{1x}\Delta_{2x} - \Delta_{2x}\Delta_{1x} = 0 \tag{32}$$

So, the investigated cofactor is zero.

[3] The remaining two parameters are of the meaning of switched imitances. They can be deciphered successfully also, see for example the switched definitions of the sensitivity functions but taking into account the complexity of this question they will be discussed separately.

carry out the basic cofactor definition again. The basic structure is on Fig. 3.

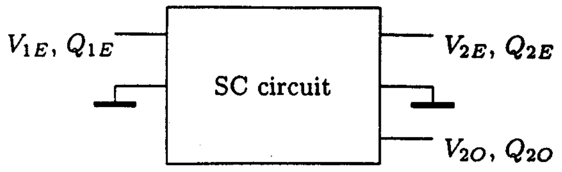

Fig. 3
Three-port model

And the general cofactor solution, eliminating Q_{10}, V_{10} from (3), is, therefore:

$$\begin{bmatrix} V_{1E} \\ V_{2E} \\ V_{2O} \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \Delta_{11}\Delta_{21}\Delta_{\bar{2}1} \\ \Delta_{12}\Delta_{22}\Delta_{\bar{2}2} \\ \Delta_{1\bar{2}}\Delta_{2\bar{2}}\Delta_{\bar{2}\bar{2}} \end{bmatrix} \begin{bmatrix} Q_{1E} \\ Q_{2E} \\ Q_{2O} \end{bmatrix} . \tag{40}$$

The chain parameters (28) can be written in the form of

$$\begin{aligned} V_{1E} &= a_{11}V_{2E} + a_{1\bar{1}}V_{2O} + a_{12}Q_{2E} + a_{1\bar{2}}Q_{2O} \\ Q_{1E} &= a_{21}V_{2E} + a_{2\bar{1}}V_{2O} + a_{22}Q_{2E} + a_{2\bar{2}}Q_{2O} \end{aligned} , \tag{41}$$

where the individual elements are defined analogically in relation to the previous cases as follows:

$$\left. \begin{aligned} a_{11} &= \frac{\Delta_{11}}{\Delta_{12}} \\ a_{1\bar{1}} &= \frac{\Delta_{11}}{\Delta_{1\bar{2}}} \\ a_{21} &= \frac{\Delta}{\Delta_{12}} \\ a_{2\bar{1}} &= \frac{\Delta}{\Delta_{1\bar{2}}} \end{aligned} \right| Q_{2E} = Q_{2O} = 0 \quad . \tag{42}$$

$$\left. \begin{aligned} a_{22} &= -\frac{\Delta_{22,\bar{2}\bar{2}}}{\Delta_{12,\bar{2}\bar{2}}} \\ a_{2\bar{2}} &= -\frac{\Delta_{\bar{2}\bar{2},22}}{\Delta_{1\bar{2},22}} \end{aligned} \right| V_{2E} = V_{2O} = 0$$

Parameter a_{12} can be evaluated from the equations

$$\begin{aligned} \Delta\Delta_{1\bar{2}}V_{1E} &= Q_{2E}\Delta_{1\bar{2},21} + Q_{2O}\Delta_{1\bar{2},\bar{2}1} \\ 0 &= Q_{2E}\Delta_{22,1\bar{2}} + Q_{2O}\Delta_{\bar{2}2,1\bar{2}} \end{aligned} . \tag{43}$$

In the results from (43), we find cofactors of higher order which correspond well with analogue definitions of a_{12} and the solutions in (33). The case of an undefined output node can be managed analogically. In this case we obtain, in fact, two independent sets of chain parameters.

5.6 Computer-aided Evaluation

In the future, it will be possible to implement the above approaches on the computer, using either a classical cofactor algorithm (such as COCOSC or SPASO for the modified method), or its numerical form, where Faddeyev algorithm or some eigenvalue method can be applied. The modified voltage method, particularly in combination with the above mentioned algorithms, can be used for effective semisymbolic evaluation of charge transfers. This will be discussed in more detail in another article.

6. Examples of Evaluation

Finally, three simple practical examples of chain parameters and charge transfers will be presented. The first and third ones are concerned with the investigated imitance convertor, where the chain parameters are evaluated manually from the basic equations, using algebraic cofactors and performing the modified voltage method to establish the charge transfer functions. The second one is a bilinear RC simulation, evaluating chain parameters in the matrix form.

6.1 Imitance Convertor

Here the chain parameters of the SC bilinear convertor (see Fig. 4) will be derived, first in the manual way, and then using the cofactor formulation.

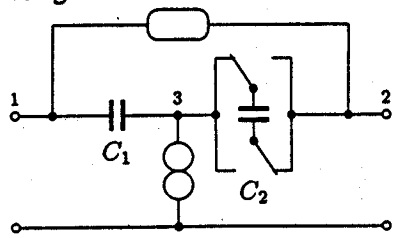

Fig. 4
Imitance convertor

The reduced admitance matrix of this circuit is (see also [4])

	$1+2$	3	$\bar{1}+\bar{2}$	$\bar{3}$
1	C_1	$-C_1$	$C_1z^{-\frac{1}{2}}$	$-C_1z^{-\frac{1}{2}}$
2	C_2	$-C_2$	$-C_2z^{-\frac{1}{2}}$	$C_2z^{-\frac{1}{2}}$
$\bar{1}$	$C_1z^{-\frac{1}{2}}$	$-C_1z^{-\frac{1}{2}}$	C_1	$-C_1$
$\bar{2}$	$-C_2z^{-\frac{1}{2}}$	$C_2z^{-\frac{1}{2}}$	C_1	C_1+C_2

. (44)

In this case, the admitance parameters do not exist, and, therefore, we have to establish the chain parameters directly:

1 From the circuit structure we can easily see

$$\begin{aligned} V_{1E} &= V_{2E} \\ V_{1O} &= V_{2O} \end{aligned} \tag{45}$$

$$a_{11} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad a_{12} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{46}$$

2 Rewriting the remaining equations from the matrix (42) and eliminating V_3 voltages in both phases, we obtain finally

$$\mathbf{a}_{21} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{a}_{22} = \begin{bmatrix} \frac{C_1}{C_2}\frac{1+z^{-1}}{1-z^{-1}} & -\frac{2C_1}{C_2}\frac{z^{-1}}{1-z^{-1}} \\ -\frac{2C_1}{C_2}\frac{z^{-1}}{1-z^{-1}} & \frac{C_1}{C_2}\frac{1+z^{-1}}{1-z^{-1}} \end{bmatrix} . \quad (47)$$

The same result can be obtained using cofactor definition (35), derived in the previous chapter.

6.2 RC Circuit

In this example, the chain parameters of the RC simulation are derived from the admitance parameters

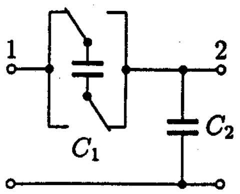

Fig. 5
RC circuit

Reduced basic matrix

	1	2	$\bar{1}$	$\bar{2}$
1	C_1	$-C_1$	$C_1 z^{-\frac{1}{2}}$	$-C_1 z^{-\frac{1}{2}}$
2	$-C_1$	$C_1 + C_2$	$-C_1 z^{-\frac{1}{2}}$	$(C_2 - C_1) z^{-\frac{1}{2}}$
$\bar{1}$	$C_1 z^{-\frac{1}{2}}$	$-C_1 z^{-\frac{1}{2}}$	C_1	$-C_1$
$\bar{2}$	$-C_1 z^{-\frac{1}{2}}$	$(C_2 - C_1) z^{-\frac{1}{2}}$	$-C_1$	$C_1 + C_2$

(48)

Rewritten to the admitance parameters, swapping apropriate columns and rows

	1	$\bar{1}$	2	$\bar{2}$
1	C_1	$C_1 z^{-\frac{1}{2}}$	$-C_1$	$-C_1 z^{-\frac{1}{2}}$
$\bar{1}$	$C_1 z^{-\frac{1}{2}}$	C_1	$-C_1 z^{-\frac{1}{2}}$	$-C_1$
2	$-C_1$	$-C_1 z^{-\frac{1}{2}}$	$C_1 + C_2$	$(C_2 - C_1) z^{-\frac{1}{2}}$
$\bar{2}$	$-C_1 z^{-\frac{1}{2}}$	$-C_1$	$(C_2 - C_1) z^{-\frac{1}{2}}$	$C_1 + C_2$

(49)

And chain parameters are established using Y - A

$$\mathbf{a}_{11} = \mathbf{y}_{21}^{-1}\mathbf{y}_{22}$$

$$\mathbf{a}_{11} = \begin{bmatrix} 1 + \frac{C_2(1+z^{-1})}{C_1(1-z^{-1})} & \frac{C_2 z^{-\frac{1}{2}}}{C_1(1-z^{-1})} \\ \frac{C_2 z^{-\frac{1}{2}}}{C_1(1-z^{-1})} & 1 + \frac{C_2(1+z^{-1})}{C_1(1-z^{-1})} \end{bmatrix} \quad (50)$$

$$\mathbf{a}_{12} = \mathbf{y}_{21}^{-1}$$

$$\mathbf{a}_{12} = \frac{1}{C_1^2(1-z^{-1})} \begin{bmatrix} -C_1 & C_1 z^{-\frac{1}{2}} \\ C_1 z^{-\frac{1}{2}} & -C_1 \end{bmatrix} \quad (51)$$

$$\mathbf{a}_{21} = \mathbf{y}_{12} - \mathbf{y}_{11}\mathbf{y}_{21}^{-1}\mathbf{y}_{22}$$

$$\mathbf{a}_{21} = \begin{bmatrix} C_2 & C_2 z^{-\frac{1}{2}} \\ C_2 z^{-\frac{1}{2}} & C_2 \end{bmatrix} \quad (52)$$

$$\mathbf{a}_{22} = -\mathbf{y}_{11}\mathbf{y}_{21}^{-1}$$

$$\mathbf{a}_{22} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (53)$$

6.3 Modified Method

Using the modified voltage method, we can obtain the charge transfer matrix (17), which is the inverse of parameter a_{22}, as will be demonstrated below. The rewritten matrix is now

	1 + 2	3	Q_2	$\bar{1}+\bar{2}$	$\bar{3}$	$Q_{\bar{2}}$
1	C_1	$-C_1$	0	$C_1 z^{-\frac{1}{2}}$	$-C_1 z^{-\frac{1}{2}}$	0
2	C_2	$-C_2$	1	$-C_2 z^{-\frac{1}{2}}$	$C_2 z^{-\frac{1}{2}}$	0
	1	0	0	0	0	0
$\bar{1}$	$C_1 z^{-\frac{1}{2}}$	$-C_1 z^{-\frac{1}{2}}$	0	C_1	$-C_1$	0
$\bar{2}$	$-C_2 z^{-\frac{1}{2}}$	$C_2 z^{-\frac{1}{2}}$	0	C_1	$C_1 + C_2$	1
	0	0	0	1	0	0

. (54)

The reason the switch is added is to perform the output condition of the charge transfer: to set the voltage to zero. The charge through this switch can now be calculated

$$H_{EE} = \frac{\Delta_{1Q_2}}{\Delta}\; H_{EO} = \frac{\Delta_{1Q_{\bar{2}}}}{\Delta}, \qquad H_{OE} = \frac{\Delta_{\bar{1}Q_2}}{\Delta}\; H_{OO} = \frac{\Delta_{\bar{1}Q_{\bar{2}}}}{\Delta}, \quad (55)$$

therefore

$$\mathbf{H} = \begin{bmatrix} -\frac{C_2(1+z^{-1})}{C_1(1-z^{-1})} & -\frac{2C_2 z^{-1}}{C_1(1-z^{-1})} \\ -\frac{2C_2 z^{-1}}{C_1(1-z^{-1})} & -\frac{C_2(1+z^{-1})}{C_1(1-z^{-1})} \end{bmatrix} . \quad (56)$$

The inverse rule $\mathbf{H} = \mathbf{a}_{22}^{-1}$ can be shown easily by evaluating the factor of

$$\det \mathbf{H} = \left(\frac{C_2}{C_1}\right)^2 . \quad (57)$$

7. Conclusion

Solving the analogy of two-port parameters in the Z domain, we tried to sketch a more generalized approach to SC circuit analysis which can be successfully applied, for example, for general amplifier use (OTA amplifiers, etc.) or for switched current technology, which has a lot of similarities with SC technology. A full solution of SC chain parameters and charge transfer was presented here, and some simple examples were given. The same approach can be easily developed for different two-port parameters or mixed transfers, which are used, for example, in sensitivity analysis.

8. References

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