

# GENERALIZED FREQUENCY DOMAIN LMS ADAPTIVE FILTER

František DOHNAL  
Department of Technical Cybernetics  
and Military Robotics  
Military Academy Brno  
PS13-K308, 612 00 BRNO  
Czech Republic

## Abstract

*The most significant problems of acoustic echo canceller (AEC) realizations are high computational complexity and insufficient convergence rate of the applied adaptive algorithms.*

*From the analysis of the frequency domain block adaptive filter [2,3] realization and the modified subband acoustic echo canceller [6] the generalized frequency domain adaptive filter [8,9] has been derived. The result of simulations is demonstrated the efficiency of this algorithm for a stationary noise and real speech signal excitation.*

## Keywords:

digital adaptive filter, digital signal processing, acoustic echo canceller

## 1. Introduction

The acoustic echo control arises as a real need in hand-free telephony or teleconference communication systems. The presence of any acoustic echo of our voice in the incoming far-end signal produces a undesired effect in natural half-duplex communication and an important loss of the intelligibility and the quality in full-duplex communications.

To solve this problem, the present wide-band audioterminals try to incorporate an additional solution based on the implementation of a parallel adaptive filter to the acoustic path. The goal is to provide an echo replica on its output to cancel locally the acoustic echo. However, due to the duration of a typical channel impulse response for a reverberant room (from 0.1 to 0.6 seconds), the adaptive filter length may be several thousand taps at a sampling rate of 16 kHz. Obviously, this fact makes difficulty its real-time implementation when using filtering schemes.

The time-domain LMS algorithm [1], in that context, suffers from both high computational complexity

and slow convergence. The use of the frequency domain block adaptive filter [2,3] essentially reduces computational complexity and increases convergence rate. Of course, in implementations of the frequency domain block adaptive filter, the size of the block must be set to twice the number of filter tap weights. The associated processing delay, equal to the block size, appeared to be prohibitive in acoustic echo cancellers. The basic method of solving this problem is to segment the impulse response into small blocks. This idea has been employed in the subband acoustic echo cancellers [4,5,6].

The analysis of the realization of the frequency domain block adaptive filter and the modified subband acoustic echo canceller [6] where DFT filter bank was employed to filter bank realization [7] has been performed. From this analysis has followed that this subband acoustic echo canceller do not accept the effect of the multiplicative modifications in the DFT filter bank, that has been taken into account in the fast convolution methods used for the frequency domain block adaptive filter realization. This effect has brought the decrease of the convergence rate and increase of the steady-state mean square error by time domain aliasing [7].

From these facts the new adaptive filter for the acoustic echo canceller [8,9] has been derived and these new schemes will be referred to as Generalized Frequency Domain Adaptive Filter (GFDAF).

This algorithm is derived in the section 2. In the section 3, the simulation results of this algorithm for a stationary noise and a real speech signal excitation is presented. The section 4 is devoted to the computational complexity analysis.

## 2. Algorithm derivation

In the following, uppercase symbols will denote frequency-domain variables, lowercase symbols will stand for time-domain variables, and boldface will denote vectors and matrixes. Superscript  $T$  will denote a transpose and superscript  $H$  a complex conjugate transpose of the vector or matrix.  $F$  will denote the symmetric  $N \times N$  matrix whose  $k_{th}$ ,  $l_{th}$  element is  $F_{kl} = \exp(-j(2\pi/N)kl)$ . The matrix  $F$  transforms any vector of  $N$  samples into the vector of its DFT coefficients.

We will assume to use the modified structure of a subband acoustic echo canceller proposed by Chen [6] for the algorithm derivation. The structure of the modified subband AEC is shown in Fig. 1.

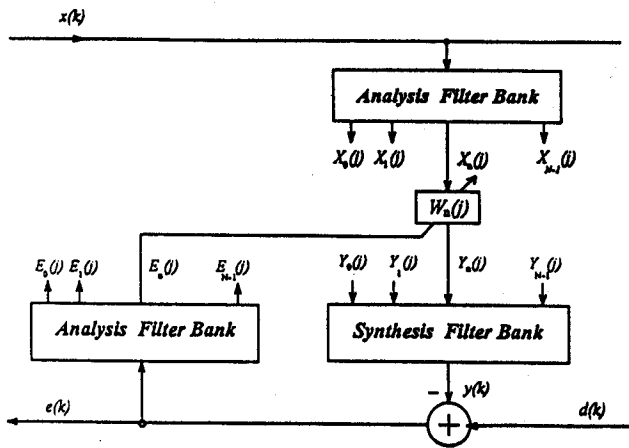


Fig.1 Modified structure of subband acoustic canceller

We will consider to using of the DFT filter bank [7] based on Weighted Overlap and Add (WOLA) reconstruction algorithm for the analysis and synthesis filter bank. On this circumstance, the expression for the DFT filter bank analyzer is given

$$X_n(j) = \sum_{r=0}^{N-1} h(-r)x(r+jR)W_N^{-nr} \quad \text{for } n=0,1,\dots,N-1 \quad (1)$$

and expression for the DFT filter bank synthesizer

$$y(r+jR) = f(r) \frac{1}{N} \sum_{n=0}^{N-1} Y_n(j)W_N^{nr} \quad \text{for } n=0,1,\dots,N-1 \quad (2)$$

where  $h(k)$  denote "so called" analysis and  $f(k)$  synthesis window,  $R$  is the decimated ratio,  $W_N^{-nr} = \exp(-j(2\pi nr/N))$  and  $W_N^{nr} = \exp(j(2\pi nr/N))$ .

The properties of the DFT filter bank are strongly dependent on the choice of the number of bands  $N$ , the decimation ratio  $R$ , and the designs of the analysis and synthesis filters (windows)  $h(k)$  and  $f(k)$ . The analysis and synthesis filter must be designed, dependly on required properties of the filter bank, to eliminate the effects of the frequency domain and time domain aliasing [7].

For the simplifying, we will assume to use the rectangular window for the analysis  $h(k)$  and synthesis filter  $f(k)$ . The decimation ratio  $R$  is equal to the block length  $L$ . Then, we can rewrite the expression for the DFT filter bank analyzer (1) to the form

$$X_n(j) = \sum_{r=0}^{N-1} x(jL+r)W_N^{-nr} \quad \text{for } n=0,1,\dots,N-1 \quad (3)$$

and expression for the DFT filter bank synthesizer (2) to the form

$$y(jL+r) = \frac{1}{N} \sum_{n=0}^{N-1} Y_n(j)W_N^{nr} \quad \text{for } n=0,1,\dots,N-1 \quad (4)$$

By the DFT matrix definition  $F$  and its inverse  $F^{-1}$ , we can rewrite the equations for DFT filter bank analyzer and synthesizer to matrix form

$$X(j) = F x(jL) \quad y(jL) = F^{-1} Y(j) \quad (5)$$

where  $N \times 1$  time domain vectors  $x(jL)$  and  $y(jL)$  are defined

$$x(jL) = [x(jL), x(jL+1), \dots, x(jL+(N-1))]^T \quad (6)$$

$$y(jL) = [y(jL), y(jL+1), \dots, y(jL+(N-1))]^T \quad (7)$$

and  $N \times 1$  frequency domain vectors  $X(j)$  and  $Y(j)$  are defined

$$X(j) = [X_0(j), X_1(j), \dots, X_{N-1}(j)]^T \quad (8)$$

$$Y(j) = [Y_0(j), Y_1(j), \dots, Y_{N-1}(j)]^T \quad (9)$$

The frequency domain output  $Y(j)$  of the subband AEC, in time  $k=jL$ , can be then written in the matrix form as

$$Y(j) = \sum_{i=0}^{K-1} X(j-i)W_i(j) \quad (10)$$

In equation (10), the output  $Y(j)$  is given by the sum of the product of the input signal short-time spectrum  $X(j)$  and the short-time spectrum of weight vectors  $W_i(j)$  for  $i=0,1,\dots,K-1$ . The product  $X(j-i)W_i(j)$  corresponds to time domain convolution.

From this has followed that the modified subband AEC do not accept the multiplicative modification in the DFT filter bank which causes a distortion by the time domain aliasing. To eliminate this distortion, the duration  $N_h$  of the analysis filter  $h(k)$ , the duration  $N_f$  of the synthesis filter  $f(k)$  and a number of nonzero coefficients  $N_w$  of each weight vector  $W_i(j)$  must satisfy the condition [7] for elimination of the time domain aliasing

$$N_h + N_f + N_w - 1 \leq 2N \quad (11)$$

This condition is achieved by an appropriately design the time alignment of  $h(k)$  and  $f(k)$  relative to the range of  $W_i(j)$  so that the undesired aliasing terms are windowed out. If we choice the time alignment so that in the overlap save method for the fast convolution then

$$N_h = N \quad N_f = L \quad N_w = M = N - N_f + 1 \quad (12)$$

and  $N \times 1$  augmented vectors  $y^a(j)$ ,  $w_i^a(j)$ ,  $e^a(j)$ ,  $d^a(j)$  and  $N \times N$  matrix  $x^a(j)$  in a time domain could be defined as in the FBLMS algorithm [2,3].

From these facts the modified adaptive filter [8,9] has been derived and it has been named the Generalized Frequency Domain Adaptive Filter - GFDAF. The output  $y(j)$  of the GFDAF algorithm, in time  $k=jL$ , is given from (10) by using the projection operator  $p_L$ , which to keep only  $L$  outputs corresponding to the linear convolution, as

$$y(j) = p_L \{y^a(j)\} = p_L \{F^{-1} Y(j)\} \quad (13)$$

where

$$Y(j) = \sum_{i=0}^{K-1} Y_i(j) = \sum_{i=0}^{K-1} X(j-i)W_i(j)$$

From the subband *AEC* adaptive equation [5] and the *FBLMS* adaptive equation [2,3] we can define the *LMS* adaptive algorithm for *GFDAF*. Then  $i_{th}$  weight vector  $W_i(j+1)$ , in block time  $j+1$ , is given by the adaptive equation

$$\text{for } i=0,1,\dots,K-1$$

$$W_i(j+1) = P_{M,0}(W_i(j) + 2\mu X(j-i)^H E(j)) \quad (14)$$

with the error vector

$$E(j) = P_{0,L}(D(j) - Y(j)) \quad (15)$$

where  $D(j)$  is the  $N$ -point *DFT* of the augmented desire signal vector  $d^a(j)$ .

The two  $N \times N$  circulant matrixes  $P_{M,0}$  and  $P_{0,L}$  are defined as

$$P_{M,0} = F \begin{bmatrix} I_M & 0 \\ 0 & 0 \end{bmatrix} F^{-1}$$

$$\text{and } P_{0,L} = F \begin{bmatrix} 0 & 0 \\ 0 & I_L \end{bmatrix} F^{-1} \quad (16)$$

The equivalent order  $M'$  of the *GFDAF* algorithm is given by a product of the order  $K$  of the subband *ADF* and the block length  $L$ , then  $M'=KL$ .

The above derived algorithm solves the problems with the time domain aliasing effect and it is possible to improve the algorithm properties by a better elimination of the frequency domain aliasing. The better elimination of the frequency domain aliasing is possible to reach by the reduction of decimated ratio ( $R \leq N/4$ ) and the using of analysis window with the better stopband attenuation, for example the Hamming window [7].

The *GFDAF* algorithm is the generalization of the modified subband acoustic echo canceller [6] and the Multidelay Block Frequency Domain Adaptive Filter [10]. Note that frequency domain adaptive filters are possible to consider as the special case of the *GFDAF* with the order of the subband adaptive filter equal to one.

### 3. Simulations

The properties of the design algorithm were evaluated by computer simulations with the modelled acoustic echo path impulse response for both stationary and nonstationary signals.

The echo path impulse response was modelled by the known image method for simulating small rectangular rooms described in [11]. The acoustic echo path impulse response of 4096 taps was modelled for the room dimensions 3.4m x 4.4m x 2.5m and distance between microphone and loudspeaker equal to 2 meters. The simulated echo signals, picked up in near-end room, are

the results of filtering an input signal with modelled echo path impulse response. For all experiments, a white noise with 40 dB *SNR* has been added to the echo signal that simulates the noise in near-end room.

The white noise, the stationary noise with an average speech spectrum (*USASI* noise) and the real speech signal was used as the input signal. The convergence properties of the algorithm has been evaluated by Echo Return Loss Enhancement (*ERLE*) averaged per 128 samples, that is defined as

$$ERLE(k) = 10 \log \left( \frac{\sum_{i=0}^{127} [e(k-i)]^2}{\sum_{i=0}^{127} [d(k-i)]^2} \right) \text{ [dB]} \quad (17)$$

The convergence behaviour of the *GFDAF*, for the white noise and *USASI* noise excitations, is in Fig.2. The order of the *GFDAF* is  $M'=1024$ , when the block length  $L=128$  and the order of subband *ADF*'s  $K=16$ . From Fig.2. is seen, that convergence behaviour of this algorithm is almost the same for both white noise and *USASI* noise, therefore almost independent on the eigenvalue spread of the input signal autocorrelation matrix. This property is very important for the real speech excitation of an adaptive filter.

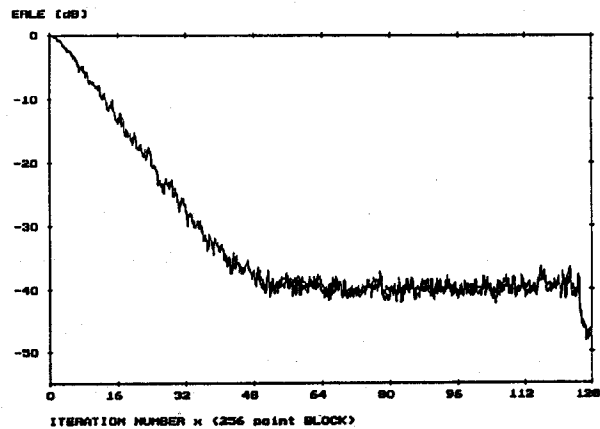


Fig.2 Convergence behaviour of the *GFDAF* for stationary signals

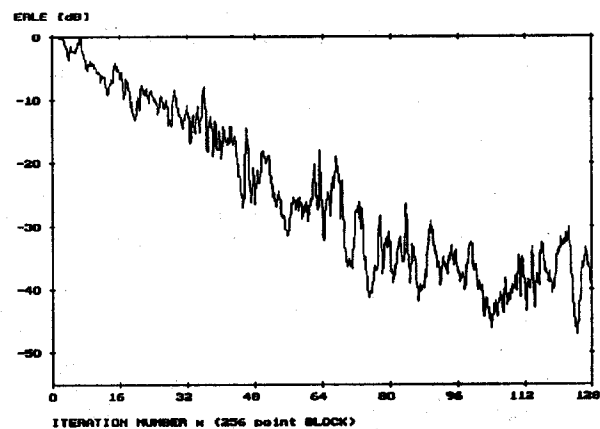


Fig.3 Convergence behaviour of the *GFDAF* for a real speech signal

In Fig.3. is the convergence behaviour of the GFDAF, for the real speech signal excitation with duration 4 seconds (sampling rate 8 kHz). The order of the GFDAF is  $M=1024$  ( $L=128$  and  $K=16$ ). Evidently the GFDAF algorithm converges for real speech signal very well and could be suitable for in acoustic echo canceller using.

#### 4. Complexity of the algorithm

The computational complexity of the GFDAF algorithm results from the computational complexity of  $N$ -points FFT (inverse FFT) of a real sequence that requires  $(N/4)\log_2 N$  radix-2 butterflies +  $N/2$  complex additions. Each radix-2 butterfly needs four real multiplications in a hardware realization and six instructions in a software realization on signal processor WE DSP32C [12].

The length of FFT for the GFDAF realization is equal to double of the block length  $L$ . The computation of one output sample, for equivalent order of the GFDAF algorithm  $M=K L$  (where  $K$  is the order of subband adaptive filters), requires for

- a hardware realization  $(6 + 4K)\log_2(2L) + 20K$  real multiplications
- a software realization  $(9 + 6K)\log_2(2L) + (6 + 24K)$  DSP instructions.

Then, for example, a theoretical computational complexity for the modelled acoustic echo path length 256 ms and the processing delay 16 ms is equal to the approximately 22 Mops for 16 kHz sampling rate, corresponds to ADF's order  $M=4096$  ( $L=256$  and  $K=16$ ).

From the computational complexity analysis follow the realization possibility of the GFDAF algorithm on modern signal processors (WE DSP32C and TMS 320C30) in a real time.

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#### About author,...

František DOHNAL was born in Znojmo, Czechoslovakia, in 1961. He received the M.E. degree in electrical engineering in 1985 and the CSc. (Ph.D.) degree in 1993 from Military academy Brno. He is currently Assistant professor at the department of Technical Cybernetics and Military Robotics of the Military Academy Brno. His research interests include digital signal processing, adaptive systems, artificial intelligence and neural networks.