ACTIVE TWO-PORT EQUIVALENT NOISE PARAMETERS

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Abstract

A method of the calculation of active two-port equivalent noise parameters from n measured noise figures for different values of signal source output admittance is given.

Keywords:

noise figure, noise parameters, noise matching

1. Introduction

Any linear active two-port contains noise sources caused by different physical phenomena which occur at the output of the two-port in the form of an output noise power.

To solve the noise properties of the active two-port with parameters $\mathbf{Y}_{ik}$ an equivalent circuit is used. In this equivalent circuit the noisy two-port is replaced by a two-port without noise with the same transfer parameters and with connected noise sources depending only on the two-port internal noise properties and independent of the connected passive elements. These noise sources represent the effect of all internal noise components regardless of their physical basis, number and location inside the test device. Because the equivalent noise sources are partially based on the same primary phenomenon, there commonly exists a dependence between them which can be expressed by a complex correlation coeficient.

The equivalent noise parameters of the two-port are used with advantage instead of the noise sources and correlation coeficient because it is simpler to use them in a theoretical solution of the active two-port noise properties.

Equivalent noise parameters (ENP) are important in the calculation of the optimum signal source admittance (impedance) in regard to the optimum noise matching at the input of the active two-port under test.

2. Calculation of the noise figure from the active two-port ENP

The *noise figure F* of the amplifier can be expressed by equation, [1] - [4],

$$F = F_{\min} + \frac{R_n}{G_G}\left[(G_G - G_{Gopt})^2 + (B_G - B_{Gopt})^2\right]. \quad (1)$$

In this case, the noise properties of the active two-port are characterized by four *noise parameters.* $F_{\min}$ is the noise figure for the noise matching and susceptance matching. R_n is the equivalent noise resistance of the voltage noise source. G_{Gopt} and B_{Gopt} are the conductance and susceptance of the signal source output admittance, which cause the noise figure to reach its absolute minimum. G_G and B_G are any values of the signal source admittance, at which the noise figure of the two-port is evaluated.

3. Determination of the active two-port ENP by the least-squares method

Every measurement is less or more inaccurate. The errors may be *systematic*, *random* and *gross*.

Systematic errors are caused by a certain, usually known reason, they shift the results in the same direction and often by a constant value. They are caused e.g. by a defective measuring instrument, by incorrect calibration of the measuring instrument, by an improper measurement technique, etc. In the case of the noise figure measurement an additional error occurs, namely the unsteadiness of the noise power. It is then necessary to select properly the integration time interval of the noise power measurement. It is possible to detect these errors and remove them to some extent.

Random errors are indefinite and their reasons are unknown. These errors can be evaluated by statistical methods. During the noise power measurement the errors are caused by random interfering signals which have not been suppressed by shielding or filtering of the power supply voltage.

The *least-squares method* gives the minimum sum of the square deviations of the measured values from the approximation curve $y=f(x)$

$$E=\sum_{i=1}^{n}[y_i-f(x_i)]^2=\min, \qquad (2)$$

where y_i is the set of measured values.

When an experimental curve is approximated by a polynomial of the n-th order using the least-squares method, the coefficients of the polynomial are calculated by solving a set of linear equations. The solution of the approximation by means of the least-squares method becomes more complex if the approximation curve $y=f(x;A_1,A_2,\dots,A_n)$ contains nonlinear parameters. But even in this case it is often possible to solve the task relatively simply.

To evaluate the four unknown active two-port noise parameters, namely $F_{\min}$, R_n, G_{Gopt} and B_{Gopt}, it is necessary to solve the nonlinear equation (1) of a nonsymmetrical paraboloid in which F_i, G_{Gi} and B_{Gi} are variables. For different values G_{Gi} and B_{Gi}, $i=1,2,\dots,n$, it is possible to measure the noise figures F_i with accuracy approximately 10 % to 15 %. The accuracy of the determination of the conductance and susceptance of the signal source admittance is 1 % to 2 % and this inaccuracy will not be considered.

The task to evaluate the unknown noise parameters $F_{\min}$, R_n, G_{Gopt}, B_{Gopt}, cannot be solved by a classical method - successive separation of unknown variables. Because of the dispersion of the measured values of the noise figures F_i beside the nonsymmetrical paraboloid, Fig.1, caused by the measurement errors, the task will be solved by smoothing the experimental results by the least-squares method.

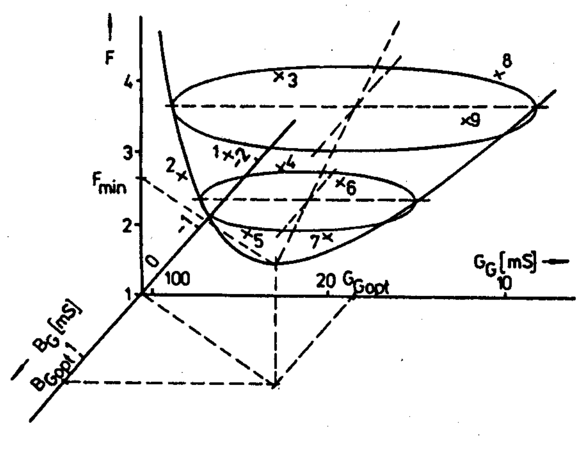

Fig.1.
Nonsymmetrical paraboloid of the noise figure in the three-dimensional co-ordinate system

The nonlinear approximation function is first linearized, [2]. Equation (1) is modified and rearranged, [5], [6], [7],

$$F=F_{\min}-2R_nG_{Gopt}+\frac{G_G^2+B_G^2}{G_G}R_n+$$

$$+\frac{1}{G_G}R_n\left(G_{Gopt}^2+B_{Gopt}^2\right)-\frac{B_G}{G_G}2R_nB_{Gopt}\,.$$

Next, the following substitutions are used

$$A=F_{\min}-2R_nG_{Gopt}$$

$$B=R_n$$

$$C=R_n\left(G_{Gopt}^2+B_{Gopt}^2\right)$$

$$D=-2R_nB_{Gopt}\,. \qquad (3)$$

These substitutions give the linearized function which can be solved by the least-squares method

$$F=A+\left(G_G+\frac{B_G^2}{G_G}\right)B+\frac{1}{G_G}C+\frac{B_G}{G_G}D\,, \qquad (4)$$

$$F_i=f(G_{Gi},B_{Gi};A,B,C,D)\,.$$

If the values of parameters A, B, C, D are known, equations (3) yield

$$F_{\min}=A+4BC-D^2\,,\; R_n=B\,,$$

$$G_{Gopt}=\frac{4BC-D^2}{2B}\,,\; B_{Gopt}=-\frac{D}{2B}\,. \qquad (5)$$

Partial derivatives of equation (4) are of the form

$$\frac{\delta F}{\delta A}=1\,,\; \frac{\delta F}{\delta B}=G_G+\frac{B_G^2}{G_G}\,,\; \frac{\delta F}{\delta C}=\frac{1}{G_G}\,,\; \frac{\delta F}{\delta D}=\frac{B_G}{G_G}$$

$$\left(\frac{\delta F}{\delta A}\right)_i=1\,,\; \left(\frac{\delta F}{\delta B}\right)_i=G_{Gi}+\frac{B_{Gi}^2}{G_{Gi}}\,,$$

$$\left(\frac{\delta F}{\delta C}\right)_i=\frac{1}{G_{Gi}}\,,\; \left(\frac{\delta F}{\delta D}\right)_i=\frac{B_{Gi}}{G_{Gi}}\,. \qquad (6)$$

Applying equ.(2) to eqs.(4) and (6) yields

$$\sum_{i=1}^{n}\left[F_i-A-\left(G_{Gi}+\frac{B_{Gi}^2}{G_{Gi}}\right)B-\frac{1}{G_{Gi}}C-\frac{B_{Gi}}{G_{Gi}}D\right]=0\,,$$

$$\sum_{i=1}^{n}\left[F_i-A-\left(G_{Gi}+\frac{B_{Gi}^2}{G_{Gi}}\right)B-\frac{1}{G_{Gi}}C-\frac{B_{Gi}}{G_{Gi}}D\right]\bullet$$

$$\bullet\left(G_{Gi}+\frac{B_{Gi}^2}{G_{Gi}}\right)=0\,,$$

$$\sum_{i=1}^{n}\left[F_i-A-\left(G_{Gi}+\frac{B_{Gi}^2}{G_{Gi}}\right)B-\frac{1}{G_{Gi}}C-\frac{B_{Gi}}{G_{Gi}}D\right]\frac{1}{G_{Gi}}=0\,,$$

$$\sum_{i=1}^{n}\left[F_i-A-\left(G_{Gi}+\frac{B_{Gi}^2}{G_{Gi}}\right)B-\frac{1}{G_{Gi}}C-\frac{B_{Gi}}{G_{Gi}}D\right]\frac{B_{Gi}}{G_{Gi}}=0\,.$$

Further modification gives a set of four linearized equations which enable the evaluation of the unknown parameters A, B, C and D:

$$A A_{11} + B A_{12} + C A_{13} + D A_{14} = A_{15} \ ,$$
$$A A_{21} + B A_{22} + C A_{23} + D A_{24} = A_{25} \ ,$$
$$A A_{31} + B A_{32} + C A_{33} + D A_{34} = A_{35} \ ,$$
$$A A_{41} + B A_{42} + C A_{43} + D A_{44} = A_{45} \ . \qquad (7)$$

Because some of the sums A_{11} to A_{45} are the same, the following equations can be used to solve the equation set (7)

$$A_{11} = \sum_{i=1}^{n} 1 = n \ , \ A_{12} = A_{21} = \sum_{i=1}^{n} \left(G_{Gi} + \frac{B_{Gi}^2}{G_{Gi}} \right) ,$$

$$A_{13} = A_{31} = \sum_{i=1}^{n} \frac{1}{G_{Gi}} \ , \ A_{14} = A_{41} = \sum_{i=1}^{n} \frac{B_{Gi}}{G_{Gi}} \ ,$$

$$A_{15} = \sum_{i=1}^{n} F_i \ , \ A_{22} = \sum_{i=1}^{n} \left(G_{Gi} + \frac{B_{Gi}^2}{G_{Gi}} \right)^2 ,$$

$$A_{23} = A_{32} = \sum_{i=1}^{n} \frac{1}{G_{Gi}} \left(G_{Gi} + \frac{B_{Gi}^2}{G_{Gi}} \right) ,$$

$$A_{24} = A_{42} = \sum_{i=1}^{n} \frac{B_{Gi}}{G_{Gi}} \left(G_{Gi} + \frac{B_{Gi}^2}{G_{Gi}} \right) ,$$

$$A_{25} = \sum_{i=1}^{n} F_i \left(G_{Gi} + \frac{B_{Gi}^2}{G_{Gi}} \right) , \ A_{33} = \sum_{i=1}^{n} \frac{1}{G_{Gi}^2} \ ,$$

$$A_{34} = A_{43} = \sum_{i=1}^{n} \frac{B_{Gi}}{G_{Gi}^2} \ , \ A_{35} = \sum_{i=1}^{n} F_i \frac{1}{G_{Gi}} \ ,$$

$$A_{44} = \sum_{i=1}^{n} \left(\frac{B_{Gi}}{G_{Gi}} \right)^2 , \ A_{45} = \sum_{i=1}^{n} F_i \frac{B_{Gi}}{G_{Gi}} \ . \qquad (8)$$

Based on equations (8), on the program [8] for the solution of the linear equations by the Gauss-Jordan method and on equations (5), the program FRGB 5 was created to calculate the active two-port noise parameters from the measured values of the noise figures for different signal source admittances. The number of measured noise figures for the program FRGB 5 is eight to twelve to reach the actual values of the noise parameters with the largest probability and to eliminate the measurement errors.

At the beginning of the calculation by the program FRGB 5 the number of measured noise figures ($n = 8$ to 12), their values F_1 to F_{12}, the conductances G_{G1} to G_{G12} and the susceptances B_{G1} to B_{G12} of the signal source admittance are entered, Tab.1. The results of the calculation comprise the values of the noise parameters, calculated and measured noise figures for the given admittances $\mathbf{Y}_{Gi}$, differences and square deviations. Then there is this information:

- the sum of deviations — SUM DF
- mean value of deviations — STRED DF
- the sum of absolute deviations — ABS(SUM DF)
- mean value of absolute deviations — ABS/M
- the sum of squared deviations — E
- mean squared deviation — M

Tab.1. Calculation of the noise parameters of the transistor KF 525.

NPT CALCULATION
TRANSISTOR TYPE KF 525
COLLECTOR CURRENT 1 (MA)
CENTER FREQUENCY FS = 10 (MHZ)
NOISE BANDWIDTH BS = 4375 (HZ)
NUMBER OF MEASURED POINTS M = 9
NUMBER OF UNKNOWN PARAMETERS N = 4

F1 = 3.65	G1 = 3.04 E-03 (S)	B1 = 3.71 E-03 (S)
F2 = 1.55	G2 = 1.49 E-03 (S)	B2 =-9.99 E-04 (S)
F3 = 8.57	G3 = 1.01 E-03 (S)	B3 = 3.71 E-03 (S)
F4 = 5.36	G4 = 4.64 E-04 (S)	B4 = 1.31 E-03 (S)
F5 = 2.87	G5 = 6.76 E-04 (S)	B5 =-1.45 E-03 (S)
F6 = 5.36	G6 = 9.38 E-03 (S)	B6 = 3.71 E-03 (S)
F7 = 8.13	G7 = 1.01 E-03 (S)	B7 = 2.63 E-03 (S)
F8 = 7.05	G8 = 1.49 E-03 (S)	B8 = 3.71 E-03 (S)
F9 = 4.03	G9 = 3.04 E-03 (S)	B9 = 1.80 E-03 (S)

CALCULATION OF FMIN, RN, GGOPT, BGOPT
FMIN = 2.24
RN = 318.67 (OHM)
GGOPT = 1.10 E-03 (S)
BGOPT = -9.43 E-04 (S)

FVYP	FNAM	DF	DF²
4.90	3.64	1.25E+00	1.58E+00
2,27	1.55	7.16E-01	5.13E-01
9.06	8.57	4.95E-01	2.45E-01
6.01	5.36	6.51E-01	4.24E-01
2.44	2.86	-4.23E-01	1.79E-01
5.30	5.36	-5.78E-02	3.34E-03
6.25	8.13	-1.87E+00	3.51E+00
6.90	7.04	-1,48E-01	2.19E-02
3.42	4.03	-6.15E-01	3.78E-01

SUM DF = 3.20E-11 STRED DF = 3.55E-12
ABS(SUM DF) = 6.24 ABS/M = .69
SUM DF² = 6.86
E = 6.87 M = .76

4. Conclusion

Statistical smoothing of the measured noise figures Fig.1, for different signal source admittances can be evaluated according to the sum of squared deviations or the mean squared deviation.

The comparison with the solutions of some other authors, e.g. [9], [10], shows better statistical smoothing of the measured values of the nonsymmetrical paraboloid by the proposed method. The equivalent noise parameters can be used to optimize noise properties of the electronic circuits.

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