THE VECTOR MODIFICATION OF HUYGHENS-FRESNEL PRINCIPLE

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Abstract

The present approach to the diffractive element design is based on Kirchhoff scalar theory of diffraction. Predictions made by this theory become unreliable if the diffraction of polarized light is evaluated. The paper presents the ***vector correction*** *of Huygens-Fresnel principle and suggests a method for fast evaluation of rapidly oscillating integrals.*

Keywords:

diffraction, vector theory, asymptotic expansion.

1. Introduction

The crucial issue in designing any diffractive optical element (DOE) is to find appropriate phase transfer function of DOE so that given incoming beam of light is properly (and efficiently) transformed to the desired outcomming beam of light. As present technological processes usually can not entirely reproduce the desired shape of DOE (due to image resolution, quantization limitations etc.), the designer must evaluate the influence of the actual optical element to the incoming light beam. The design process thus consists of several steps, each of them corrects the errors of previous one. Possessing of reliable tool for error estimation is therefore necessary.

The present approach to diffraction element design is based mainly on the Kirchhoff scalar theory of diffraction. This approximation holds for large apertures and unpolarized light only. If DOE is used for polarized monochromatic light handling, predictions made by Kirchhoff theory become unreliable. The actual result is not invariant to the rotation of polarization plane in contrast to the theoretical evaluation of this problem. The difference may have a significant influence to the proper device functionality.

This paper presents computer study of DOE influence to the light beam using "vector correction" to the Kirchhoff scalar theory of diffraction and suggests a quicker evaluation method for oscillating double integrals.

2. Scalar theory of diffraction

Kirchhoff scalar theory of diffraction has been derived from the Green integral theorem for two scalar functions inside and on a closed surface S. Assuming point source of light the amplitude of the diffracted light U_P behind the phase-transforming DOE can be formulated by following equation

$$U_P = -\frac{ikA}{4\pi}\oint T\frac{e^{ik(r+s)}}{rs}(\mathbf{n}.\mathbf{n}_s - \mathbf{n}.\mathbf{n}_r)dS, \tag{1}$$

where A is the initial amplitude of the light source, k is the wave number, vector $\mathbf{r}$ connects initial source to the DOE surface element, vector $\mathbf{s}$ connects point of interest to the same element of DOE, $\mathbf{n}_r$, $\mathbf{n}_s$ are their respective unit vectors and $\mathbf{n}$ is outward normal to the DOE. Function T describes the phase-transforming characteristics of the DOE. To describe Fresnel zone plate which focuses light from point in the distance $-a$ to the point in the distance $+b$ the following function can be used

$$T = e^{-i\pi t}, \tag{2}$$

where

$$t = \operatorname{sgn}\sin[k(\sqrt{\rho^2+a^2} + \sqrt{\rho^2+b^2} - a - b)], \tag{3}$$

ρ is the distance between surface element and the optical axis of the set-up.

3. Vector modification of Huygens Fresnel principle

Following Huygens-Fresnel principle we can consider every point of a wave-front to be an elementary source of a secondary spherical wavelets, and the wave-front at any later instant may be regarded as the mutual interference field of these wavelets. Our attempt to overcome these limits in correct assessment of light-DOE interaction is based on *modified* Huygens-Fresnel principle:

Every point of a wave-front can be considered to be an elementary dipole emanating electromagnetic radiation. The wave-front at any later instant may be regarded as the mutual interference field of these wavelets.

Thus, in our modification the term *electric dipole* replaces the original term *elementary light source.*

The field radiated by an elementary dipole is rigorously described by the equation

$$\mathbf{E}=\frac{e^{ikR}}{4\pi\varepsilon_0 R}\{\frac{(\mathbf{p}_0\mathbf{R})\mathbf{R}}{R^2}(\frac{3}{R^2}-\frac{3ik}{R}-k^2)-\mathbf{p}_o(\frac{1}{R^2}-\frac{ik}{R}-k^2)\},\quad(4)$$

$$\mathbf{H}=\frac{e^{ikR}}{4\pi R}\{\frac{-ik}{R}-k^2\}\frac{\mathbf{p}_o\times\mathbf{R}}{R}.\quad(5)$$

For the sake of simplicity let us abandon the small contribution to the overall field amplitude

$$k^2>>\frac{k}{2}>>\frac{1}{R^2}.\quad(6)$$

After proprietary simplification the total field amplitude and its derivative along the outward normal **n** is

$$\mathbf{E}=\frac{e^{ikR}(-ik)}{4\pi\varepsilon_0 R}(\frac{1}{R}-ik)(\frac{[\mathbf{RR}]\cdot\mathbf{p}}{R^2}-\mathbf{p}),\quad(7)$$

$$\frac{\partial\mathbf{E}}{\partial n}=\frac{e^{ikR}k^2\cos(\widehat{\mathbf{n},\mathbf{R}})}{4\pi\varepsilon_0 R}(\frac{1}{R}-ik)(\frac{[\mathbf{RR}]\cdot\mathbf{p}}{R^2}-\mathbf{p}),\quad(8)$$

where brackets [,] denote tenzor variables known also as "diadic multiplication". Let us substitute into Green's theorem

$$\int_V(\mathbf{U}\Delta\mathbf{E}-\mathbf{E}\Delta\mathbf{U})dV=\oint_S(\mathbf{U}\cdot\frac{\partial\mathbf{E}}{\partial n}-\mathbf{E}\cdot\frac{\partial\mathbf{U}}{\partial n})dS\quad(9)$$

these equalities

$$\mathbf{U}=\frac{e^{ikr}(-ik)}{4\pi\varepsilon_0 r}(\frac{1}{r}-ik)(\frac{[\mathbf{rr}]\cdot\mathbf{p}}{r^2}-\mathbf{p}),\quad(10)$$

$$\frac{\partial\mathbf{U}}{\partial n}=\frac{e^{ikr}k^2\cos(\widehat{\mathbf{n},\mathbf{r}})}{4\pi\varepsilon_0 r}(\frac{1}{r}-ik)(\frac{[\mathbf{rr}]\cdot\mathbf{p}}{r^2}-\mathbf{p}),\quad(11)$$

$$\mathbf{E}=\frac{e^{iks}(-ik)}{4\pi\varepsilon_0 s}(\frac{1}{s}-ik)(\frac{[\mathbf{ss}]\cdot\mathbf{q}}{s^2}-\mathbf{q}),\quad(12)$$

$$\frac{\partial\mathbf{E}}{\partial n}=\frac{e^{iks}k^2\cos(\widehat{\mathbf{n},\mathbf{s}})}{4\pi\varepsilon_0 s}(\frac{1}{s}-ik)(\frac{[\mathbf{ss}]\cdot\mathbf{q}}{s^2}-\mathbf{q}).\quad(13)$$

After straightforward but tiring calculation the final equation emerges

$$\mathbf{U}_Q=\frac{3ik}{32\pi^2\varepsilon_0}\oint_S\{\frac{e^{ik(r+s)}}{rs}(\frac{1}{r}-ik)(\frac{1}{s}-ik)\cdot p\cdot(\frac{[\mathbf{rr}]}{r^2}-[1])\cdot(\frac{[\mathbf{ss}]}{s^2}-[1])(\cos(\widehat{\mathbf{n},\mathbf{s}})-\cos(\widehat{\mathbf{n},\mathbf{r}}))\}dS.\quad(14)$$

Further simplification gives quite tidy formula

$$\mathbf{U}_q=\frac{-3ik^3}{32\pi^2\varepsilon_0}\oint_s\{\frac{e^{ik(r+s)}}{rs}\mathbf{p}\cdot(\frac{[\mathbf{rr}]}{r^2}-[1])\cdot(\frac{[\mathbf{ss}]}{s^2}-[1]).(\cos(\widehat{\mathbf{n},\mathbf{s}})-\cos(\widehat{\mathbf{n},\mathbf{r}}))\}dS.\quad(15)$$

Let us compare this equation with Kirchhoff formula

$$\mathbf{U}_q=\frac{-ik}{4\pi}\oint_s\frac{Ae^{ik(r+s)}}{rs}(\cos(\widehat{\mathbf{n},\mathbf{s}})-\cos(\widehat{\mathbf{n},\mathbf{r}}))dS.\quad(16)$$

Both equations should give close results, as physical reality is the same. The relation between radiation of point source and elementary dipole is

$$A=\frac{pk^2}{4\pi\varepsilon_0}\quad(17)$$

Substitution reveals difference factor 3/2 between Kirchhoff's description and vector modification. In our opinion this factor might be an artifact of quite complex simplification during derivation process of final formula.

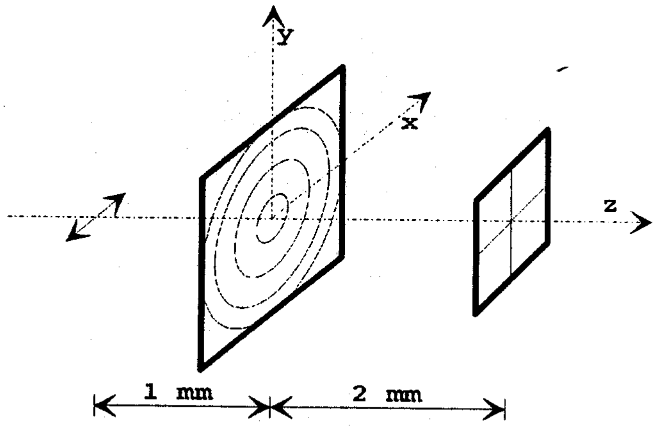

Figure 1: Model example set-up

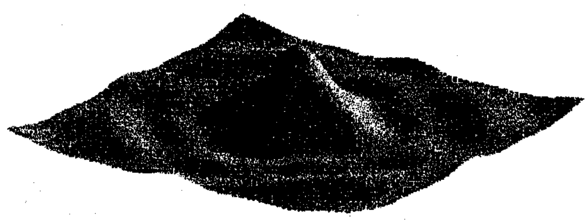

Figure 2: Intensity profile calculated by the scalar theory

4. Model example

To compare the result predicted by the Kirchhoff scalar theory with that one given by our vector modification, we tried to evaluate a simple example. The square (1mm x 1mm) Fresnel zone plate was taken and placed at the beginning of coordinate system. It was designed to focus light from point [0,0,-1 mm] to the point

[0,0,2 mm] (Fig.1). Computer evaluation of this problem took about 120 hours of computer time (IBM-PC, 386SX + 387, 25 MHz). We used 10 PCs mutually interconnected to speed up the whole process. The necessary computing time restricted the maximum dimension of the evaluated Fresnel zone plate. A simple calculation of computing time for bigger lenses convinced us that this task can not be solved by use of brutal force.

Fig.2 and Fig.4 show the intensity profile calculated using equation (1). The displayed area is a square 10μm x 10μm. We can see the symetricity of the profile. The dimensions of the central maximum are about 3.5μm x 3.5μm.

Fig.3 and Fig.5 show the same area, but calculated using our vector modification. The source electromagnetic dipole is oriented along the x-axis of the coordinate system. We can see the central maximum is no longer symetric, its dimensions are about 2.5μm × 1.5μm.

Figure 3: Intensity profile calculated by the vector modification

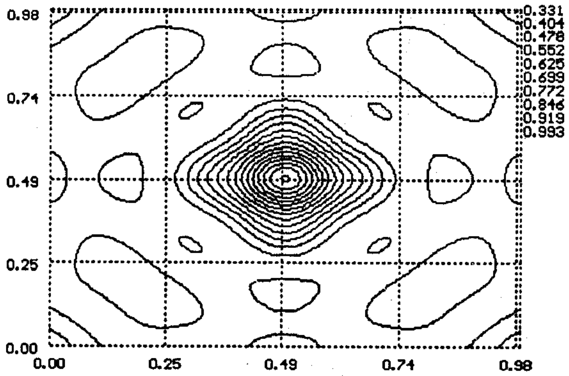

Figure 4: The topographic map of the scalar intensity profile

There are peculiar rectangular patterns visible on these figures. The striking fact is totally different orientation of them. While the scalar case has a rhombic orientation, the vector one has the barrier perpendicular to the aperture edges. It resembles to a kind of scaled-down projection of the lens aperture to the plane of calculation. The significantly different shape of the main central peak is clearly visible. Following the intensity distribution along the x-axis in Fig.3 we can see the dominant secondary maxima; they are absent along the y-axis.

The projection of the aperture shape to the focal plane and the shape of the main maximum support reasonable hypothesis that there are different focal lengths for x and y-directions. This hypothesis complies both with the experiment and an intuitive theoretical model of diffraction. Our vector modification evidences theoretically that axially symetric diffractive lenses have an inherited astigmatic aberration when focusing polarized light and that the focal length in the plane of polarization is longer than that one in the perpendicular plane.

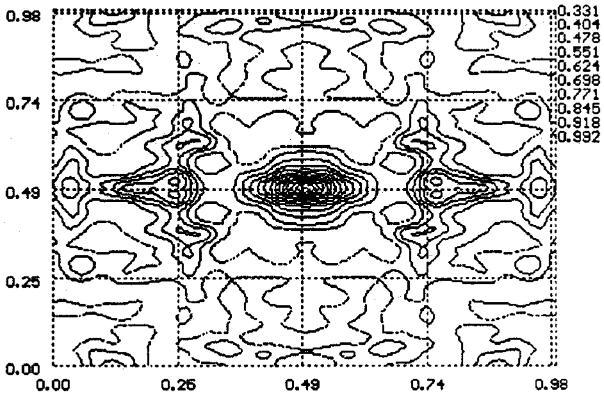

Figure 5: The topographic map of the vector intensity profile

5. Asymptotic expansion of integral

Method of the asymptotic expansion of integrals is very prospective for speeding up calculation of diffraction phenomena. Its application is not elementary, though. Not only there is a need for quite complex mathematics, but also there are omittings and mistakes at the papers of foundators of this method. The extremely good feature of this method is that it complexity and computer time requirment are not depending on the size of DOE. The method consists in replacement of whole integral by sum of only few contributions that arise from certain specially chosen *critical points*. Evaluation of the integral

$$u(Q) = \iint_{\Omega} g(x,y) e^{ikf(x,y)} dxdy \qquad (18)$$

shows that for large k the exponential varies so rapidly that the contributions from the various elements dx, dy to the integral almost completely cancel each other, so that the net result is extremely small. This cancellation, however, will not be effective if function f does not vary with x and y. Therefore the contribution from the vicinity of any point where f is stationary must be calculated separately. Such points where

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0 \qquad (19)$$

are called *critical points of the first kind*. In addition, the cancellation may also become ineffective in the neighborhood of the limits of integration, which gives rise to *critical points of the second* and *third kinds*. Their contributions represent the diffraction at the edge of the aperture. By these considerations it is seen that the integral (18) may be written as a sum of a number of contributions, each arising from the vicinity of some critical point.

To find the values of the contribution u_P of any critical point P let functions f and g be expanded into the series around the point P:

$$f(x,y) = f_{00} + f_{10} \cdot s + f_{01} \cdot t + f_{11} \cdot st + f_{20} \cdot s^2 + f_{02} \cdot t^2 + \cdots,$$
$$g(x,y) = g_{00} + g_{10} \cdot s + g_{01} \cdot t + g_{11} \cdot st + g_{20} \cdot s^2 + \ldots$$

It is useful to eliminate the term f_{11} in the exponential. This can be done by means of linear transformation, called affine transformation, of coordinates. A second order polynom describes a conic. Rotating of coordinate system to be parallel to conic axis has a consequention in extinction of the term f_{11}. The right angle of rotation α is given by equation

$$tg(2\alpha) = \frac{f_{11}}{f_{20} - f_{02}}$$

The calculation gives contribution from the critical point of first kind in the form

$$u_{p1} = \frac{i\pi e^{ikf'_{00}}}{k\sqrt{f'_{20} f'_{02}}} \left(g'_{00} + \frac{i g'_{20}}{2kf'_{20}} + \frac{i g'_{02}}{2kf'_{02}} \right).$$

Critical points of the second kind are points on the curve C bounding the domain of integration Ω at which

$$\frac{\partial f}{\partial l} = 0,$$

where dl is an element of arc of the bounding curve C. The appropriate transformation of the coordinate system makes one axis parallel to the boundary at the critical point. This rotation simplifies further calculations. Complete contribution from the critical point of the second kind is

$$u_{p2} = e^{ikf'_{00}} \frac{\sqrt{-i\pi}}{k^{\frac{2}{3}} f_{10} \sqrt{f_{02}}} \left(g'_{00} + g'_{10} \frac{i}{kf'_{10}} + g'_{20} \frac{-2}{k^2 f_{10}^2} + g'_{02} \frac{i}{2kf'_{02}} \right) \quad (25)$$

If boundary of the integration domain C is not an analytic curve, it can not be taken as axis of t' and we must consider the non-analytic points of C separately. We confine ourselves to the corner, i.e. the points where the tangent varies discontinuously; they will be called *critical points of the third kind*. Contribution is

$$u_{p3} = \frac{-e^{ikf'_{00}}}{k^2 f'_{10} f'_{01}} \Big(g'_{00} + g'_{10} \frac{i}{kf'_{10}} + g'_{01} \frac{i}{k^2 f'_{01}} + g'_{20} \frac{-2}{k^2 f_{10}^2} + g'_{11} \frac{-1}{k^2 f'_{10} f'_{01}} + g'_{02} \frac{-2}{k^2 f_{10}^2} \Big) \quad (26)$$

Application of this method to our purpose has several limitations. Due to the function T the subintegral function is not continuous within the integration domain, this fact breaks the conditions of this method. Dividing the whole domain to zones with constant phase shift we can fulfill these conditions. Each zone of the DOE will be therefore computed separately and the total effect will be summed. Thus the necessary computing time is dependent only to the number of Fresnel zones in the zone plate which increases with the d/f ratio and not with the size of the element.

To compare the significant difference between diffraction patterns predicted both by Kirchhoff scalar theory and modified Huygens-Fresnel principle we are currently evaluating simple optical element using both theories. The final judgement will be naturally made by measurement of element optical properties. Taking into account the required precision of measured values, we decided to design, produce, compute and measure a relatively simple dielectric grating fabricated of the PMMA layer 0.628 micron thick placed on the silica vafer. The periode of the grating is 1.2 micron, the distance between two ridges is 0.6 micron. The grating is windowed by the square aperture 3 × 3 milimetres and was designed for operation with red He-Ne laser. Computational study of polarized light interaction with this test grating is carried out using workstation HP-712 running first version of our C-language program. The software for diffraction simulation will be tested in both full an asymptotic version.

The mutual comparison of results obtained by several very different methods should bring new light to the problem of diffractive phenomena on dielectric microstructures, cut off the blind ends of the research in this field and definitely illuminate the prospective directions of investigation.

6. References

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About authors,...

Jan Popelek was born in Opava, Czechoslovakia in 1966. He graduated in 1989 with a RNDr. (MSc equivalent degree) in Optoelectronics from Olomouc University. From 1989 to 1991 he was a research assistant with Institute of Metallurgical Processes of Czechoslovak Academy of Sciences in Ostrava, working on visualisation (interferometry and optical tomography). From December 1991 to June 1992 he was a visiting researcher at Phillips Intrernational Technology Centre Leuven, Belgium, researching advanced optical link for digital audio. In 1993 he finished studies towards his Ph.D. degree in Optoelectronics at the Dept of Microelectronics, Technical University of Brno. At the time beeing, he is writing his Ph.D. thesis at the Dept of Electronics, Technical University of Ostrava. His main fied of interest behind playing violin is integrated optics, particularly the theory of difractive optical elements design.

Robert Follner was born in Jeseník, Czechoslovakia in 1971. He graduated in 1995 with a Ing. (MSc equivalent degree) in Microelectronics from Technical University of Brno. His diploma thesis tackled fast implementation of equations used for evaluation of light intensity behind a thin diffractive optical element. Today (end of 1995) he takes his Ph.D. courses at the Dept of Microelectronics, Technical University of Brno. He is recognized unix guru and his main hobby is Internet surfing.

THE SEMINAR ON DESIGN AND DIAGNOSTICS OF ELECTRONIC CIRCUITS AND SYSTEMS '95

The Proceedings of Design and Diagnostics of Electronic Circuits and Systems '95 Seminar brought the latest papers on research and development in the DDECS '95 branch. The DDECS '95 Seminar was organised by the Department of Electronics, FEI, VŠB-Technical University of Ostrava in September 1995 in Rožnov pod Radhoštěm. The opening anniversary of VŠB in Ostrava in 1945 was celebration these days. This was the opportunity to remember the history of school. The faculty of Electrical Engineering was the fifth faculty established at the VŠB - Technical University of Ostrava.

The history of Diagnostic Seminars is longer then ten years. The DDECS continues the ten-year tradition of "Diagnostics of Microprocessors" initiated by Prof. Ing. Jan Hlavička, DrSc., honorary chair of DDECS '95 Seminar. The previous seminars were very useful and popular. The seminar Design and Diagnostics of Electronic Circuits and Systems '95 wants to follow them. The DDECS '95 integrate the design and diagnostics. The topics deal with both the traditional and the new areas...

The DDECS '94 was devoted to VHDL Design Tools. The DDECS '94 was held in the framework of the TEMPUS project. The lecturer was Prof. Paul Raes, who works in the VHDL field from the beginning of existence of IEEE Std 1076-1987. Under the influence of the lectures of DDECS '94 was in immediately starting of, regular VHDL courses in pre-graduate as well as post-graduate education at the Department of Electronics. These regular courses are instructed every year. The knowledge gained through proper scientific research and intensive co-operation between the VŠB-TU, the other technical universities and enterprises which is even on the international level, are promptly integrated into the teaching of the courses.

The DDECS '95 had its own advantage and liability. For specialists more then ten-years tradition was an advantage. However for organisers it was a liability because of the very good quality of previous seminars. The Diagnostics of Microprocessors III (in 1981) had its own aim: Application of self-tests in the diagnostics. All the seminars had their own objects. The DDECS '95 has the aim too: The VHDL modelling in diagnostics and the Boundary-Scan Test by IEEE Std 1149.1-1990. The both aims are very frequently mentioned in technical issues. In the six sections of the Seminar was concerned into areas (i) High-End Design and Diagnostic Means for ASICs, (ii) Generation of External- and Self-Tests for ASICs, (iii) Design and Diagnostics of FPLs, (iv) Special Diagnostic Methods and Means, (v) DSP Systems, and (vi) Reliability. In the sections it was lectured more then thirty papers.

The DDECS '97 is planed on May 1997 as a part of TEMPUS Project "Education in Quality Control in Electrical Industry" S_JEP 9468, which is collaborated by Department of Microelectronics, Faculty of Electrical Engineering, Czech Technical University Prague (Contractor), Department of Microelectronics, Faculty of Electrical Engineering and Computer Science, Technical University of Brno (Coordinator), and Department of Electronics, Faculty of Electrical Engineering and Computer Science, VŠB-Technical University of Ostrava. The foreign co-operating universities: University Bournemouth, Metropolitan University Leeds, University of Hull in Great Britain, and University of Grenoble in France. The DDECS '97 Seminar will be held at the Northern Moravia in Beskydy Mountains.

Doc. Ing. Karel Vlček, CSc.