Adaptive Goertzel's Algorithm for DFT Computation with Higher Accuracy

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Abstract

In this paper the adaptive Goertzel's algorithm applied for required DFT coefficient computation in the harmonic analyser is described. This algorithm utilizes effective combination of the synchronous DFT and window method. The experiments with the adaptive Goerzel's algorithm given in this paper have shown that in the case of a harmonic analysis of periodical triangular wave it can provide the accuracy of the harmonic computation better than 0.01%.

Keywords:

Goertzel's algorithm, DFT, harmonic analysis.

1. Introduction

There are many instances when signal processing involves the measurement of spectrum. In a large percentage of applications, the problem of measuring the spectrum corresponds to finding the z transform of a finite record of the sequence at a large number of points equally spaced around the unit circle. These measurements correspond to measuring the DFT of the finite sequence and are generally most efficiently implemented by an FFT.

There are some applications where only a selected number of values of the DFT are desired, but the entire DFT is not required. In such a case, the FFT algorithm may no longer be more efficient than a direct computation of the desired values of the DFT.

The direct computation of the DFT may be formulated as a linear filtering operation on the input data sequence [3]. The linear filters takes the form of a parallel bank of resonators where each resonator selects one of the frequencies $\omega_k = \frac{2\pi k}{N}$, k=0,1,...,N-1 corresponding to the N frequencies in the DFT. Moreover, N does not need to be a composite integer like it is for the standard FFT algorithms.

The most frequently used method for the direct computation of the DFT is Goertzel's algorithm [4]. This algorithm requires more mathematical operations than a standard FFT algorithm. The advantage of the Goerzel's algorithm in comparison with the conventional FFT algorithm is that it enables to compute the DFT coefficients for any value of N (not only for power of two) without necessity to change the algorithm structure.

In this paper the adaptive Goertzel's algorithm developed for the purpose of spectral analysis of power supply is described. This algorithm utilizes effective combination of the synchronous DFT and window method [3]. The presented adaptive Goertzel's algorithm has been effectively applied in the harmonic analyser HARAN-30 [1].

The harmonic analyser HARAN-30 [1] should analyse and record the harmonics of the power supply at steady-state up to the 30-th (or 50-th) harmonic. The steady-state measured voltage is a periodical signal with fundamental frequency from the interval 49.0Hz to 50.5Hz. With regard to its application field the measurement accuracy should be better than 0.1% normalised to the first harmonic.

The overall error of equipment consists of errors caused by hardware errors (mainly A/D converter and input filter) as well as the error of method used for harmonic computation. The hardware errors were minimised by appropriate hardware implementation. With regard to that facts the minimum accuracy of the harmonic computation method was set to 0.01% in the case of periodical triangular wave analysis (triangular wave is typically used in error specification of commercial power analysers [2]). All numerically critical parts of the algorithm were implemented in floating point and therefore the errors due to inaccuracy of computation were negligible. At the choice of harmonic computation methods it was necessary to take into account the constant sampling frequency $F_{sp} = 7812.5Hz$. The value of F_{sp} was set in dependence on hardware configuration and it was not possible to change it. The harmonic computation was carried out by the discrete Fourier transformation (DFT).

In this paper the adaptive Goertzel's algorithm applied for required DFT coefficient computation is described. This algorithm utilizes effective combination of the synchronous DFT and window method.

2. Synchronous DFT

The synchronous DFT method is based on analysis of integer multiple of the input signal periods. Under condition of processing of integer multiple of the input signal periods this method allows to achieve the zero error of method. Generally the sampling frequency is not exactly the integer multiple of the fundamental frequency of an input signal. The error resulting from this fact is inversely proportional to the sampling frequency and is represented by spectral leakage of neighbour spectral components.

For the purpose of evaluation of the synchronous DFT error for constant F_{sp} a numerical simulation of the computation of individual harmonics for triangular wave was made. In our experiment fundamental frequency was randomly varied from 49.0Hz to 50.5Hz. The results of this experiment represented by the maximum relative errors of the calculation are given in the Tab.1. The errors are normalised to the first harmonic for the 300 triangular waves at $F_{sp} = 7812,5Hz$ and p input signal periods ($p = 1,2,3,4$). We can see from the Tab.1 that the synchronous DFT application with constant F_{sp} does not enable to reach the required accuracy.

3. Window function method

The advantage of this method lies in the fact that the spectral leakage can be greatly reduced by using window functions. This method allows to analyse the periodical signal without necessity to determine integer multiple of its periods. The excellent review of the useful window functions can be found e.g. in [3]. In our experiments we dealt with the following window functions ($n = 0, 1, ..., N-1$):

Triangular *(TR)*

$$w(n) = 1 - \frac{\left|n - \frac{N-1}{2}\right|}{\frac{N-1}{2}}$$

Hamming *(HA)*

$$w(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right)$$

Blackman-Harris

$$w(n) = \sum_{m=0}^{k-1} a_m (-1)^m \cos\left(\frac{2\pi nm}{N-1}\right)$$

for 3 term Blackman Harris (3B) with -18dB/dec roll-off

$$a_0 = 0.42, \quad a_1 = 0.50, \quad a_2 = 0.08,$$

for 4 term Blackman Harris (4B) with -92dB highest sidelobe level

$$a_1 = 0.35875, \quad a_1 = 0.48829,$$
$$a2 = 0.14128, \quad a3 = 0.01168.$$

The magnitude spectra of the above given windows are depicted at the Fig.1. The typical property of window function is the larger width of the main lobe due to power decreasing in side lobes. In order to get good accuracy of the computation it is convenient to apply 2 periods, 2 periods, 3 periods and 4 periods of the analysed signal for HA, TR, 3B and 4B, respectively.

For the purpose of the evaluation of the error of the harmonic computation with windowing a numerical simulation for triangular wave was made. In our experiment fundamental frequency was randomly varied from 49.0Hz to 50.5Hz. The results of this experiment represented by the maximum relative errors of the calculation are given in the Tab.2. The multiple of $N = 157 \times p$ was used as a window length.

triangular wawe 49.0-50.5 Hz 300 realizations	Synchronous DFT			
	1 period	2 periods	3 periods	4 periods
relative error[%]	0.097	0.059	0.043	0.033

Tab.1

triangular wawe 49.0-50.5 Hz 300 realizations	The window function			
	HA	TR	3B	4B
relative error[%]	0.393	0.586	0.101	0.134

Tab.2

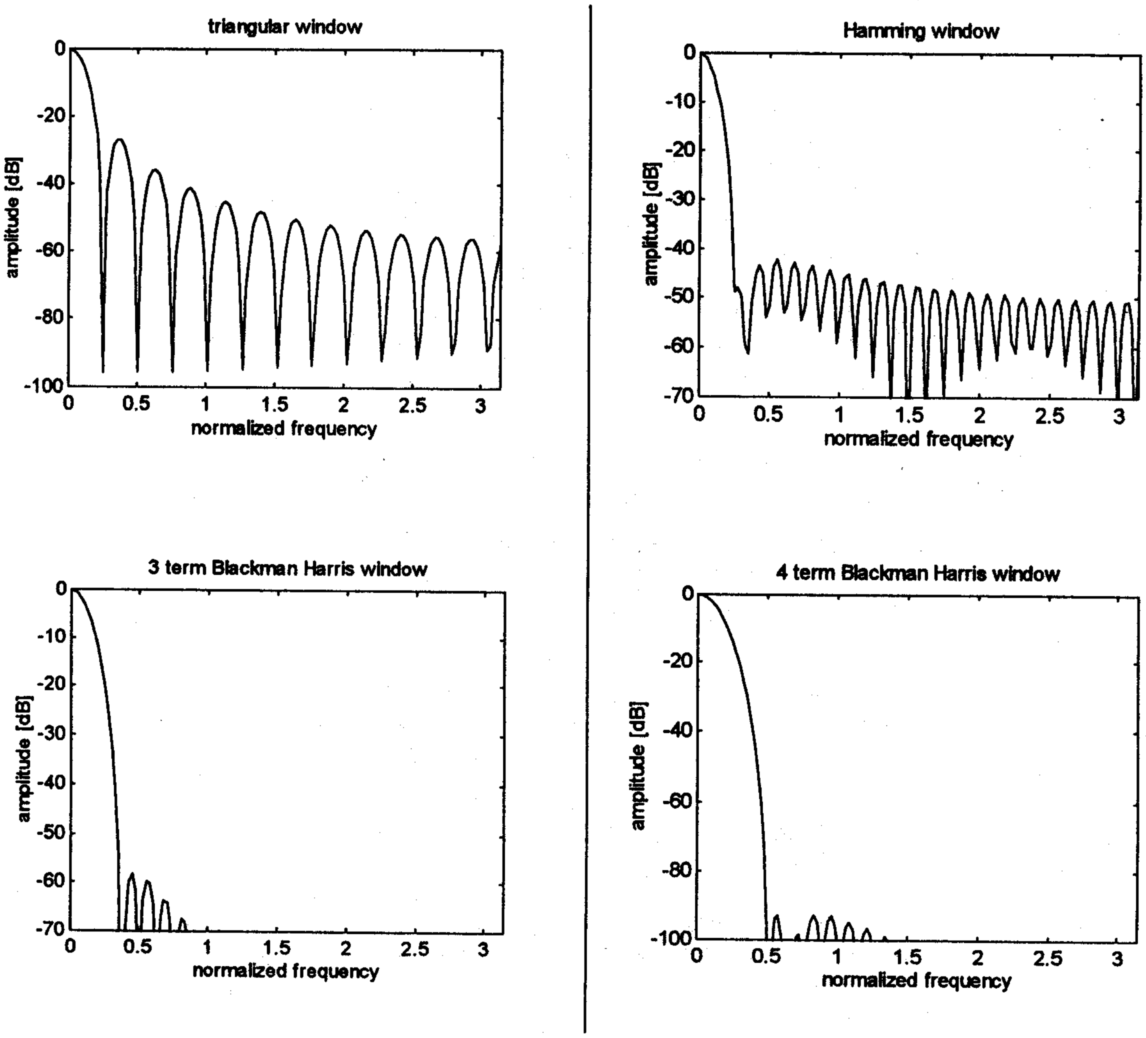


Fig.1 Various time domain windows and their magnitude spectra a) Triangular window b) Hamming window c) 3 term Blackman Harris window d) 4 term Blackman Harris window

It is the synchronous window for the signal frequency $F_{sig} = 49.75Hz$ (approximately in the middle of the interval <49.0 Hz, 50.5 Hz>) and $F_{sp} = 7812.5Hz$. We can see from the Tab.2 that the window function application does not enable to reach the required accuracy.

4. Adaptive Goertzel's algorithm

The adaptive Goertzel's algorithm exploits the advantages of the synchronous DFT as well as the window functions for the further improvement of the accuracy. Its block structure is depicted in the Fig.2. The adaptive Goertzel's algorithm consists of the following steps:

a) Determining the integer multiple of the input signal periods with minimum possible error.

For the purpose of determining the integer multiple of the input signal periods a correlation analysis was effectively used. This method is accurate even for the greatly corrupted input signal. Let N_{opt} is optimum number of samples within p periods of the input signal corresponding to the estimated integer multiple of the input signal periods. Value of the parameter p depends on the chosen window function. It can be shown that N_{opt} is from interval $\langle N_{\min}, N_{\max}\rangle$ where $N_{\min} = \left\lfloor \frac{pF_{sp}}{50.5} \right\rfloor$, $N_{\max} = \left\lceil \frac{pF_{sp}}{49.0} \right\rceil$ and $\lceil x \rceil$ is the smallest integer larger than or equal to x and $\lfloor x \rfloor$ is the highest integer lower than or equal to x. Based on correlation analysis it can be shown that N_{opt} corresponds to the value N for which $K(N)$ defined as

$$K(N) = \sum_{n=0}^{M} x(n)x(N+n)$$

is maximum, where M is the number of the samples based on which the autocorrelation is computed. For instance, $p = 4, N_{\min} = 618, N_{\max} = 638$ and $M = 158$ for $4B$. After evaluation of N_{opt}, it is also

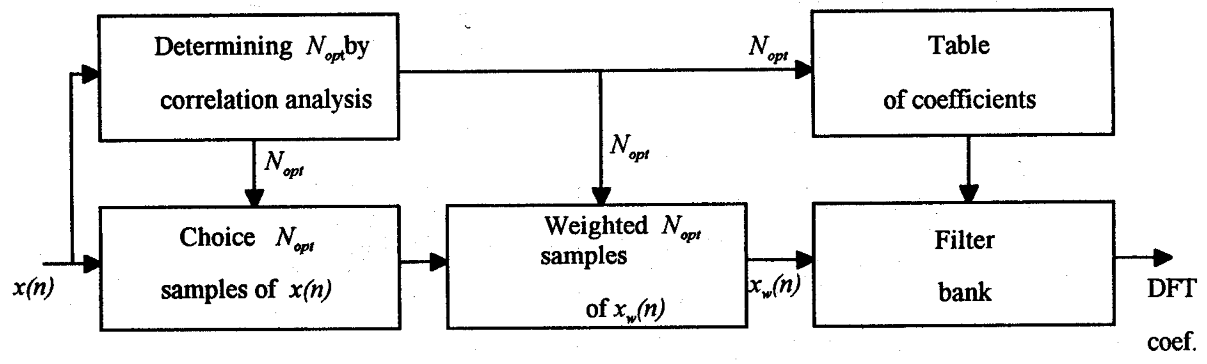


Fig. 2 Principle of the calculation of the DFT coefficients by adaptive Goertzel's algorithm

possible to compare the energy in the individual periods and to decide if the stationary element of the signal is analysed and to exclude transient phenomena from the analysis.

b) Application of window function $w(n)$ to N_{opt} samples of the input signal $x(n)$:

$$x_w(n) = w(n)x(n) \quad n = 0, 1, \ldots N_{opt} - 1$$

c) Computation of N_{opt} point DFT of the weighted input signal $x_w(n)$.

The computation of the DFT was performed by the Goertzel's algorithm [4]. It allows, to compute the DFT coefficient with the bank of the second order recursive digital filters with adaptively changed coefficients . The adaptive Goertzel's algorithm can be described as follows:

$$v_k(n) = 2\cos\left(\frac{2\pi kp}{N_{opt}}\right) v_k(n-1) - v_k(n-2) + x_w(n),$$

$$v_k(-1) = v_k(-2) = 0,$$

$$n = 0, 1, 2, \ldots N_{opt} - 1, \quad k = 1, 2, \ldots K_{\max}$$

$$A_k = \sqrt{\left[v_k^2(N_{opt}-1) + v_k^2(N_{opt}-2) - 2v_k(N_{opt}-1)v_k(N_{opt}-2)\cos\left(\frac{2\pi kp}{N_{opt}}\right)\right]},$$

where the coefficients $\cos\left(\frac{2\pi kp}{N_{opt}}\right)$ $k = 1, 2, \ldots K_{\max}$ of the filter bank are adaptively changed in dependence on the value of N_{opt}. In the last expression A_k is magnitude of the *kth* input signal harmonic and $K_{\max} = 30$ or 50.

Our experiences have shown that the computation of the selected DFT coefficients on single chip microcontrollers by means of the Goertzel's algorithm is in many cases more effective than the FFT algorithm application.

For the purpose of evaluation of the error of the harmonic computation by the adaptive Goertzel's algorithm the numerical simulation for the triangular wave was made. In our experiment fundamental frequency was randomly varied from 49.0Hz to 50.5Hz. The results of this experiment represented by the maximum relative errors of the calculation are given in the Tab.3.

Following the Tab.3 we can see that the adaptive Goertzel's algorithm with 4B window satisfies requirements by which the relative error is less than 0.01%. Therefore we decided to implement this algorithm of the harmonic analysis in the equipment HARAN-30.

5. The HARAN-30 description

The equipment HARAN-30 has been designed as a standard circuitry of the single-chip microcontroller PHILIPS 80C552. The equipment contains backup real time clock and backup data memory where the results of analysis are recorded as well as registration of power supply drop outs longer

triangular wawe 49.0-50.5 Hz 300 realizations	The adaptive Goertzel's algorithm			
	HA/p=2/	TR/p=2/	3B/p=3/	4B/p=4/
relative error[%]	0.089	0.014	0.010	0.007

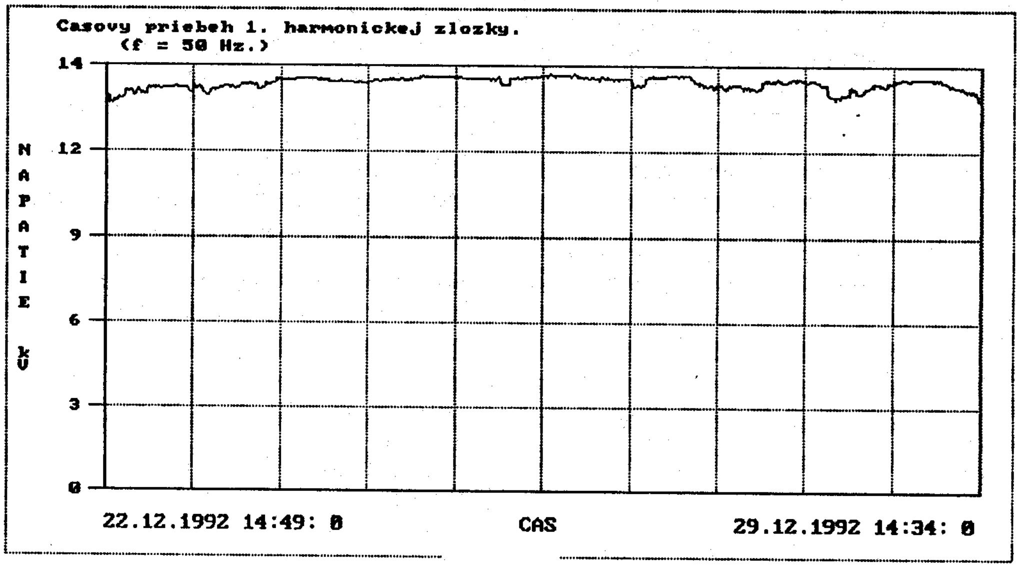
Tab.3

than 1 second. For setting of commands and for listing of stored data there are three buttons and LCD display. The 16-bits sigma delta converter MOTOROLA DSP565ADC16 [5] is used as an input circuit. It enables the significant reduction of hardware complexity and A/D conversion error minimisation. In our equipment we utilized effectively the following properties of DSP56ADC16:

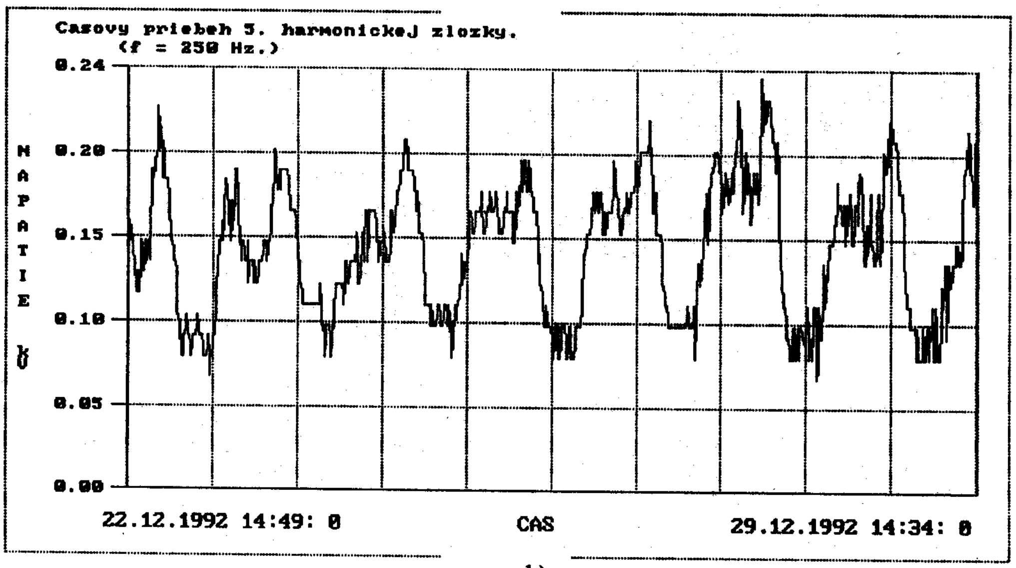
- 64 times oversampling (this feature allows to use a simple RC input filter, the need for the precise devices can be eliminated)
- single supply voltage +5V (the same as a digital part of equipment)
- fully differential inputs (the range is +/-2V)
- on-chip reference voltage source
- in-band ripple less than 0.001 dB (it is negligible concerning required accuracy)

The basic features of the equipment HARAN-30 are:

- computation of the first 30 harmonics of power supply in monitoring mode (the input voltage is analysed in adjustable interval from 1 up to 15 minutes with 1 minute step);
- computation of the first 50 harmonics of power supply at one shot measurement;



a)



b)

Fig. 3 The output of PC visualization software for high power supply analysis with 15 minutes intervals
a/ record of the 1-st harmonic
b/ record of the 5-th harmonic

- displaying and recording of the relative level of the analysed harmonics with resolution 0.001;
- recording the results of 672 measurements (one week with 15 minutes' time interval);
- recording the date and time of power supply drop outs longer than 1 second;
- possibility to transfer of the data to the PC-AT computer through the optically isolated RS232 interface for further processing;
- visualization of the measurement results using the PC-AT computer (e.g. the results of the analysis of the 1-st and 5-th harmonic for 15 minutes measuring interval of high power supply measurement with the measuring transformer. are shown in the Fig.3. The 24 hours "periodicity" of the power supply load is evident from 5-th harmonic which is not evident from 1st harmonic);
- supply of the equipment and measurement of the power supply from the same power cord;
- backup of the storing data and of the real time clock for the time at least 1 month without power supply
- compact design;
- induced noise-tolerant of the equipment
- low cost.

6. Conclusion

The harmonic analyser HARAN-30 have been designed under contract for the Electro-Power Company. This equipment satisfies desired parameters in full range. It has been shown that the equipment error is less than 0.1% in evaluating of the relative levels of power supply harmonics. The tests have shown that equipment conception and control software of the equipment are fault tolerant with regard to industrial disturbance.

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