To the Possibility of Calculation of Phase Synchronism Active Range in Two-Modes Laser

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Abstract

The contribution intends to promote further application possibilities of Donocik's method for stability investigation of dynamical systems and presents a nontrivial application - the attempt to calculate a phase synchronism active range of two oscillating modes in a laser by attraction regions of stationary states of a nonlinear system testing. The emphasis is laid upon a final discussion.

Keywords

laser oscillating modes, active phase synchronism, functional stability

1. Introduction

The initial mathematical-physical model suitable for direct investigation of the existence and conditions of self-locking and ceasing of spontaneous phase synchronism takes the form [2,3]

$$\ddot{\varphi}+(\kappa+\gamma)\dot{\varphi}-\left[\frac{\dot{\varphi}^2}{2}-\delta\dot{\varphi}+2(g^2Nd_0-\kappa\gamma)\right]\cdot \cdot\frac{2B\cdot\sin^2\varphi+A\cdot\sin 2\varphi}{10g^2\kappa T+2A\cdot\cos^2\varphi+B\cdot\sin 2\varphi}=2\gamma\delta. \quad (1)$$

Applying the Functional Stability rules [1,2,3] we can gradually attain the relation acceptable for Donocik's Theory Criterion [1]. The general expression for stationary states attraction regions can be used solving mentioned laser model. So called active synchronism of laser modes represents the necessary and sufficient conditions of the self-transition from an asynchronous into synchronous state. The whole procedure has been published and conditions of the passive synchronism presented [3] therefore they are not repeated here, once more.

The solution starts with a supposition that the following relations derived in [3] are relevant for our case, namely

$$\lim_{t\to\infty}\left\{\begin{array}{l}\mu'\doteq\dot{\varphi}_0\exp[-(\kappa+\gamma)t]+\dfrac{2\gamma\delta}{\kappa+\gamma}\cdot\{1-\exp[(\kappa+\gamma)t]\}+\\ \displaystyle\int_0^t\exp[-(\kappa+\gamma)(t-\tau)]\cdot\left[-\delta\mu'+\frac{1}{2}\mu'^2+2(g^2S_0-\kappa\gamma)\right]\cdot\\ \cdot\dfrac{2B\cdot\sin^2\varphi(\tau)+A\cdot\sin 2\varphi(\tau)}{10g^2\kappa T+2A\cdot\cos^2\varphi(\tau)+B\cdot\sin 2\varphi(\tau)}\cdot d\tau\\ \\ 1>\displaystyle\int_0^t\{\exp[-(\kappa+\gamma)(t-\tau)]\}\cdot(-\delta+\mu')\cdot\\ \cdot\dfrac{2B\cdot\sin^2\varphi(\tau)+A\cdot\sin 2\varphi(\tau)}{10g^2\kappa T+2A\cdot\cos^2\varphi(\tau)+B\cdot\sin 2\varphi(\tau)}\cdot d\tau\end{array}\right\} \quad (2\text{ a , b})$$

where $\mu' = \dot{\varphi}(t)$ and

φ is the phase angle difference of the two oscillating modes,

$\dot{\varphi}$ is the time derivative of the difference of phase angles (chosen functional $\mu = \lim_{t\to\infty}\dot{\varphi}(t)$),

$\dot{\varphi}_0$ is $\dot{\varphi}(t)$ for t = 0,

κ is the electromagnetic field damping constant of the relevant mode,

γ is the damping constant of the excited active medium,

δ is the angular frequency deviation of the relevant own resonator mode out of the angular frequency of the atomic resonant line centre (i.e. $\delta = \omega - \nu$),

ω is the laser resonator own mode angular frequency of a non-loaded cavity,

ν is the angular frequency of atomic resonance line centre of the mentioned two-level model,

g is the interactive coefficient of the oscillating mode field with the excited active field, T is the time constant of the unsaturated stationary value stabilization of population inversion of the active medium level $S_0 = Nd_0$,

N is the number of atoms of the active medium,

d_0 is the unsaturated stationary value of population inversion of active medium levels related to 1 atom of the medium,

S_0 is the total unsaturated stationary value of population inversion of active medium levels related to all atoms of the medium,

A,B are symbols representing relations

$$A = 2g^2T\cdot\frac{(\kappa_1+\kappa_2)+T(\omega_1-\omega_2)^2}{1+T^2(\omega_1-\omega_2)^2}$$

$$B = 2g^2T\cdot\frac{(\omega_1-\omega_2)[1-T(\kappa_1+\kappa_2)]}{1+T^2(\omega_1-\omega_2)^2}. \quad (3)$$

Here indeces 1,2 correspond to the first and second oscillating mode quantities, respectively.

2. Synchronism of Two Oscillating Modes Calculation

If a preliminary hypothesis is accepted that only two regimes of operation may occur in the region beyond the threshold of population inversion, i.e. a passive and an active range of synchronism, then the studied active range is just the complement of region of attraction of asynchronism stationary state. This whole region must generally create a subregion of the passive range. The transition from the asynchronous state into synchronism cannot be realized out of the region of passive synchronism.

The stationary state time derivative of the difference of phase angles can be expressed [2,3] in the stationary asynchronous state (classical two modes operation) by expresssions

$$\mu = \dot{\varphi}_{AS} = \dot{\varphi}_{2,AS} - \dot{\varphi}_{1,AS} = 2\Delta_{AS} = \lim_{t\to\infty}\mu' = konst \neq 0 \quad (4)$$

and from this

$$\varphi_{AS} = 2\Delta_{AS}t + \varphi_0, \quad (5)$$

where φ_0 is the initial value of the difference for t = 0.

If the system is active in an immediate neighbourhood of the mentioned state then $\dot{\varphi}(t)$ can be supposed to have a constant value and therefore the expressions in square brackets of equations (2a,b) can be pointed out of the integral. After this rearrangement we can gain

$$\lim_{t\to\infty}\left\{1>(-\delta+\mu')\cdot\frac{\mu'-\dot{\varphi}_0\exp[-(\kappa+\gamma)t]-\frac{2\gamma\delta}{\kappa+\gamma}\{1-\exp[-(\kappa+\gamma)t]\}}{-\delta\mu'+\frac{1}{2}\mu'^2+2(g^2S_0-\kappa\gamma)}\right\}$$

and from this

$$1>(-\delta+2\Delta_{AS})\cdot\frac{\Delta_{AS}-\frac{\gamma\delta}{\kappa+\gamma}}{-\delta\Delta_{AS}+\Delta_{AS}{}^2+g^2S_0-\kappa\gamma}, \quad (6)$$

what represents the condition (necessary and sufficient), so that two oscillating modes in asynchronous state might exist. Respecting this fact the conclusion can be made that phase synchronism self-locking may arise beyond the population inversion S_0 threshold values, when (6) is not valid and the inverse relation is fulfiled

$$1<\frac{(-\delta+2\Delta_{AS})\cdot(\Delta_{AS}-\frac{\gamma\delta}{\kappa+\gamma})}{-\delta\Delta_{AS}+\Delta_{AS}^2+g^2S_0-\kappa\gamma}, \quad (7)$$

We decide if for $S_0 > S_{0,pas}$ the denominator has a positive or a negative value solving the equation

$$g^2S_0-\kappa\gamma+\Delta_{AS}(\Delta_{AS}-\delta)=0 \quad (8)$$

and accordingly with physical principles we have - see Haken [4]

$$\Delta_{AS}=\frac{\gamma\delta}{\kappa+\gamma}+K(S_0-S_{0,pr}) \quad (9)$$

where K is a constant (positive or zero) and $S_{0,pr}$ is the population inversion threshold value in the form [4]

$$S_{0,pr}=\frac{\kappa\gamma}{g^2}\cdot(1+\frac{\delta^2}{(\kappa+\gamma)^2}). \quad (10)$$

Substituting (9), (10) into (8) we obtain

$$(S_0-S_{0,pr})\cdot\left[g^2+\frac{\delta K(\gamma-\kappa)}{\kappa+\gamma}+K^2(S_0-S_{0,pr})\right]=0 \quad (11)$$

and from this

$$S_{0,1}=S_{0,pr},\quad S_{0,2}=S_{0,pr}-\frac{1}{K^2}\cdot\left[g^2+\frac{\delta K(\gamma-\kappa)}{\kappa+\gamma}\right] \quad (12)$$

It stands to reason that $S_{0,1} = S_{0,pr}$ is greater than $S_{0,2}$. The value $S_{0,2}$ is under the threshold and is out of a physical sense. All the values beyond the threshold fulfil the condition that (8) has a positive value. Considering these conclusions we can achieve the result [2]

$$S_0 > S_{0,pr} + \frac{g^2}{K^2} \tag{13}$$

and taking for K [2]

$$K = \frac{1}{5} \cdot \frac{g^2}{\kappa + \gamma} \cdot \frac{2\delta T}{1 + (2\delta T)^2} \tag{14}$$

we have got a critical value of the whole unsaturated population inversion $S_{0,k}$ which is expresssed by the right side of (13)

$$S_{0,k} = S_{0,pr} + \frac{25(\kappa + \gamma)^2}{g^2} \cdot \frac{\left(1 + 4\delta^2 T^2\right)^2}{4\delta^2 T^2}. \tag{15}$$

Two-modes oscillating state has changed the quality suddenly with respect to the chosen functional with an arbitrary small $S_{0,k}$ exceeding.

3. Analysis of the Result

The active synchronism can be determined by condition (15) in the situation when S_0 is generally a subregion of the passive synchronism only in the case that the parameters κ, γ, δ, T, g are chosen independently. In [2] the proof is given that it is not like this. The relation has been found for an arrangement of the parameters when phase synchronism is not followed exceeding $S_{0,k}$ in the form

$$2\delta T > \sqrt{\frac{10(\kappa + \gamma)^2 T}{\gamma + 10(\kappa + \gamma)^2 T}}. \tag{16}$$

The active synchronism condition is not therefore represented by (15). The assumption of two possible states in laser resonator (asynchronous and synchronous) is obviously questionable. The stay may occur that beyond population inversion threshold one oscillating mode ceases suddenly. The chosen functional is not then defined. Nevertheless, this type of a step-wise change is registered by the theory as an abrupt qualitative change. Condition (15) represents the transition into monomode (one-frequency) operation. It is not caused by the self-phase synchronism of the two modes but by one mode ceasing. This conclusion corresponds fully with the results [5 to 7], saying that the two-modes (nonmonochromatic) or monomode (monochromatic) operation can be reached with respect to the parameters. The angular frequency of monochromatic radiation is adjusted closely to the proper resonator mode being characterized under the fixed parameters of active medium by the minimum damping in the resonator, minimum distance from the atomic resonant line centre and by the strongest interaction with the excited active field. As far as the parameters of both the modes are equal then the mode vanishing is given by a fluctuation of parameters (e.g. temperature). Any quotation has not been registered on the phase synchronism of two modes and the evaluation of the problem in [4] is supposed to be incorrect. Both the authors, Haken and Ostrovskij, however, consult different models only formally [2]. The method of qualitative analysis is in both the cases the linearization of models in the immediate neighbourhood of their stationary states and the resulting investigation of Ljapunov's asymptotic stability conditions.

An angular frequency deviation of one mode increases, which is made by the amplitude enlargement of the second mode (with S increasing caused by laser-exciting). The gain decreases and as the case may be the resonator losses increase. The amplitude addition with S_0 enlargement is less than without a mutual interaction, at first (amplitude stabilization in the immediate region beyond the threshold) up to the value $S_{0,k}$, exceeding of which is followed by one mode vanishing (energy "overflowing").

The resulting angular frequency deviation of oscillations out of the atomic resonant line centre is finally stabilized after the mode ceasing at the minimum value, determined by (9)

$$\Delta_{min} = \frac{\gamma\delta}{\kappa + \gamma}. \tag{17}$$

It is the basic difference from the supposed synchronous state, the deviation is not equal zero and is placed out of the centre of the atomic resonant line.

To the question of the synchronism active range existence:

The conclusion may be stated, that the passive range has been found, can be kept permanently during operation and of course can theoretically exist, after all. Relations (13), (15) are evidently necessary conditions but not sufficient ones. The synchronous state transition is described with the initial model what can be documented - see the calculation of synchronous operation of phase-locked oscillators. According to [8] the equation has the form

$$a\ddot{\varphi} + b\dot{\varphi} + c \cdot \sin\varphi = d, \quad a, d \neq 0, \tag{18}$$

where a, b, c, d are real constants and φ has the meaning corresponding to (1). Equation (1) takes in connection with (18) much more complicated goniometric nonlinearity only. As (18) describes the transition from asynchronous to synchronous operation state of phase-locked oscillators then our model (1) is also selected correctly and must be valid.

As to the functional: If it did not reflect the laser behaviour qualitative changes, we should not find any condition of passive synchronism most probably. The relevant integrals in (2a,b) have been approximated with the assumption that the relevant stationary state of the system is in an immediate neighbourhood. It is not fully elementary condition but is not substantial for the active synchronism condition investigation.

Used approximation of integrals (2a,b) is sufficient qualitatively, but the expressions for angular frequency deviations of oscillating modes out of the atomic line centre (taken over literature [4]) as well as the expression for amplitudes of both the modes dependent on S_0 create most probably a very extensive simplification. Necessary relations ought to be of higher orders so that the deductions may be taken in account about further principles than an "overflowing" effect (the validity of (9) is limited). It is not a trivial matter, however, to find a more precise expression in an analytical form and a necessity would arise to derive the basic equation with respect to the asumption that the amplitudes of the field of oscillating modes are not constant in time.

4. Practical Results and Conclusion

The previous analysis is very important for applications. The abrupt changes of the state of operation which can occur with the systematic change of the laser excitation, are carried out as the changes of output radiation power density. If a high selective (monomode) optic-fibre is applied for one mode radiation transmission the state may happen that when the critical threshold value is over just this mode ceases. Second mode cannot participate the transmission in spite of the fact that it has twice higher energy in comparrison with the original mode - the reason is many times higher damping. Then the losses during communication may occur even though whole the energy of radiation has not changed. The switched voltage of optothyristor can be influenced by a power density of laser radiation, similarly.

In these cases knowledge of an arrangement of parameters could enable a suitable location of critical states. The results are supposed to be verified with a relevant measurring system. Theoretical conclusions of the contribution as well as the previous works [1,3] are of original contents. With the functional stability theory the results are achieved being able to become the basis of a real practical experiment.

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