A NEW CLASS OF NONLINEAR FILTERS: MICROSTATISTIC VOLTERRA FILTERS

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Abstract

In this paper a new subset of the time-invariant microstatistic filters so-called microstatistic Volterra filters are proposed. This class of nonlinear filters is based on the idea of the conventional microstatistic filter generalization by substituting Wiener filters applied in the conventional microstatistic filter structure by Volterra filters. The advantage of the microstatistic Volterra filters in comparison with the Wiener filters, Volterra filters and conventional microstatistic filters is the fact that in the case of non-Gaussian signal processing the microstatistic Volterra filters can outperform Wiener filters, Volterra filters or conventional microstatistic filters. The validity of this basic property of the microstatistic Volterra filters is verified by a number of computer experiments. The disadvantage of the microstatistic Volterra filters is their relatively high computational complexity.

Keywords:

Nonlinear filters, Volterra filters, microstatistic filters.

1. Introduction

Threshold decomposition has shown to be a powerful method for the analysis and design of a wide variety of robust nonlinear filters. Traditionally, nonlinear filters using threshold decomposition utilized either ranked order, maxima and minima, or Boolean functions of the thresholded samples inside an observation window to make the estimate of a desired signal. Lately, the class of microstatistic filters has been introduced where the sum of all linear estimates at the threshold level constitute the final output of the filter [1,2]. This class of nonlinear filters we will call conventional microstatistic filters (CMFs). There are many advantages to the use of linear estimates rather than Boolean functions at the threshold level. The most significant of these is that linear system theory can be used for the optimization and analysis of this class of nonlinear filters. It was shown in [1] that the Wiener filter solution is included in the class of the CMFs and, in general, it is a suboptimal solution over the class of possible filters.

Another quite different but also constructive and versatile approach to nonlinear filtering is to utilize the filter structure in the form of a truncated discrete Volterra series [3]. In practice, the Volterra series can be regarded as a Taylor series with memory. This class of nonlinear estimators which is known as Volterra filters (VFs) is also attractive since it can deal with a general class of nonlinear systems while its output is still linear with respect to the VF parameters. Therefore like in the case of the CMF linear system theory can be also used for the optimization and analysis of the VFs. It follows directly from the VF theory that the Wiener filters (WF) represent the first-order VF. At the design of the CMF and VF the mean-square error criterion is usually applied.

It follows from the facts presented above that the CMF as well as the VF have a number of common positive properties. For both classes of these nonlinear filters it can be shown that in the case of non-Gaussian signal processing or in the case of nonlinear system modeling they can outperform the WF [1-3]. Based on these facts we will introduce in our paper a new class of the time-invariant nonlinear filters so-called microstatistic Volterra filters (MVF).

In the case of the MVF the input signal is decomposed by using M-level block threshold decomposer into M signals. The i-th component of the decomposed signal is fed into the i-th Volterra filter. The output of the MVF is then represented by the sum of the outputs of the all VF. The new proposed class of microstatistic filters includes the positive features of the CMF as well as the VF. The WF, VF and CMF solution are included in the class of the MVFs and, in general, they are suboptimal solutions over the class of possible filters. The significant property of the MVF is the fact that linear system theory like in the case of the CMF and VF can be used for the optimization and analysis of this class of nonlinear filters. It is also expected that MVF will outperform the WF,VF and CMV.

The remainder of this paper is organized as follows. In the next section, a short description of the new proposed class of the nonlinear filters is given.

The design of the optimum time-invariant MVF by using the mean-square error criterion is discussed in the Section 3. Section 4 is intent on experimental verification of the basic properties of the MVF. In this section a brief comparison of the basic properties of the MVF, WF, VF and CMF is also given. Finally, concluding remarks are made in Section 5.

2. Definition of Microstatistic Volterra Filters

As it was outlined in the introduction of this paper microstatistic Volterra filtering is based on the idea of the CMF generalization which lays in the substitution of the WFs applied in the CMF structure by the VFs. A block scheme of the MVF obtained by that way is given in the Fig.1 where $x(n)$ and $y(n)$ are the input and output signal of the MVF, respectively. It can be seen from this figure that the MVF consists of an M-level block threshold decomposer of the input signal $x(n)$ and M VFs. The number of the VFs corresponds to the number of output signals of the decomposer. The i-th output signal of the decomposer is fed into the i-th VF ($VF_i(Q,N)$). The output of the MVF $y(n)$ is then given by the sum of a constant term h_0 and the outputs of all VF. The constant term h_0 has to be applied in the MVF structure in order to obtain an unbiased MVF output. Henceforth, the individual parts of the MVF will be described a bit more in detail.

2.1 Block Threshold Decomposition

The performance of the M-level decomposer of the input signal of the filter can be described by the expressions

$$D[x(n)] = [x_L(n)x_{L-1}(n)\dots x_1(n)x_{-1}(n)\dots x_{-L+1}(n)x_{-L}(n)]^t \quad (1)$$

where $D[.]$ represents operation of the decomposition of the signal $x(n)$ into a set of the M signals $x_k(n)$

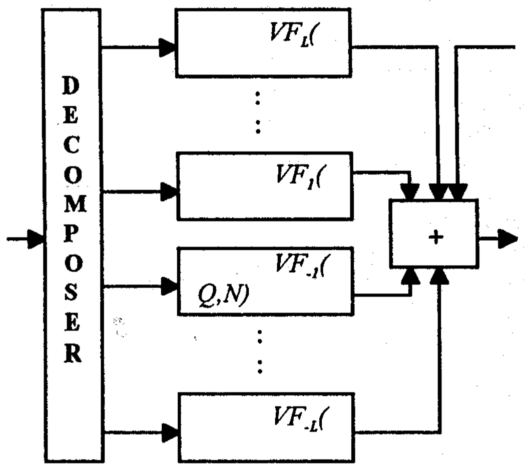

Fig.1 Microstatistic Volterra filter.

where $M = 2L$. The connection between $x(n)$ and $x_i(n)$ is given by

$$x_k(n) = D_k[x(n)] \quad (2)$$

where $D_k[.]$ denotes the decomposition operation for the k-th level of the decomposer. In the field of the microstatistic filter the most popular decomposition method is the well-known block threshold decomposition (BTD). In the case of the BTD the threshold sample $x_k(n)$ is uniquely determined from $x(n)$ by

$$x_k(n) = D_k[x(n)] = \begin{cases} 0 & \text{if} \quad x(n) < l_{k-1} \\ x(n) - l_{k-1} & \text{if} \quad l_{k-1} < x(n) \le l_k \\ l_k - l_{k-1} & \text{if} \quad l_k < x(n) \end{cases} \quad (3)$$

for $x(n) \ge 0$ and $1 \le k \le L$, and where $l_L = \infty$. The BTD for negative values is given by

$$x_k(n) = D_k[x(n)] = \begin{cases} 0 & \text{if} \quad x(n) > l_{-k+1} \\ x(n) - l_{-k+1} & \text{if} \quad l_{-k+1} > x(n) \ge l_{-k} \\ l_{-k} - l_{-k+1} & \text{if} \quad l_{-k} > x(n) \end{cases} \quad (4)$$

for $x(n) < 0$ and $1 \le k \le L$, and where $l_{-L} = -\infty$. Further, the threshold values are confined as

$$-\infty = l_{-L} < \quad \dots \quad < l_{-1} < l_1 < \quad \dots \quad < l_L = \infty \quad (5)$$

2.2 Volterra Filters

From the Fig.1 we can see that the signals $x_k(n)$ ($k = \pm 1, \ \pm 2, \ \dots \ L$) obtained by the BTD are fed into the VF_k. The mathematical model of the VF_k of the Q-th order memory of which is N samples length ($VF_k(Q,N)$) is given by

$$y_k(n) = \sum_{j=1}^{Q} \sum_{p_1=0}^{N-1} \dots \sum_{p_j=p_{j-1}}^{N-1} h_{jp_1p_2\dots p_j}(k) x_k(n-p_1) x_k(n-p_2) \dots x_k(n-p_j) \quad (6)$$

where $x_k(n)$ and $y_k(n)$ are the input signal and the output of the VF_k, respectively. The right side of (6) is called the truncated Volterra series. The sequence $h_{jp_1p_2\dots p_j}(k)$ is called the Volterra kernel of the j-th order.

Let us define the Volterra kernel coefficient vector $\mathbf{H}_k$ containing all coefficients of the VF_k given by (6) and the vector $\mathbf{X}_k(n)$ containing the input signal samples $x_k(n)$ as well as their products used in (6). Assume also that elements of $\mathbf{H}_k$ and $\mathbf{X}_k(n)$ are arranged in lexicografical ordering. Then by using the vectors $\mathbf{H}_k$ and $\mathbf{X}_k(n)$ the expression (6) can be rewritten in these forms

$$y_k(n) = \mathbf{H}_k^T \mathbf{X}_k(n) = \mathbf{X}_k^T(n)\mathbf{H}_k \ . \quad (7)$$

It follows straight-lined from the expression (6) that the first order VFs represent the well-known nonrecursive WFs. Besides, it is evident from (6) or (7) that the output of the VF_k is linear function with respect to the elements of the Volterra kernels.

2.3 Microstatistic Volterra Filter Output

The output of the MVF given in the Fig.1 is represented by the sum of h_0 and the outputs of all VF_k. Then, the output of the MVF can be expressed as

$$y(n) = h_0 + \sum_{k=1}^{L} [y_k(n) + y_{-k}(n)] \quad (8)$$

Now, let us define the MVF coefficient vector $\mathbf{H}$ containing constant term h_0 and all vectors $\mathbf{H}_k$ and the vector $\mathbf{X}(n)$ containing number 1 and all vectors $\mathbf{X}_k(n)$ as follows

$$\mathbf{H} = [h_0 \ \ \mathbf{H}_L^T \ \ \mathbf{H}_{L-1}^T \ldots \mathbf{H}_1^T \ \ \mathbf{H}_{-1}^T \ldots \mathbf{H}_{-L+1}^T \ \ \mathbf{H}_{-L}^T]^T \quad (9)$$

$$\mathbf{X}(n) = [1 \ \ \mathbf{X}_L^T(n) \ \ \mathbf{X}_{L-1}^T(n) \ldots \mathbf{X}_1^T \ \ \mathbf{X}_{-1}^T \ldots \mathbf{X}_{-L+1}^T \ \ \mathbf{X}_{-L}^T(n)]^T . \quad (10)$$

Then, by using the expressions (7)-(10) the output of the MVF given in the Fig.1 can be expressed in the form

$$y(n) = \mathbf{H}^T \mathbf{X}(n) . \quad (11)$$

From this expression we can see that the output of the MVF is still linear function with respect to the MVF coefficients although the MVF is a nonlinear filter. With regard to that fact the linear system theory can be used for the optimization and analysis of the MVFs. Besides, it follows from the above mentioned considerations that the WF, VF as well as CMF are specific subsets of the class of the MVF.

3. Optimum Microstatistic Volterra Filter Design

Let us assume that the input signal of the MVF $x(n)$ and a desired signal $d(n)$ are stationary random processes. Now, we want to find the coefficients of an optimum MVF which minimize the mean-squared error (MSE) between desired signal $d(n)$ and the filter output $y(n)$. Then, the optimum MVF coefficients are obtained as the solution that minimizes the cost function

$$MSE(\mathbf{H}) = E[e^2(n)] = E[(d(n) - y(n))^2] \quad (12)$$

where $E[.]$ denotes the expectation operator. Substituting (11) into (12) the $MSE(\mathbf{H})$ can be expressed in this form

$$MSE(\mathbf{H}) = E[d^2(n)] - 2\mathbf{H}^T\mathbf{P} + \mathbf{H}^T\mathbf{R}\mathbf{H} \quad (13)$$

where

$$\mathbf{P} = E[d(n)\mathbf{X}(n)] \qquad \mathbf{R} = E[\mathbf{X}^T(n)\mathbf{X}(n)] \ . \quad (14)$$

In the expressions (14), $\mathbf{P}$ is the cross-correlation vector composed of the samples of the higher order cross-correlation functions of $d(n)$ and $x_k(n)$ and $\mathbf{R}$ is the symmetric nonnegative definite correlation matrix consisting of the samples of the higher order correlation functions of the threshold signals $x_k(n)$. With regard to that facts it can be found that the function $MSE(\mathbf{H})$ given by (14) has the only one minimum which has to satisfy the condition

$$\mathbf{R}\mathbf{H}_{OPT} = \mathbf{P} \quad (15)$$

where $\mathbf{H}_{OPT}$ is the coefficient vector of the optimum MVF. It follows from the last expression that the design of the optimum MVF lays in the solution of the linear algebraical equation system (15). Under condition that the matrix $\mathbf{R}$ is a regular one the vector $\mathbf{H}_{OPT}$ can be computed by

$$\mathbf{H}_{OPT} = \mathbf{R}^{-1}\mathbf{P} \ . \quad (16)$$

When the coefficient vector of the MVF equals its optimum value $\mathbf{H}_{OPT}$, the MSE attains its minimum value, $MSE(\mathbf{H}_{OPT})$, defined by

$$\begin{aligned} MSE(\mathbf{H}_{OPT}) &= E[d^2(n)] - \mathbf{H}_{OPT}^T\mathbf{P} = \\ &= E[d^2(n)] - \mathbf{H}_{OPT}^T\mathbf{R}^{-1}\mathbf{H}_{OPT} \end{aligned} \quad (17)$$

	Threshold levels							
	l_4	l_3	l_2	l_1	l_{-1}	l_{-2}	l_{-3}	l_{-4}
the 1st experiment	∞	1.50	1.00	0.50	-0.50	-1.00	-1.50	$-\infty$
the 2nd experiment	∞	1.50	1.00	0.50	-0.50	-1.00	-1.50	$-\infty$
the 3rd experiment	∞	9.00	6.00	3.00	-3.00	-6.00	-9.00	$-\infty$

Table 1 Threshold levels used in the experiments

4. Experimental Results

In order to visualize the differences in performance for the MVF given in this paper and the WF, VF and CMF, three computer experiments were made. The desired signal and signal to be filtered were generated as stationary processes and as the performance indices a signal to noise ratio (SNR) and MSE were used. All statistical characteristics required for the solution of the various filters were estimated from the data on 1000 samples.

In all experiments the 8-level block threshold decomposer was used. Its threshold levels were set experimentaly by the such a way as to maximize the SNR. The threshold levels used in our experiments were the same for the CMF and MVF and they are given in the Table 1. In the case of the VF and MVF the second-order Volterra filters were used in all experiments.

In the first and second experiment the desired signal consisted of the six sine waves with frequencies 0.113, 0.131, 0.156, 0.163, 0.188 and 0.213 of sampling frequency. The amplitudes of the sine waves were 1.0, 2.8, 0.7, 1.0, 0.95 and 1. The signal to be filtered was obtained by the same way as the desired one but the amplitudes and frequencies of its sine waves were corrupted by noise. In the first experiment the SNR and MSE before filtering were -2.2dB and 10.25, respectively. In the case of the second experiment it was 5dB and 1.92. The dependences of the SNR and MSE on the filter memory for the WF, VF, CMF and MVF are given in the Fig.2 - Fig.5. From these figures we can see that the MVF can outperform the other tested filters.

In the third experiment the desired signal $d(n)$ was generated by using a nonlinear Volterra-like MA model.Then, the desired signal generation can be described by

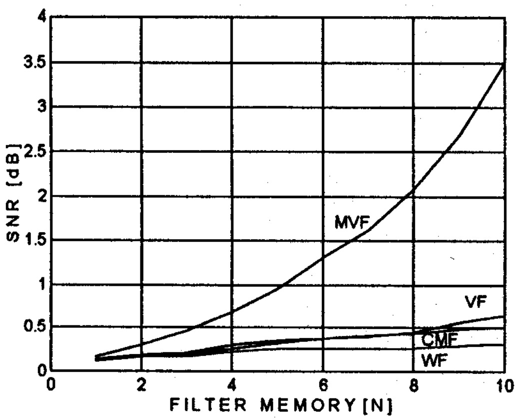

Fig.2 The 1st experiment. Performance index SNR versus filter memory.

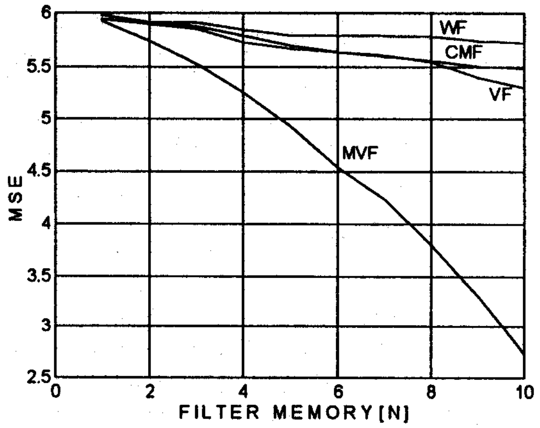

Fig.3 The 1st experiment. Performance index MSE versus filter memory.

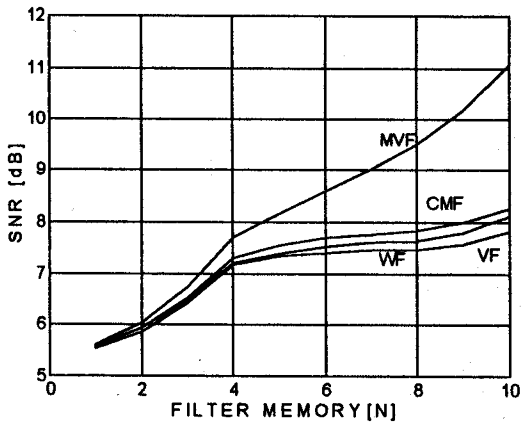

Fig.4 The 2nd experiment. Performance index SNR versus filter memory.

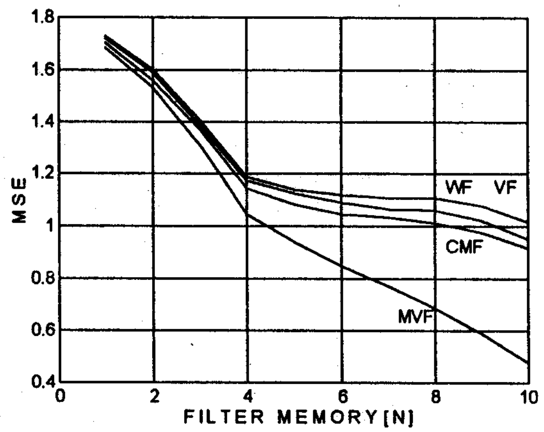

Fig.5 The 2nd experiment. Performance index MSE versus filter memory.

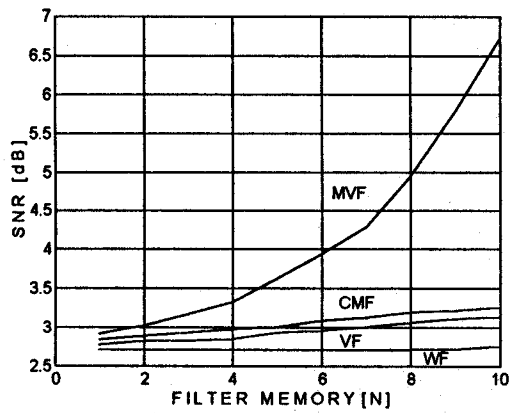

Fig.6. The 3rd experiment. Performance index SNR versus filter memory.

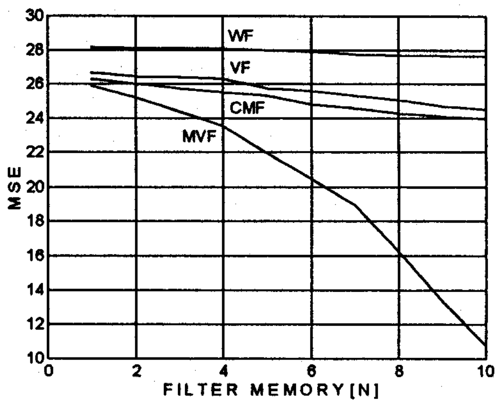

Fig.7. The 3rd experiment. Performance index MSE versus filter memory.

$$d(n) = \sum_{i=0}^{4} g_{1_i} w(n-i) + \sum_{i=0}^{4}\sum_{j=0}^{4} g_{2_{ij}} w(n-i)w(n-j) ,$$

where $w(n)$ is a white zero-mean and pseudorandom signal with the gaussian distribution and the coefficients g_{1_i} and $g_{2_{ij}}$ are defined as follows

$$[g_{1_0}, g_{1_1}, g_{1_2}, g_{1_3}, g_{1_4}]^T = [0.1, -1.5, 0.5, 0.2, 0.3]^T ,$$

$$\begin{bmatrix} g_{2_{00}} & g_{2_{01}} & g_{2_{02}} & g_{2_{03}} & g_{2_{04}} \\ g_{2_{10}} & g_{2_{11}} & g_{2_{12}} & g_{2_{13}} & g_{2_{14}} \\ g_{2_{20}} & g_{2_{21}} & g_{2_{22}} & g_{2_{23}} & g_{2_{24}} \\ g_{2_{30}} & g_{2_{31}} & g_{2_{32}} & g_{2_{33}} & g_{2_{34}} \\ g_{2_{40}} & g_{2_{41}} & g_{2_{42}} & g_{2_{43}} & g_{2_{44}} \end{bmatrix} =$$

$$= \begin{bmatrix} -1.6843 & 1.7896 & 1.4848 & 1.7578 & -0.4474 \\ -0.0528 & 0.3108 & 0.0968 & -0.4900 & 0.1324 \\ -0.4548 & 1.4167 & 1.1610 & 1.3724 & -0.1093 \\ -1.4258 & -0.3957 & -2.4103 & -0.6508 & 1.5108 \\ 0.7436 & -2.1980 & 1.1449 & 1.4343 & -1.1135 \end{bmatrix} .$$

A filtered signal was obtained corrupting the desired signal with additive Gaussian noise. The SNR and MSE before filtering were -0.7dB and 59.20, respectively.

The dependences of the choosen performance indices on the filter memory for the WF, VF, CMF and MVF are depicted on the Fig.6 and Fig.7. From these figures we can see that also in this case the MVF outperform the other tested filters.

5. Conclusion

In this paper a new subset of the microstatistic filter so-called MVFs have been proposed. This new class of nonlinear filters is based on the idea of the CMF generalization by substituting WFs applied in the CMF structure by the VFs. The advantage of the MVF in comparison with the WF, VF and CMF is the fact that in the case of non-Gaussian signal processing the MVF can outperform WF, VF or CMF. On the other hand, the disadvantage of the MVFs is their substantially higher computational complexity. With regard to the two very important facts it can be said that the MVF can be applied with advantage in this field of signal estimation where the applications of the WF, VF or CMV cannot provide the desired quality of signal processing.

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