THE COMPUTATION OF FORWARD SCATTERED FUNCTIONS FOR PRUPPACHER - PITTER RAINDROP FORMS AT 37 GHZ

Lubor BAŽANT
Dept. of Electromagnetic Field
Czech Technical University of Prague
Technická 2, 166 27 Prague
Czech Republic
e-mail: bazant@cvut.feld.cz

Abstract

The amplitude scattered functions were computed for Pruppacher - Pitter raindrop forms at 37GHz by using the Multiple MultiPole numerical method. Before doing that the measurement of the complex permittivity of rain water had been performed at the same frequency and rain water complex permittivity values had been determined.

Keywords

rain, attenuation, scattered function, permittivity

1. Introduction

When the microwave terrestrial link is suggested the attempt is to guarantee definite signal level at the receiving antenna with certain level of the radiated power at the transmitting antenna. From this reason the influence of a medium is investigated where a signal is transferred. One of the most important factors which generally causes signal quality decreasing at 37 GHz frequency range is rain.

In this field there are several papers discussing the rain influence on the differential attenuation, the differential phase shift and the cross polarization factors [1], [2], [3], [4], [5]. Although these autors′ published results don't precisely answer to the results of practise measurements. That may be from these reasons: the first one - the shape of raindrops - most authors use spherical or spheroidal raindrop shapes, the second one - the dielectric properties of rain water - there is a nonunicity at published expressions and values of the complex permittivity of water [6], [7], [8], and above all these published values of the complex permittivity aren't values of true rain water.

The new computer simulations have been started at 37 GHz from these reasons and mainly because of result verifying of the short distance (60 metres) experimental measurement of the rain attenuation and the cross polarization [9] performed at the same frequency at our department.

The main aim of this research is to get precise value of the rain water complex permittivity and the values of the amplitude forward scattering functions at 37 GHz together with the determination of the differential attenuation, the differential phase shift and the cross polarization factors (according to known relationships already) at future.

2. The Waveguide Measurement of the Complex Permittivity of Rain Water

To obtain the values of the relative complex permittivity of rain water the waveguide measurement of the relative complex permittivity of rain water was performed with the measure equipment as shown in Fig. 1.

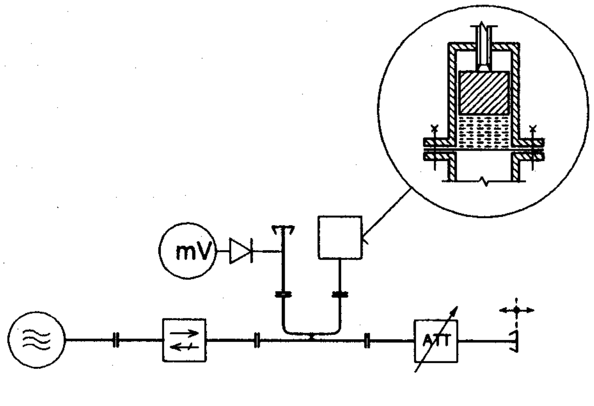

Fig. 1 The arrangement of the measure equipment

The principle of this measurement is identical with the waveguide bridge measurement of the solid substances complex permittivity, but magic T is substituted by the 3dB symmetrical directional coupler. The method of the waveguide bridge measurement is based on the input impedance determination, what is determined from the phase value and the standing wave amplitude. By means of these quantities the relative complex permittivity is expressed as

$$\varepsilon' = \frac{\lambda_g^2}{\lambda_g^2 + \lambda_m^2} \cdot \left[\left(\frac{\lambda_m}{2\pi} \right)^2 \cdot \left(\beta^2 - \alpha^2 \right) + 1 \right] \quad (1)$$

$$\varepsilon'' = \frac{2\lambda_g^2}{\lambda_g^2 + \lambda_m^2} \cdot \left(\frac{\lambda_m}{2\pi} \right)^2 \cdot \alpha \cdot \beta \quad , \quad (2)$$

where λ_g the wavelength in the waveguide, λ_c is the wavelength cut-off of the certain waveguide, α, β are the constants of the attenuation and the phase shift. The quantities of α, β may be obtained from the next expression describing the relationship between α, β and λ_g, r - standing wave ratio, l_0 - the position of the standing wave first minimum from measured substance and d - the thickness of the measured sample.

$$\frac{tgh[(\alpha + j\beta) \cdot d]}{[(\alpha + j\beta) \cdot d]} = = -j \cdot \frac{\lambda_g}{2\pi \cdot d} \cdot \frac{1 - jr \cdot tg\left(\frac{2\pi}{\lambda_g} \cdot l_0 \right)}{r - j \cdot tg\left(\frac{2\pi}{\lambda_g} \cdot l_0 \right)} \quad (3)$$

The full description of this method and detailed derivation are described in [10]. Considering the function $\frac{tgh(z)}{z}$ on the left side of eq. (3), where z is complex value, isn't simple function, the certain function value may be obtained for infinitesimal number of points. The correct root of the transcendental equation solution is chosen from the repeated occurrence of same value in roots sets computed for the different thickness of the measured samples. The measurement had been performed for eight samples of rain water and for the same number of the distilled water samples. The measured data had been put into PERM program, which has computed possible roots of equation (3). The resultant value of the relative complex permittivity of rain water was evaluated as ε_r = 31.32 - 36.76j under 20 °C. These computed results agree fairly well with Ray expression, somewhat little with Saxton - Lane expression. An inaccuracy of the results, what hadn't exceed 5 % of the average value of the relative complex permittivity of rain water and may be consider as negligible for next using, had been obviously caused by the water absorption along the waveguide side and the side of the piston of the sliding short.

3. Raindrop Diffraction, the Scattered Function

When the rain influence on the signal transfer is investigated, it's necessary to solve the elementary problem of the electromagnetic wave diffraction by single raindrop [11] at first. The elementary diffraction is illustrated in Fig. 2.

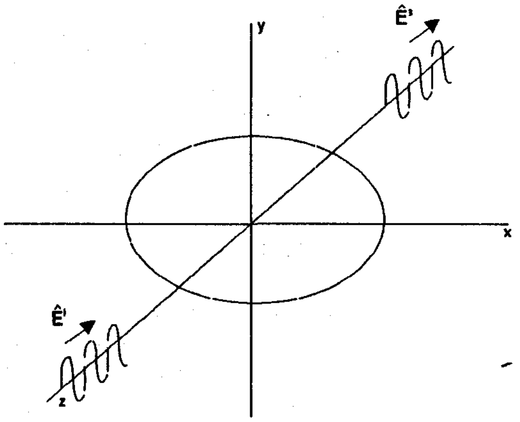

Fig. 2 The orientation of the diffraction problem for calculation of the scattered functions

The electric field of a unit plane wave is impressing on the raindrop, it's described by

$$\hat{\vec{E}}^i = \vec{e} \cdot e^{(-jk_0 r \vec{K}_1 \cdot \vec{K}_2)} \quad . \quad (4)$$

The electric field of a scattered wave, in the far field region, is then written as

$$\hat{\vec{E}}^s = \hat{f}(\vec{K}_1, \vec{K}_2) \cdot r^{-1} \cdot e^{(-jk_0 r)} \quad , \quad (5)$$

where $\vec{e}$ is a unit vector specifying the polarization state of the incident wave, $\vec{K}_2$ is a unit vector directed from the origin to the observation point, $\vec{K}_1$ is a unit vector directed toward the propagation direction of the incident wave, k_0 is the free - space propagation constant, r is distance from the origin to the observation point and $\hat{f}(\vec{K}_1, \vec{K}_2)$ is the amplitude scattered function. $\hat{\vec{E}}^s$ may be obtained from the solution of the boundary - value problem on the surface of a raindrop. Because of using of nonaxisymetrical forms of the Pruppacher - Pitter raindrops here isn't possible to use the precise analytic solution and the numerical method must be used for the computation of the scattered field.

4. Multiple MultiPole Method

To solve diffraction problem the Multiple MultiPole method have been chosen here. The method description, the application and the software support are given at next chapters.

4.1 The Principle of MMP Method

The numerical method of the multiple multipole is based on the multiple multipoles [12], what are the linear combinations of the elementary solutions (thin wire expansion, spherical, cylindrical, rectangular and cylindrical waveguide solutions of the Helmholtz equation). Every elementary solution, according to [13], may be described by series of the finite set of basis or expansion functions f_i multiplied by coefficients c_i (6).

$$f = \sum_i c_i \cdot f_i \tag{6}$$

The solution of arbitrary problem consists in determination of unknown coefficients of single multipoles, where these coefficients are determined from the comparison of the inner and outer electromagnetic field solutions at the boundary surface points.

4.2 The MMP Method Application on the Raindrop Diffraction Problem

In the raindrop diffraction case the solution of the electromagnetic field out of raindrop is written as

$$v = v_0 + v_1 \ , \tag{7}$$

where v_0 is potential of electric field of incidenting plane wave, v_1 is potential of the inhomogenity in the plane wave propagation caused by raindrop (the scattered field). Its expression is consecutive

$$v_1^{s(c)} = \sum_i \left(\sum_m \sum_n h_n^{(2)}(kr) \cdot P_n^m(\cos\theta) \cdot \begin{matrix}\sin\\ \cos\end{matrix}\, m\varphi \right) . \tag{8}$$

That's the linear combination of the independent solutions of the outside problem of the Helmholtz equation in spherical coordinates (r,θ,φ). $h_n^{(2)}(kr)$ is the modificated Henkel function of the second kind and of the n-th order, $P_n^m(\cos\theta)$ is the associated Legendre function of the n-th order and of the m-th degree. The solution of the potential of electric field inside a raindrop is then given by the linear combination of the independent solutions of the inside problem of the Helmholtz equation in spherical coordinates (r,θ,φ)

$$v_2^{s(c)} = \sum_i \left(\sum_m \sum_n j_n(kr) \cdot P_n^m(\cos\theta) \cdot \begin{matrix}\sin\\ \cos\end{matrix}\, m\varphi \right) , \tag{9}$$

where $j_n(kr)$ is the modificated Bessel function of the n-th order. Further, the boundary conditions must be performed at every point of the raindrop surface. The electric potential must be continuous on a spherical area

$$v_0 + v_1 = v_2 \tag{10}$$

and the flux of the electric induction must be continuous on a spherical area

$$\varepsilon_0 \cdot \left(\frac{\partial v_0}{\partial r} + \frac{\partial v_1}{\partial r} \right) = \varepsilon_0 \cdot \varepsilon_{r(water)} \cdot \frac{\partial v_2}{\partial r} \ . \tag{11}$$

Generally, every point of the surface is represented by elementary orientated area and consequently it's determined by its origin, one normal E_n component and two tangential E_t, E_t' components of the electric field. After the introducing to the boundary conditions (10), (11), these equations are obtained for every point of surface

$$E_{0t} + E_{1t} = E_{2t} \tag{12}$$

$$E'_{0t} + E'_{1t} = E'_{2t} \tag{13}$$

$$\varepsilon_0 \cdot \left(E_{0n} + E_{1n} \right) = \varepsilon_0 \cdot \varepsilon_{r(water)} \cdot E_{2n} \tag{14}$$

and similar three equations are possible to get for the magnetic field. That's leading to the thin matrix of the coefficient, what's written in (15).

$$\left[C_{imn}^{c(s)E(H)} \right] \cdot \left[f_{imn}^{c(s)E(H)} \right] = 0 \tag{15}$$

$C_{imn}^{c(s),E(H)}$ is thin coefficients matrix, $f_{imn}^{c(s)E(H)}$ is the matrix of the eigenfunctions, i is number of the expansions, m is the degree, n is the order, $c(s)$ means the cosinus (sinus) in the single members of the summa of solution (8), (9), $E(H)$ means the solution of the electric or the magnetic field.

4.3 MMP - Modelling

The whole problem have been solved by means of the 3D Electrodynamic Wave Simulator - program [14], what had been developed by Ch. Hafner's team at ETHZ Zürich and what is based on multiple multipole method. The 3D shape of raindrops was generated by the program, the position and number of multipoles were generated manually and consequently tested by the program from the reason of the multipole independence checking. There is valid for multipole generating, generally, the more independent multipoles and higher order and degree of multipoles are used, the more exact results are obtained. Input files have been created for raindrops of the diameter from 0.5 to 7.5mm with 0.5 mm step. One of the raindrop models, together with the location of multipoles is shown in Fig. 3.

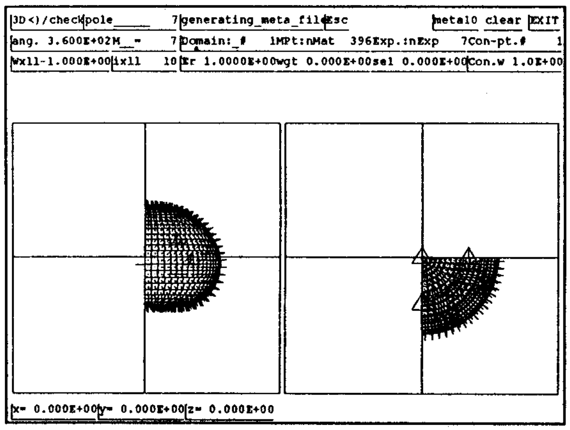

Fig. 3 The model of the raindrop of 5.5mm diameter with multipole location

5. Raindrop Scattering Results

To obtain the scattered functions of single raindrops, the electromagnetic scattered far field must be determined. Further, according to eq. (5), the scattered functions may be computed. For illustration, the time average values of the scattered E-field in near region of the raindrop of 5.5 mm diameter is shown in Fig. 4, when the vertical polarization of E-field of the incident 37GHz wave is used.

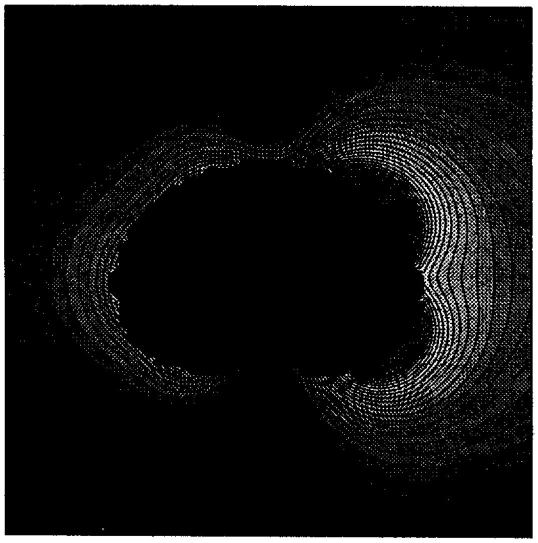

Fig. 4 The time average values of the scattered E-field of the raindrop of 5.5 mm diameter at 37 GHz. The plane wave of vertical E-field polarization is incidenting from left side. The relative complex permittivity of rain water is 31.32 - 36.76j.

The time average values of the total E-field of the same raindrop are shown for comparison in Fig. 5 and the total E-field in time depending domain in Fig. 6. The plane wave is incidenting from left side in all cases.

The forward scattered functions of Pruppacher - Pitter form raindrops were computed for the vertical polarization of incidental E - field at 37 GHz. The result values are shown in Tab.1 for the raindrops of the effective radius from 0.25 mm to 7.5 mm and of computed

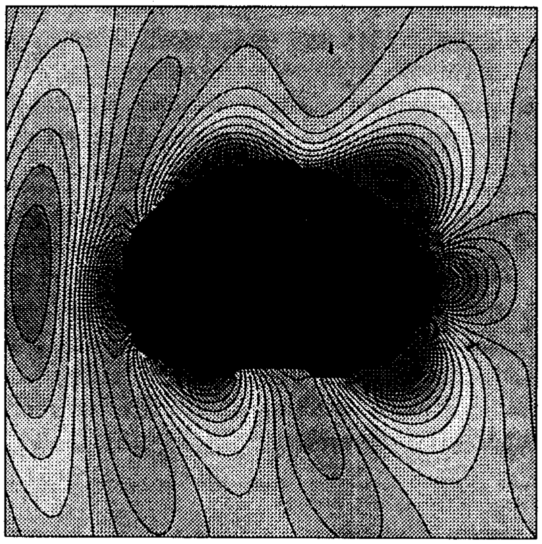

Fig. 5. The time average values of the total E-field of the raindrop of 5.5 mm diameter at 37 GHz. The plane wave of vertical E-field polarization is incidenting from left side. The relative complex permittivity of rain water is 31.32 - 36.76j.

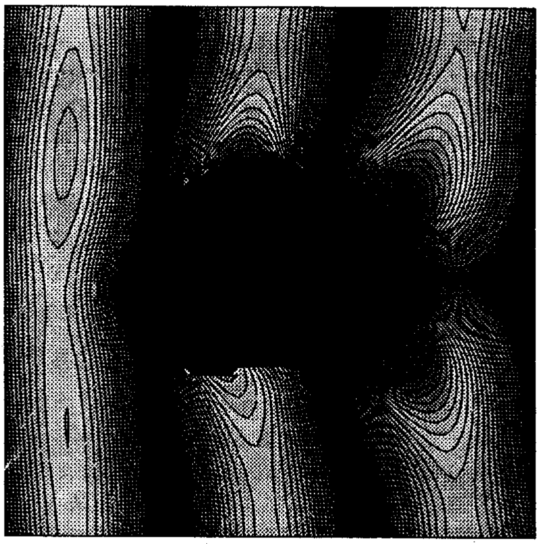

Fig. 6 The time depending values of the total E-field of the raindrop of 5.5 mm diameter at 37 GHz. The plane wave of vertical E-field polarization is incidenting from left side. The relative complex permittivity of rain water is 31.32 - 36.76j.

relative complex permittivity of the rain water 31.32 - 36.76j.

Table 1. Computed forward scattered functions of Pruppacher - Pitter form raindrops at 37 GHz.

Effective raindrop radius (mm)	Re $(f) \times 10^3$	Im $(f) \times 10^3$
0.25	0.00034574	-0.00052959
0.50	0.0013741	0.021577
0.75	0.057925	0.061656
1.00	0.12992	0.10593
1.25	0.20548	0.18982
1.50	0.24883	0.32831
1.75	0.20947	0.56908
2.00	0.06930	0.97990
2.25	0.10460	-1.58291
2.50	0.24225	-2.28584
2.75	0.37727	-2.95826
3.00	0.49904	-3.54995
3.25	0.80884	-4.15776
3.50	0.97481	-4.69716

References

[1] OGUCHI, T.: Attenuation and Phase Rotation of Radio Waves Due to the Rain: Calculation at 19.3 and 34.8 GHz, Radio Sci., 8(1), 1973, pp. 31-38.

[2] OGUCHI, T.-HOSOYA, Y.: Scattering Properties of Oblate Raindrops and Cross Polarization of Radio Waves Due to Rain, 2, Calculation at Microwave and Millimeter Wave Region, J. Radio Res. Lab. (Japan), 21(105), 1974, pp.191-259.

[3] MORRISON, J. A.-CROSS, M. J.: Scattering of a Plane Electromagnetic Wave by Axisymmetric Raindrops, Bell Syst. Tech. J., 53(6), 1974, pp. 955-1019.

[4] CHU, T. S.: Rain Induced Cross Polarization at Centimeter and Millimeter Wavelengths, Bell Syst. Tech. J., 53(8), 1974, pp. 1557-1579.

[5] MORGAN, M. A.: Finite Element Computation of Microwave Scattering by Raindrops, Radio Sci., 15(6), 1980, pp. 1109-1119.

[6] SAXTON, J. A.-LANE J. A.: Electrical Properties of Sea Water, Wireless Eng., 29(349), 1952, pp. 269-275.

[7] RAY P. S.: Broadband Complex Refractive Indices of Ice and Water, Applied optics, 11(8), 1972, pp 1836.

[8] KLAIN, L. A.-SWIFT, C. T.: Improved Model for the Dielectric Constant of Sea Water at Microwave Frequencies, IEEE Trans, AP-25, 1, 1977, pp. 104-111

[9] BÁRTÍK, H.- MAZÁNEK, M.: The 37 GHz Short Distance Experiment, Proceeding of Workshop 96, II, (Brno) 1996, pp. 763-764.

[10] LIEDERMANN, K.-PAVLOVIČ, P.- SVAČINA, J.: Numerické vyhodnocení vlnovodných měření vlastností dielektrik, Slaboproudý obzor, 51(5), 1990, pp. 194-199

[11] FIŠER, O.: Interakce elektromagnetického pole s dešťovým prostředím se zřetelem na predikci útlumu deštěm družicových spojů (Kandidátská disertační práce), Czech Technical University, Prague, 1986.

[12] STRATTON, J., A.: Electromagnetic Theory, New York, McGraw-Hill Comp., 1941.

[13] HAFNER, CH.: On the Design of Numerical Methods, IEEE Ant. Prop. Mag., 35(4), 1993, pp. 13-20.

[14] HAFNER, CH.-BOLMHOLT, L.: The 3D Electrodynamic Wave Simulator, New York, John Wiley and Sons, 1993.

Acknowledgements. The author wish to thank Dr. Ch. V. Hafner from ETHZ Zürich for his help with the 3D Electrodynamic Wave Simulator - program and consultations of MMP theory. This research have been supported by the GAČR grant No. 102 - 93 - 0937.

About authors...

Lubor BAŽANT was born in Pardubice, Czech Republic, in 1969. He received the MSc. degree in electrical engineering from CTU Prague, in 1992. At present, he is PhD student at CTU Prague, Dept. of Electromagnetic Field. His research interests include the theory and computer simulation of wave propagation at millimeter wave region.