DESIGN OF IIR FILTERS BY IMPULSE RESPONSE OPTIMISATION

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Abstract

The contribution describes a program system to design digital IIR filters by optimisation of impulse response. Besides briefly presenting the already published method, it emphasizes the necessity to regularize the problem by changing the requested impulse response into a causal one by reshuffling its values. The program enables to design the filters starting from either frequency or time domain response and also from a manually prescribed pole configuration which is particularly useful for teaching purposes. *)

Keywords:

IIR digital filters, optimisation, filter design, polynomial equations

1. Introduction

The design of IIR digital filters is usually based on the well established theory of approximations as used in continuous-time system design, the most common method being the bilinear transform directly converting the analogue filters into similar digital ones [1,2]. Nevertheless, the nowadays available computing power enables to design the digital filters independently on any continuous-time patterns, by means of optimization methods. The most straightforward method is based on least-square optimisation of frequency characteristic where the criterium of optimality is a sum of squared differences between samples of the desirable and the obtained characteristics. Unfortunately, the methods leads to a system of nonlinear equations that is difficult to solve. An alternative approach consists in minimizing the differences between the given and realised impulse responses as described in [1]. On the cost of limiting the designed systems to purely recursive (AR) filters, it leads to a system of linear equations the solution of which does not make any problem. The only real (and probably not discussed yet) problem is how to reshuffle the filter coefficients in order to obtain the best possible filter parameters with a given filter order.

2. Design method

The transfer function of the filter to be designed is supposed to be purely recursive,

$$H(z) = \frac{b_0}{1 - a_1.z^{-1} - a_2.z^{-2} - a_3.z^{-3} - \ldots - a_N.z^{-N}} \,. \quad (1)$$

In order to determine the filter coefficients a_i, let us cascade the filter with desirable transfer function $H_d(z)$ and a system described by 1/H(z) where H(z) is the transfer function of the designed filter. It is obvious that if H(z) were designed optimally, the cascade would be a unity transfer block, transforming the unit impulse u(n) into the same u(n). In reality, H(z) will differ from its ideal pattern and consequently the response of the cascade to u(n) will be a different signal v(n). With respect to (1), obviously the Z-transform of v(n) is

$$V(z) = H_d(z).\frac{1 - \sum_{k=1}^{N} a_k.z^{-k}}{b_0} \quad (2)$$

which after inverse Z-transform gives

$$b_0.v(n) = h_d(n) - \sum_{r=1}^{N} a_r.h_d(n-r) \,. \quad (3)$$

Imposing the obvious constraint v(0)=1 so that $b_0=h_d(0)$ where $h_d(n)=Z^{-1}\{H_d(z)\}$, the remaining terms of the v-sequence should approximate zero and the optimality criterium

$$E = \sum_{n=1}^{\infty} v^2(n) \quad (4)$$

is to be minimised in the N-dimensional space of filter coefficients a_i, $i \in \langle 1,N \rangle$. Substituting for v(n) from (3) we obtain

$$E = \frac{1}{b_0^2}.\sum_{n=1}^{\infty} [h_d(n) - \sum_{r=1}^{N} a_r.h_d(n-r)]^2 \quad (5)$$

which, after partial differentiation with respect to a_i and setting to zero leads to

*) The program is a result of an MSc diploma project the supervisor of which was the second author.

$$\sum_{r=1}^{N} [a_r . \sum_{n=1}^{\infty} h_d(n-r).h_d(n-i)] = \sum_{n=1}^{\infty} h_d(n).h_d(n-i) \quad (6)$$

which represents a linear system of N equations for a_i, $i \in <1,N>$. The system coefficients are clearly based only on the known values of the desired impulse response but to become computable, the infinite sums must be approximated by the sums of finite number L of terms. This is also necessary from the point of view of the filter specification which can only be given by a finite number of $h_d(n)$ values, say for $n \in <0,L>$. Cutting the sums this way should not influence the results substantially as thanks to the (also required) stability of the desirable filter the error sum must be finite (and small in practical cases).

When starting from a desired frequency characteristic, the vector of desirable impulse response values must be first calculated by DFT of the sampled frequency characteristic. As the phase part of this characteristic is usually supposed to be zero, the obtained impulse response is non-causal. It is then necessary to "causalise" it by inserting a suitable delay as the most straightforward way of simply using the periodicity of DFT result and taking the values of h_d starting from t=0 leads to high response values at late time (for high n) following almost zero values in h_d. This cannot be reasonably approximated by a low order filter which would lead to a poor design.

It should be noted here that (though it is not directly connected with the design) a substantial part of the program is devoted to computation of the pole locations of the designed system on the basis of H(z), i.e. as dependent on the vector $[a_i]$. Finding poles as complex roots of a higher order polynomial turned out to be a difficult task as even the highest precision of the number representation in the used computer was insufficient. The Graff's method [3] was used recursively in order to obtain as many roots as possible (finding all roots cannot be generally guaranteed).

3. Program features

The program should serve twofold purpose: primarily it should provide a means to design AR filters in practical cases, on the other hand it should serve also as an educational tool for teaching the basics in digital signal processing. Especially with respect to the other goal, there is also a possibility to input pole locations manually and from these data to compute the transfer function, frequency characteristic and filter coefficients. This should give students a feeling for the relation between pole configuration and filter behaviour.

The program has been worked out in Borland-Pascal environment using object approach and graphical user interface based on hierarchical menu. The basic functions of the program are as follows:

1. Filter design based on desirable frequency characteristic,
2. Filter design based on desirable impulse response,
3. "Causalisation" of the desirable impulse response (see par. 2),
4. Filter gain adjustment,
5. Computation of pole locations of a given or designed filter,
6. Filter design based on given pole locations.

For any given or designed filter, the frequency characteristics and impulse responses can be calculated and plotted; a comparison of different versions (e.g. desired and designed) are possible.

An example of the filter design as enabled by the program is documented by figs.1-4.

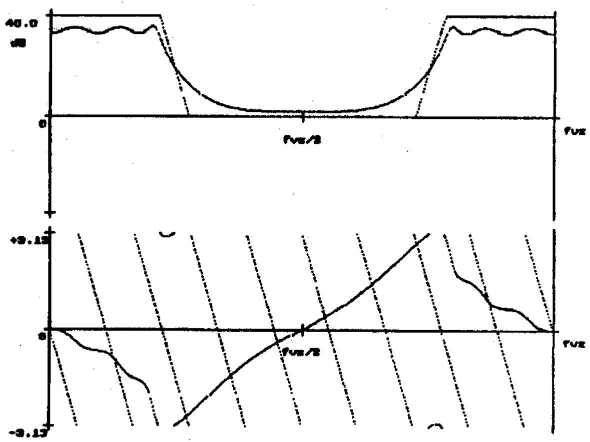

Fig.1 Desirable (piecewice linear) and resulting frequency characteristics (upper part - gain, lower part - phase)

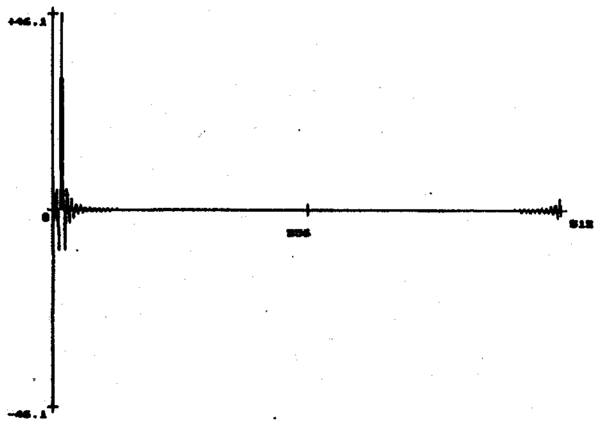

Fig.2 Desirable impulse response as calculated by DFT and "causalised" by an experimentally chosen delay

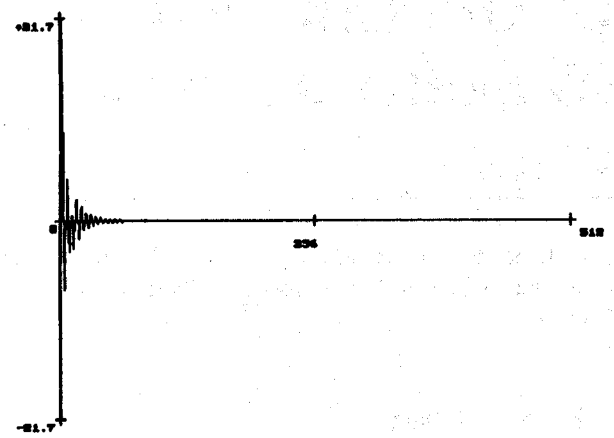

Fig.3 Impulse response of the designed filter

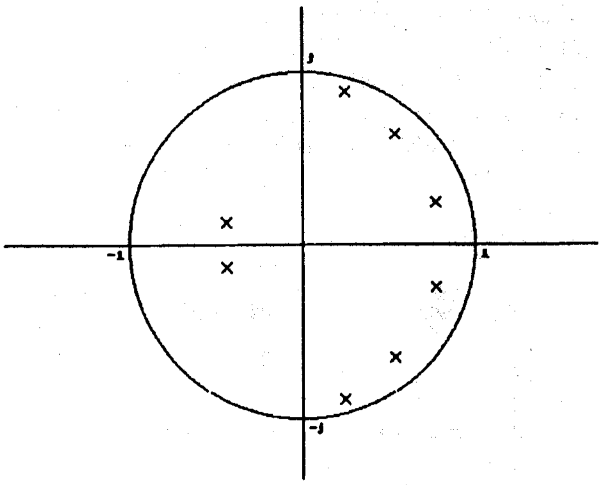

Fig.4 Computed pole configuration of the designed filter

4. Conclusions

Practical realization of the method to design IIR filters on the basis of the desired impulse response proved its feasibility. It turned out that when starting from a desired frequency characteristic, the impulse response obtained by DFT must be modified in order to cumulate the high response values in the neighbourhood of its beginning, else the designed filter properties are poor.

The program resulting from the project enables both the IIR AR filter design according to a given time- or frequency- domain response and interactive design based on intuitive pole localization as needed for educational purposes.

5. References

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