BROADBAND UTILIZATION OF EIGENVALUE TECHNIQUES

RNDr. František NEBUS
Prof. Ing. Zdeněk KŮS, CSc.
Department of Radioelectronics
Military Academy of SNP
031 19 Liptovský Mikuláš
Slovak Republic

Abstract

A large number of signal processing problems are concerned with estimating unknown parameters from sensor array measurement. This paper presents simple algorithm for estimating spatio-temporal spectrum of signals received by passive array derivated from MUSIC algorithm, based on eigenstructure of covariance matrices of received signals. Some simulation results are presented.

Keywords:

Array processing, Eigenvalue, Broadband, Covariance

1. Introduction

Digital algorithms research for signal processing of several kind of antenna arrays (AA) concerning to radiolocation, radionavigation, underwater acoustic and geophysics, cartridge final instigation, is pointed out from eighty years due to great computer development. Thanks to this, also eigenvalue techniques (ET) application for signal analysis should become in reality [1].

Main means for all known eigenvalue algorithms for Direction Of Arrival (DOA) estimation became MUltiple SIgnal Clasification (MUSIC) idea - evaluation of covariance coefficients between AA elements [2] instead of conventional beamforming methods. From mean value in time of $\underline{\mathbf{R}}$ crosscovariance coefficients one may construct covariance matrix which is base matter for eigenvalue, eigenvector decomposition and later on estimation of DOA parameters. Restrictions for chosen eigenvalue algorithm are stationary scene during data acquisition for $\underline{\mathbf{R}}$, number of signals M less then number of AA elements K, $M \leq K-1$ and difference at least in one parameter such as elevation, azimuth, polarization, frequency between each couple of received signals.

Published results concerning to ET consider narrowband signals spreading in narrow band or wideband signals problem with Coherent Signal Subspace method or ESPRIT [3], [4]. Proposed method is simple MUSIC extension to broadband problem, however good results were reached.

2. Eigenvalue techniques

Let we have equidistance rectangular planar AA with number of elements $m \,.\, n = K$, M signal sources as mentioned above, F_i represents i -th signal complex envelope, α_i ε_i, λ_i its azimuth and elevation resp. and wave length, d distance between AA elements in line (see Figure 1). For signal with noise, received at $X_{k,l}$ element we can write

$$X_{k,l} = \sum_{i=1}^{M} F_i e^{j\left\{\frac{2\pi d}{\lambda_i}\left[(k-1)\sin(\alpha_i)+(l-1)\sin(\varepsilon_i)\right]\right\}} + W_{k,l} \qquad (1)$$

and for signal vector $\vec{X}$ from whole AA

$$\vec{X} = \underline{A} \; \vec{F} + \vec{W} \quad . \qquad (2)$$

and from time snapshots of $\vec{X}$ one may construct matrix $\underline{\mathbf{R}}$

$$\underline{\mathbf{R}} = \overline{\vec{X} \; \vec{X}^{*}} = \underline{\mathbf{A}} \; \vec{F} \; \vec{F}^{*} \, \underline{\mathbf{A}}^{*} + \underline{\mathbf{W}} \qquad (3)$$

like mean value, where $\underline{\mathbf{A}}$ is matrix reflected AA and sources composition for the time being unknown, $\vec{F}$ is signal column vector, $\vec{W}$ is noise column vector, $\underline{\mathbf{A}}^{*}$ is conjugated matrix of $\underline{\mathbf{A}}$. Eigenvalue decomposition shows up μ_i eigenvalues $i=1,\ldots,K$ (consisting of M signals an K-M noise eigenvalues) next eigenvectors and consequently through identification function I_ϕ desired signal parameters

$$I_\phi = \frac{1}{a(\phi)^{*}\, E_N \, E_N^{*}\, a(\phi)} \qquad (4)$$

with very high resolution from within interesting general angle space, where $\underline{\mathbf{E_N}}$ is noise subspace generated from noise eigenvectors and $a(\phi)$ scanning vector [4]. Since I_ϕ evaluation of matrix $\underline{\mathbf{A}}$ is known, it is possible to calculate power matrix $\underline{\mathbf{P}}$ of individual signal power spectrum and their crosscorelations.

$$\underline{P} = \left(\underline{A}^{*}\,\underline{A}\right)^{-1} \underline{A}^{*} \left(\underline{R} - \mu_{min}\,\underline{I}\right) \underline{A} \left(\underline{A}^{*}\,\underline{A}\right)^{-1} \quad (5)$$

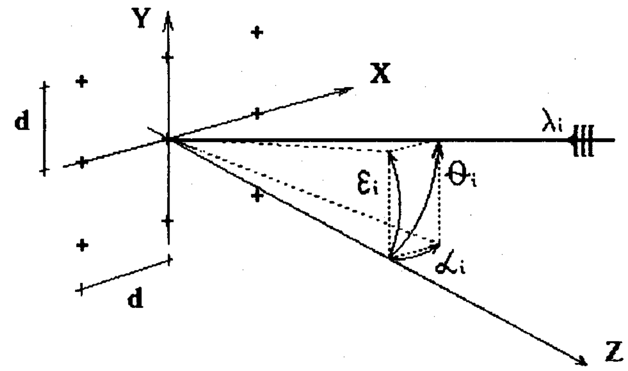

Fig.1 AA and signal composition

2.1 Broadband utilization

Mentioned conditions are for several kind systems sufficient, however especially for passive surveillance systems are not, because of very narrow frequency band. Let narrowband signals are spread within wide frequency band with centre d/λ =0,5. As matrix $\underline{A}$ contains not only all DOA parameters, but d/λ_i also, that reflects in matrix $\underline{R}$ (3). This causes two undesirable effects. Primarily, as much as signal frequency deviation from centre increases, the information validity of $\underline{R}$ deviates from truth and I_ϕ shows desired slightly shifted target and one mirror target too, in an complicated scene some interference targets (see Figure 2).

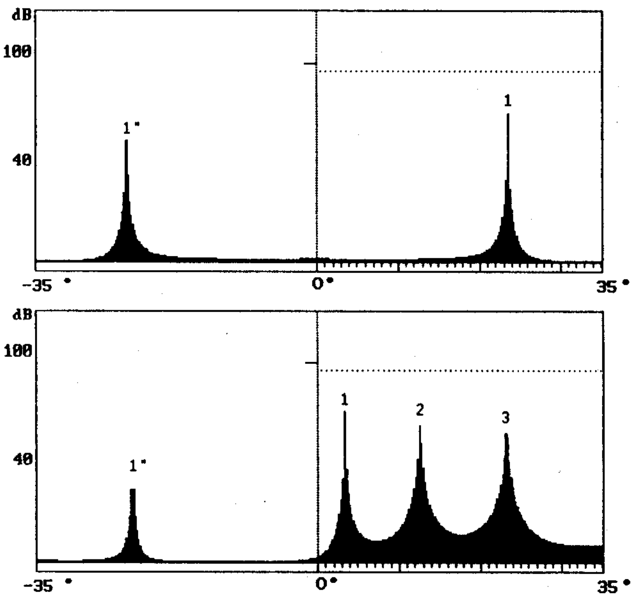

Fig. 2 False signal detection

This fact is due to complex signal data acquisition techniques, however it is possible to eliminate false targets with an appropriate signal eigenvalues estimation Γ_r criterium and is given by edge at Γ_r function graph

$$\Gamma_r = \frac{\sum_{i=1}^{r} \mu_i}{\sum_{k=1}^{K} \mu_k}, \quad r = 1,\ldots,K \quad \mu_1 \geq \mu_2 \geq \ldots \geq \mu_K . \quad (6)$$

Usually Γ_r = 0,8 is appropriate enough. Number r is the signal eigenvalue number. Secondary and more important influence has signal frequency diversification to ET accuracy, that is much more higher in comparison with conventional techniques, however more sensitive. The accuracy resolution on dependence from λ well approximates formula for azimuth plane case

$$\frac{\alpha_i - \alpha}{\alpha} = \frac{\lambda - \lambda_i}{\lambda} \quad (7)$$

where α_i, λ_i are real, α, λ are estimated parameters. This is because of $a(\phi)$ dependence on d/λ (4). One might thing that is sufficient to tune d/λ and find out the sharpest peak for I_ϕ, like is usual for ET, but it is not true. In real conditions (SNR<10), such tuning imply group of peaks acording to (7) and not a single peak (see Figure 3).

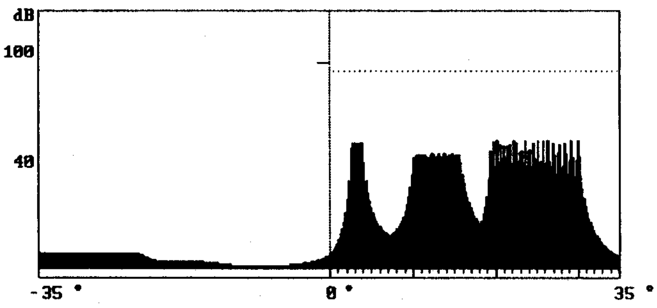

Fig.3 Peak groups due to d/λ tunning

To save very high resolution of ET in this case, it is enough to make DOA analysis only for fixed band centre d/λ =0,5 and from parallelly done frequency analysis, employed is again ET in time frequency domain with respect to condition M dim($\underline{R}$")-1, use frequency and power spectrum with crosscovariances $\underline{P}$". After comparison between $\underline{P}$" and $\underline{P}$ it is easy to estimate which target from DOA belongs to which frequency target and make corrections in DOA parameters via formula (7), or accurately with means I_ϕ evaluation. This suggestion is on Figure 4.

target density in angle and frequency domain and their crossinfluence, accuracy dependence on amount of snapshots and $\underline{\mathbf{R}}$ composition, the analysis of eigenvalue decomposition methods.

In case M = 2:

1. SNR=2,5 α=3° d/λ =0,4; 2. SNR=1,5 α=13° d/λ=0,47; five elements AA, 300 snapshots

estimated values:

1. SNR=2,38 ± 0,41 SNR"=2,42 ± 0,29 α=2,82° ± 0,82 d/λ=0,400 ± 0,079
2. SNR=1,71 ± 0,35 SNR"=1,58 ± 0,28 α=12,75° ± 0,98 d/λ=0,472 ± 0,043

In case M = 3:

1. SNR=1,5 α=3 d/λ=0,27; 2. SNR=1 α=13° d/λ=0,5;
3. SNR=0,5 α=27° d/λ=0,6; nine elements AA, 300 snapshots

estimated values

1. SNR=1,45 ± 0,09 SNR"=1,32 ± 0,10 α=2,89° ± 0,19 d/λ=0,272 ± 0,003
2. SNR=0,99 ± 0,04 SNR"=0,92 ± 0,08 α=13,09° ± 0,24 d/λ=0,503 ± 0,016
3. SNR=0,48 ± 0,03 SNR"=0,48 ± 0,07 α=25,13° ± 0,24 d/λ=0,596 ± 0,015

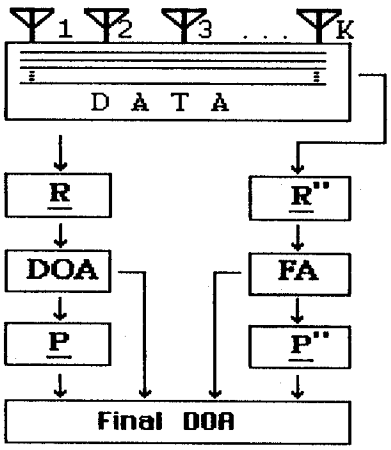

Fig.4 Proposal method for broadband utilization

The statistical results are from 100 runs for both cases without failure cases and for dim($\underline{\mathbf{R}}$")=9, additive noise is present at each omnidirectional sensor and is uncorelated from sensor to sensor. In 5 runs resp. 2 runs, were missed some of targets and in 9 resp 4 runs were detected false targets, however there were used very simple target detection algorithm only.

3. Conclusion

The analysis and simulations produced in our Radioelectronics Deptartment show, that broadband DOA application for MUSIC is possible without any frequency preprocessing (band splitting). It gives good results and keep great advantage of ET to „see" all interesting space at once, however disadvantage of great computational need increases twice and the exploitation of λ dimension of I_ϕ gives higher risk in DOA target loses (7). This proposal is useful for linear or planar AA, 2D scene, does not matter if narrowband or broadband, where instead of one with dim($\underline{\mathbf{R}}$)=$m.n$ is much more easier (because of eigenvalue decomposition problem), to take $\underline{\mathbf{R}}_\alpha$ and $\underline{\mathbf{R}}_\varepsilon$ of two rectangular linear AA with dim($\underline{\mathbf{R}}_\alpha$)=$m$, dim($\underline{\mathbf{R}}_\varepsilon$)=$n$ and after comparison of $\underline{\mathbf{P}}_\alpha$, $\underline{\mathbf{P}}_\varepsilon$ and $\underline{\mathbf{P}}$" respectively find out correct parameters.

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About autors

František NEBUS was born in 1958. He received the RNDr. degree in Faculty of Mathematics and Physics UK Bratislava in 1982 and finished Ph.D. studies in 1995 at Military Academy SNP Liptovský Mikuláš. Currently he is independent scientist with the department of Radioelectronics. His research intrests include digital signal processing from antena arrays.

Zdeněk KŮS was born in 1937. He received the MSc. and Ph.D. degrees in electrical engineering from Military Academy AZ in Brno in 1969 and 1980 respectively and associate professor and university profesor at High Military Technical School in Liptovský Mikuláš in 1983 and 1987 respectively. His field of expertise covers antena's signal processing and microwave technics.