BROADBAND UTILIZATION OF EIGENVALUE TECHNIQUES

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Abstract

A large number of signal processing problems are concerned with estimating unknown parameters from sensor array measurement. This paper presents simple algorithm for estimating spatio-temporal spectrum of signals received by passive array derived from MUSIC algorithm, based on eigenstructure of covariance matrices of received signals. Some simulation results are presented.

Keywords:
Array processing, Eigenvalue, Broadband, Covariance

1. Introduction

Digital algorithms research for signal processing of several kind of antenna arrays (AA) concerning to radiolocation, radionavigation, underwater acoustic and geophysics, cartridge final instigation, is pointed out from eighty years due to great computer development. Thanks to this, also eigenvalue techniques (ET) application for signal analysis should become in reality [1].

Main means for all known eigenvalue algorithms for Direction Of Arrival (DOA) estimation became MULTiple Signal Classification (MUSIC) idea - evaluation of covariance coefficients between AA elements [2] instead of conventional beamforming methods. From mean value in time of \( \mathbf{R} \) crossecovariance coefficients one may construct covariance matrix which is base matter for eigenvalue, eigenvector decomposition and later on estimation of DOA parameters. Restrictions for chosen eigenvalue algorithm are stationary scene during data acquisition for \( \mathbf{R} \), number of signals \( M \) less then number of AA elements \( K, M \leq K-1 \) and difference at least in one parameter such as elevation, azimuth, polarization, frequency between each couple of received signals.

Published results concerning to ET consider narrowband signals spreading in narrow band or wideband signals problem with Coherent Signal Subspace method or ESPRIT [3], [4]. Proposed method is simple MUSIC extension to broadband problem, however good results were reached.

2. Eigenvalue techniques

Let we have equidistance rectangular planar AA with number of elements \( m = n = K, M \) signal sources as mentioned above, \( F_i \) represents \( i \)-th signal complex envelope, \( \alpha_i, \beta_i, \lambda_i \) its azimuth and elevation resp. and wave length, \( d \) distance between AA elements in line (see Figure 1). For signal with noise, received at \( X_{ij} \) element we can write

\[
X_{ij} = \sum_{i=1}^{M} F_i e^{j[2\pi/\lambda((i-1)\sin(\alpha_i)+(i-1)\sin(\beta_i))]} + W_{ij}
\]  

(1)

and for signal vector \( \mathbf{X} \) from whole AA

\[
\mathbf{X} = \mathbf{A} \hat{\mathbf{F}} + \mathbf{W}
\]

(2)

and from time snapshots of \( \mathbf{X} \) one may construct matrix \( \mathbf{R} \)

\[
\mathbf{R} = \overline{\mathbf{X} \ X^*} = \mathbf{A} \hat{\mathbf{F}} \hat{\mathbf{F}}^* \mathbf{A}^* + \mathbf{W}
\]

(3)

like mean value, where \( \mathbf{A} \) is matrix reflected AA and sources composition for the time being unknown, \( \hat{\mathbf{F}} \) is signal column vector, \( \hat{\mathbf{W}} \) is noise column vector, \( \mathbf{A}^* \) is conjugated matrix of \( \mathbf{A} \). Eigenvalue decomposition shows up \( \mu \) eigenvalues \( i=1,...,K \) (consisting of \( M \) signals an \( K-M \) noise eigenvalues) next eigenvectors and consequently through identification function \( I_* \) desired signal parameters

\[
I_* = \frac{1}{a(\phi)^{*} E_n E_n^* a(\phi)}
\]

(4)

with very high resolution from within interesting general angle space, where \( E_n \) is noise subspace generated from noise eigenvectors and \( a(\phi) \) scanning vector [4]. Since \( I_* \) evaluation of matrix \( \mathbf{A} \) is known, it is possible to calculate power matrix \( \mathbf{P} \) of individual signal power spectrum and their crosscorrelations.
\[ P = \left( A^* A \right)^{-1} A^* \left( R - \mu_{\max} I \right) A \left( A^* A \right)^{-1} \]  \hspace{1cm} (5)

This fact is due to complex signal data acquisition techniques, however it is possible to eliminate false targets with an appropriate signal eigenvalues estimation \( \Gamma \), criterium and is given by edge at \( \Gamma \), function graph

\[ \Gamma_r = \frac{\sum_{i=1}^{r} \mu_i}{\sum_{i=1}^{K} \mu_i}, \quad r = 1, \ldots, K \quad \mu_1 \geq \mu_2 \geq \ldots \geq \mu_K. \]  \hspace{1cm} (6)

Usually \( \Gamma_r = 0.8 \) is appropriate enough. Number \( r \) is the signal eigenvalue number. Secondary and more important influence has signal frequency diversification to ET accuracy, that is much more higher in comparison with conventional techniques, however more sensitive. The accuracy resolution on dependence from \( \lambda \) well approximates formula for azimuth plane case

\[ \frac{\alpha_r - \alpha}{\alpha} = \frac{\lambda - \lambda_i}{\lambda} \]  \hspace{1cm} (7)

where \( \alpha_r, \lambda_i \) are real, \( \alpha, \lambda \) are estimated parameters. This is because of \( a(\phi) \) dependence on \( d/\lambda \). One might thing that is sufficient to tune \( d/\lambda \) and find out the sharpest peak for \( I_\phi \), like is usual for ET, but it is not true. In real conditions (SNR<10), such tuning imply group of peaks according to (7) and not a single peak (see Figure 3).

Fig. 1  AA and signal composition

2.1 Broadband utilization

Mentioned conditions are for several kind systems sufficient, however especially for passive surveillance systems are not, because of very narrow frequency band. Let narrowband signals are spread within wide frequency band with centre \( d/\lambda = 0.5 \). As matrix \( A \) contains not only all DOA parameters, but also \( d/\lambda \), also, that reflects in matrix \( R \) (3). This causes two undesirable effects. Primarily, as much as signal frequency deviation from centre increases, the information validity of \( R \) deviates from truth and \( I_\phi \) shows desired slightly shifted target and one mirror target too, in an complicated scene some interference targets (see Figure 2).

Fig. 2  False signal detection

To save very high resolution of ET in this case, it is enough to make DOA analysis only for fixed band centre \( d/\lambda = 0.5 \) and from parallelly done frequency analysis, employed is again ET in time frequency domain with respect to condition \( M \dim(R^{-1}) - 1 \), use frequency and power spectrum with crosscovariances \( P^\prime \). After comparison between \( P^\prime \) and \( P \) it is easy to estimate which target from DOA belongs to which frequency target and make corrections in DOA parameters via formula (7), or accurately with means \( I_\phi \) evaluation. This suggestion is on Figure 4.
target density in angle and frequency domain and their
crossinfluence, accuracy dependence on amount of
snapshots and \( R \) composition, the analysis of eigenvalue
decomposition methods.

In case \( M = 2 \):
1. \( \text{SNR}=2.5 \alpha=3^\circ \ d/\lambda=0.4 \); 2. \( \text{SNR}=1.5 \alpha=13^\circ \ d/\lambda=0.47 \);
five elements AA, 300 snapshots
estimated values:
1. \( \text{SNR}=2.38 \pm 0.41 \ \text{SNR}''=2.42 \pm 0.29 \alpha=2.82^\circ \pm 0.82\)
d/\lambda=0.400 \pm 0.079
2. \( \text{SNR}=1.71 \pm 0.35 \ \text{SNR}''=1.58 \pm 0.28 \alpha=12.75^\circ \pm 0.98\)
d/\lambda=0.472 \pm 0.043
In case \( M = 3 \):
1. \( \text{SNR}=1.5 \ \alpha=3^\circ \ d/\lambda=0.27 \); 2. \( \text{SNR}=1 \ \alpha=13^\circ \ d/\lambda=0.5 \);
3. \( \text{SNR}=0.5 \ \alpha=27^\circ \ d/\lambda=0.6 \); nine elements AA, 300
snapshots
estimated values:
1. \( \text{SNR}=1.45 \pm 0.09 \ \text{SNR}''=1.32 \pm 0.10 \alpha=2.89^\circ \pm 0.19\)
d/\lambda=0.272 \pm 0.003
2. \( \text{SNR}=0.99 \pm 0.04 \ \text{SNR}''=0.92 \pm 0.08 \alpha=13.09^\circ \pm 0.24\)
d/\lambda=0.503 \pm 0.016
3. \( \text{SNR}=0.48 \pm 0.03 \ \text{SNR}''=0.48 \pm 0.07 \alpha=25.13^\circ \pm 0.24\)
d/\lambda=0.596 \pm 0.015

![Diagram](image)

Fig.4 Proposal method for broadband utilization

The statistical results are from 100 runs for both
cases without failure cases and for \( \text{dim}(R'')=9 \), additive
noise is present at each omnidirectional sensor and is
uncorrelated from sensor to sensor. In 5 runs resp. 2 runs,
were missed some of targets and in 9 resp 4 runs were
detected false targets, however there were used very simple
target detection algorithm only.

3. Conclusion

The analysis and simulations produced in our
Radioelectronics Department show, that broadband DOA
application for MUSIC is possible without any frequency
preprocessing (band splitting). It gives good results and
keep great advantage of ET to „see“ all interesting space at
once, however disadvantage of great computational need
increases twice and the exploitation of \( \lambda \) dimension of \( I \)
gives higher risk in DOA target loses (7). This proposal is
useful for linear or planar AA, 2D scene, does not matter
if narrowband or broadband, where instead of one with
\( \text{dim}(R)=m.n \) is much more easier (because of eigenvalue
decomposition problem), to take \( R \alpha \) and \( R \epsilon \) of two
rectangular linear AA with \( \text{dim}(R\alpha)=m \), \( \text{dim}(R\epsilon)=n \)
and after comparison of \( P_{\alpha} \), \( P_{\epsilon} \) and \( P'' \) respectively find
out correct parameters.

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