Prediction of rain and water vapour attenuation at frequencies 10-30 GHz

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Abstract

This contribution discusses the attenuation of microwaves in the atmosphere and applies the results to the conditions in the Czech Republic. The simple CCIR method predicting the rain attenuation is described and the relevant rain intensity as the only meteorological parameter required is presented on a map. Five methods of evaluation of the water vapour attenuation are discussed and compared. Included is also the computation of the zenith water vapour attenuation and its distribution based on the statistical evaluation of real four-year vertical profiles of important meteorological parameters from the Prague station.

Keywords
atmospheric propagation, rain attenuation, water vapour attenuation, radioclimatology

1. Introduction

Prediction of atmospheric attenuation is a (not very exact) synonym for the determination of the (time) distribution of the attenuation. It is usual to consider the complement of the distribution. For instance, the expression \( P(A > 7 \, \text{dB}) = 0.01 \, \% \) informs us that the probability that \( A \) will be greater than 7 dB is 0.01 \( \% \) (by other words: the exceedance probability of 7 dB attenuation is 0.01\%). Considering the annual population (1 year = 525600 minutes), the equation is understood that the attenuation is greater than 7 dB for 52.6 minutes per year (and for 52554.7 minutes, the attenuation is smaller or equal to 7 dB). Attention: this distribution does not inform us about the duration of single attenuation events - it informs us about the sum of duration of attenuation events when the attenuation exceeds the given value \( A \) (it is 7 dB in our example).

The distribution of the atmospheric attenuation informs us about the conditions of the radio wave propagation through the atmosphere at the locality of interest. It is very important for the planning of technical parameters (power, antenna gain, noise conditions etc.) of wireless communications - radiorelay and/or satellite links.

The atmospheric attenuation \( A \) is a random variable. It depends preferably on following atmospheric phenomena: rain, water vapour and cloud. They can be more or less mutually dependent.

Smaller attenuation is caused by hail, snow, oxygen etc. In this contribution the multipath propagation effects, oxygen neither scintillations are not discussed.

Remark: the multipath fading is negligible on satellite links.

2. Attenuation modelling

2.1 Instantaneous attenuation

The path integral of specific attenuation along the communication link represents the value of the total instantaneous attenuation. To evaluate the specific attenuation along the path link, it is necessary to know the instantaneous profile of physical parameters. This profile is not known very often.

Formulas for the specific attenuation are derived using the scattering theory of radiowaves on atmospheric particles, cf. [7] (rain case) or [8] (water vapour case).

2.2 The distribution of attenuation

For statistical purposes, it is possible to model the approximating „statistical” path profile of essential atmospheric parameters. Also, the effective path length is an appropriate tool to comply with the purpose. There were many prediction methods developed and tested. Some examples of recommended methods are shown below.

3. The CCIR method to predict the distribution of the rain attenuation for microwave links

The well known CCIR model [6] requires only one input meteorological parameter: the (one minute average) rain intensity for the exceedance probability 0.01\%. In [1] there are results of the evaluation of 98 raingauge records on the entire territory of the Czech Republic. The available results being appropriate for the CCIR method
are 5 minutes average rain intensities \( R(P=0.00475\%) \) for the 0.00475\% exceedance probability \( P \).

In Ref. [4] there is shown a relationship between 1-
(required by the CCIR method) and 5-minutes average rain intensities (published in [1]) . The next relationship can be used for statistical evaluation only:

\[
R(1\,\text{min.\,av.}) = 0.745 \cdot R(5\,\text{min.\,av.})^{1.08} \tag{1}
\]

where \( R \) is the rain intensity [mm/h].

It is necessary to shift the obtained rain intensities in the required exceedance probability 0.01\%.

Substituting the derived 1-minute average rain intensity \( R(P=0.00475\%) \) and corresponding probability \( P=0.00475\% \) into equations (16) and (17) one can obtain \( A(P=0.00475\%) \) in order to use the algorithm in the original form (Annex) obtaining \( A(P=0.01\%) \) from:

\[
A(P = 0.01\%) = \frac{A(P = 0.00475\%)}{0.12 F(P = 0.00475\%)} \tag{2}
\]

Through the „isolines procedure“ a map of the 1-
minute average rain intensities on the 0.00475\% exceedance probability level over the Czech territory was prepared (Fig. 1). This makes the rain attenuation prediction quite easy.

Remark 1: the dispersion of these intensities is quite small as it is seen in Fig.1. The location has no great influence on the rain attenuation on our territory.

Remark 2: As the typical rain intensity corresponding to the 0.01\% exceedance probability in the Czech territory the value 32 mm/h is considered (other sources).

4. The water vapour attenuation on satellite communication links

4.1 Formulas for the water vapour attenuation

In the Annex of Ref. [2] there is published an exact algorithm to evaluate the specific water vapour attenuation in the dependence on: air temperature \( T \) [K], dry air pressure \( p \) [hPa], water vapour partial pressure \( e \) [hPa] and frequency \( f \) [GHz].

In this contribution this algorithm will be labelled as method L. Another adjusted (and simpler, labelled as method C) formula is also published in [2] validating for \( p = 1013 \pm 50 \) hPa and \( t = 15^\circ\text{C} \):

\[
\gamma_w = (0.05 + 0.0021p + \frac{36}{(f - 222)^2 + 8.5}) \cdot f^2 \cdot 10^{-4} \tag{3}
\]

where \( p \) is the water vapour density [g/m\(^3\)], \( \gamma_w \) is the specific water vapour attenuation [dB/km] and \( f \) is the frequency [GHz]. The temperature correction factor of 0.6\% per 1 °C from 15°C is recommended (attenuation is inversely proportional to the temperature).
To compare both formulas, it is necessary to convert water vapour partial pressure \( e \) into water vapour density \( \rho \) or vice versa:

\[
e = \frac{T \cdot \rho}{216}
\]

(4)

In order to obtain a simple formula for the total zenith water vapour attenuation it could be tried to neglect the expression 0.0021 \( \cdot \rho \) in equation (3) - the numerical analysis has shown that the error caused by this omission does not exceed 15\% (this manner is labeled as method CM).

In [5] an older and similar formula to evaluate the specific water vapour attenuation is published (label CS in this contribution):

\[
\gamma_s = (0.067 + \frac{3}{(f - 22.3)^2 + 7.3}) f^2 10^{-4} \rho
\]

(5)

Methods C and CS are sometimes used in practice even if the validity of formulas is limited for air pressure 1023 hPa.

Another serious problem is the frequency dependence. The authors of [3] tried to use next formula, where the total attenuation is dependent only on the frequency and on the (zenith) integrated water vapour content (also in the statistical sense):

\[
A = 0.0173 [V(P)] \frac{\gamma(f, P, \rho_{ref}, t_{ref})}{\gamma(f_{ref}, P, \rho_{ref}, t_{ref})}
\]

(6)

\[
p_{ref} = 780 \text{ hPa}
\]

(7)

\[
\rho_{ref} (P) = \frac{V(P)}{4 \text{ km}}
\]

(8)

\[
t_{ref} = 14 \cdot \ln (0.22 \cdot \rho_{ref}) + 3^\circ \text{C}
\]

(9)

where \( V(P) \) [g/m\(^2\)] is the integrated water vapour content:

\[
V(P) = \int_0^{t_{ref}} \rho (h) \, dh
\]

(10)

and \( h \) is the height, \( h_s \) is the height above sea level of the earth station, \( h_{ref} \) is the upper limit of the integration given either the level of the pressure of 300 hPa (~about 9 km) according to [3] or by the equation

\[
h_{ref} = h_{ref} \left(1 + \frac{3}{(f - 22.2)^2 + 5}\right)
\]

(11)

where \( h_{ref} = 1.6 \text{ (2.1) km in clear weather (rain)} [5].

Now it is sufficient to evaluate the distribution of the (zenith) integrated water vapour content and assign its exceedance probability \( P \) to the derived attenuation using the above mentioned formulas (method S).

The reader can obtain the water vapour attenuation on the satellite link by dividing the zenith attenuation by the sine of the elevation angle. The homogeneous horizontal structure of the atmosphere is supposed.

### 4.2 Computation of the water vapour attenuation in practice

Having had the four-year data bank of vertical profiles of the air temperature \( T \), air pressure \( p \) and relative humidity \( \varphi \), the annual distribution of the (zenith) water vapour attenuation for Prague using methods L, C, CS, CM and S was computed. The distribution curves for the frequency 20 and 30 GHz are shown in figures 2 and 3, respectively. Please note, that for physical reasons the water vapour attenuation at frequency 30 GHz is smaller than the 20 GHz one.

Fig. 2 The distribution of the zenith water vapour attenuation [A] and density [W V D] at \( f \approx 20 \text{GHz} \) in Prague.

To perform the necessary conversion of relative air humidity \( \varphi \) into water vapour partial pressure \( e \), we recommend following formula:

\[
e = \varphi \cdot e_s \quad ; \quad (0 \leq \varphi \leq 1)
\]

(12)

while the saturated water vapour partial pressure \( e_s \) [hPa] is obtainable from the temperature through the following approximation:

\[
e_s = 6.108 \cdot 10^{237.3 \varphi}
\]

(13)
where \( t \) is the temperature in °C.

**THE DISTRIBUTION OF THE ZENITH WATER VAPOUR ATTENUATION at \( f = 30 \) GHz** (PRAHUB)

![Graph showing the distribution of the zenith water vapour attenuation at \( f = 30 \) GHz in Prague.]

It is obvious that the total (zenith) water vapour attenuation is proportional to the (zenith) integrated water vapour content \( t [g/m^2] \).

**Water vapour attenuation Test of method L versus CS against data from Prague \( f = 20 \) GHz**

![Graph showing the water vapour attenuation test of the methods L and CS for the Prague data at \( f = 20 \) GHz.]

Scatterplots of total zenith water vapour attenuation computed in accordance with the accurate method L and very simple method CS were prepared. 3000 attenuation values of various real vertical profiles of atmospheric parameters in Prague were evaluated. The analysis performed for 20 GHz shows the conformity of both methods (Fig.4); in the 30 GHz case, the attenuation values given by method CS should be multiplied by factor 0.94 (Fig.5).

**Water vapour attenuation Test of method L versus CS against data from Prague \( f = 30 \) GHz**

![Graph showing the water vapour attenuation test of the methods L and CS for the Prague data at \( f = 30 \) GHz.]

**5. Conclusion**

Critical overviews have appreciated the simple CCIR method to predict the rain attenuation. The map of rain intensities (Fig.1) makes the use of this method easy.

Evaluation of 3 000 vertical profiles from Prague to compare simple water vapour prediction method CS (Ref. [5]) with more accurate formulas (method L, Ref. [2]) has confirmed the method CS for 20 GHz and has further shown the possible simple conversion improving results of method CS in the 30 GHz frequency band.

The rain attenuates microwave links for less than 5% of the year while the water vapour effects radio waves for more than 50% of year. An (relative) extremely high attenuation is caused at the water resonant frequency (22.3 GHz). The computation of attenuation has shown that the rain attenuation is much greater than the water vapour attenuation.

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6. ANNEX - Equations of Simple CCIR Method to Compute the Distribution of Rain Attenuation

Input parameters: rain intensity for the 0.01% exceedance probability $R(P=0.01\%)$, frequency, elevation $\theta$ and latitude $\omega$ in the satellite link case, path length in the terrestrial link case.

Output: distribution of the rain attenuation for exceedance probability 0.1%-0.001%

$$r_{ooi} = \frac{1}{1 + 0.045L_\theta} \quad (14)$$

$$L_\theta = (h_r - h_s)\cot g \theta \quad (15)$$

$\theta$ is the elevation angle ($\theta > 5^\circ$)

$h_r$ is the height above the sea level of the precipitation top;

$h_s$ is the height above sea level of the earth station

$h_r = 4 \text{ km if } \omega < 36^\circ$;

$h_r = 4 - 0.075 (\omega - 36^\circ) \text{ if } \omega \geq 36^\circ$;

$\omega$ is the latitude;

$L_\theta = L$ the path length in the terrestrial link case.

$$\eta(P) = a \cdot R(P)^b; \quad P = 0.01\% \quad (16)$$

Parameters $\{a\}$ and $\{b\}$ are dependent on the frequency and polarisation. They are published, for instance, in [9]. Please find some values for the circular polarisation in the next table. Horizontal (vertical) polarisation is attenuated slightly more (less) than the circular one.

<table>
<thead>
<tr>
<th>Frequency [GHz]</th>
<th>10</th>
<th>12</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>interpolate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0.0094</td>
<td>0.0177</td>
<td>0.0330</td>
<td>0.0722</td>
<td>0.1191</td>
<td>0.1789</td>
<td>logarithmically</td>
</tr>
<tr>
<td>$b$</td>
<td>1.273</td>
<td>1.211</td>
<td>1.143</td>
<td>1.083</td>
<td>1.044</td>
<td>1.007</td>
<td>linearly</td>
</tr>
</tbody>
</table>

Table 1 - constants to evaluate the specific rain attenuation

$$A(P) = \eta(P) \cdot L_\theta \cdot r_{ooi}; \quad P = 0.01\% \quad (17)$$

$$A(P) = 0.0124 (P = 0.01\%) F(P); \quad 0.001 \leq P \leq 1\% \quad (18)$$

$$F(P) = \frac{P}{(0.546 + 0.043 \log P)} \quad (19)$$

References


About the author...

Ondřej Fíšer was born in Prague in 1952. He received the M.E.E. degree in electrical engineering (1977) and Ph.D. degree (1986) from the Czech Technical University (specialisation: antennas, propagation and microwaves). He works as a scientific researcher at the Institute of Atmospheric Physics of the Academy of Sciences of Czech Republic. Professional interests: radio wave propagation through the atmosphere, radar meteorology, radioclimatology, teaching. For more see http://www.ufa.cas.cz/html/meteo/fiser.html