STATISTICAL IDENTIFICATION OF KERNELS OF DISCRETE NONLINEAR SYSTEMS

Michael A. SHCHERBAKOV
IVS, State Technical University of Penza
Krasnaya 40
Penza 440017
Russia

Abstract

A method for identification of discrete nonlinear systems in terms of the Volterra-Wiener series is presented. It is shown that use of a special composite-frequency input signal as approximation to Gaussian noise provides a computational efficiency of this method, especially for high order kernels. Orthogonal functionals and consistent estimations for Wiener kernels in the frequency domains are derived for this class of noise input. A basis of the proposed computational procedure for practical identification is the fast Fourier transform (FFT) algorithm which is used both for a generating of system stimuluses and for an analysis of system reactions.

Keywords:

Nonlinear systems, identification, Volterra-Wiener series

1. Introduction

Continued interest has been shown in the use functional series in the modelling, identification and control of nonlinear systems [1-6] since the initial work by Wiener [9]. He considered the class of causal systems that produce an output with finite mean-square value when their input is a Gaussian white noise. The output $y(t)$ of an unknown nonlinear "black-box" system can be approximated by a series of functionals $G_m[h_m,x(t)]$ of the input $x(t)$ as

$$y(t)=\sum_{m=0}^{N} G_m[h_m,x(t)] \qquad (1)$$

where h_m is the Wiener kernel of the order m.

A main difficulty encountered when one wants to apply the Wiener approach to identification problems involves the measurement of the kernels which is computationally demanding. Several methods have been presented to find the Wiener kernels of nonlinear systems from given input and output pairs. Lee and Shetzen [7] showed that the kernels can be estimated by input-output crosscorrelation. French and Butz [8] used the fast Fourier and Walsh transform algorithms for calculation the Wiener kernels. However, there are some difficulties involved in these methods:

- A white Gaussian process is unrealizable.
- Formula for kernels, $m\geq 2$, involves Dirac delta functions, when two or more kernel's arguments are equal.
- The required computation increases very rapidly with the order of the Wiener kernel being calculated.

This paper will resolve these difficulties by investigating discrete systems with special type of noise inputs generated using the FFT algorithm. We will construct the G-functionals for such inputs, and the formula for the Wiener kernels and the efficient identification algorithm in the frequency domain will be presented.

2. Forcing functions for nonlinear systems testing

It is necessary that the system stimulus must, on the one hand, be like a random noise to get maximum information about unknown system and, on the other hand, to simplify the G-functionals and the procedure of identification on the whole. Taking into consideration these circumstances, let us consider as a test input the following periodic noise approximation

$$x(n)=\sum_{k=-N_x}^{N_x} X(k)\exp j\frac{2\pi kn}{N} \qquad (2)$$

Here $X(k)=A(k)\varphi(k)$ are complex Fourier coefficients, where the amplitudes $A(k)$ determine the power spectrum of the input, and the phases $\varphi(k)$ are independent random values with uniform distribution.

For zero mean real signal, the complex valued Fourier coefficients have the following relationships: $X(0)=0$, $X(-k)=X^*(k)$. According to the Central Limit Theorem, the signal in the form of (2), being a sum of independent random quantities, has a nearly Gaussian distribution for sufficiently large N_x. For every set $\varphi_l(k)$ of the random phases, the formula (2) determines the sequence $x_l(n)$ having N samples long which may be formed by the inverse FFT of the coefficients $X_l(k)=A(k)\varphi_l(k)$.

3. G-functionals and Wiener kernels in the frequency domain

According to the proposed method of generating of the test signal, the random input process $x(n)$ is determinated by the set of input Fourier coefficients $X(k)$, $k=0,...,N_x$, and the corresponding response $y(n)$ can be characterized by the set of output Fourier coefficients $Y(k)$, $k=0,...,N_y$. Therefore, it is possible to rewrite input-output relationship for nonlinear systems in the frequency domain as

$$Y_M(k) = \sum_{m=0}^{M} G_m[H_m, X(k)] \qquad (3)$$

where $H_m(k_1,...,k_m)$ is the multidimensional discrete Fourier transformation (DFT) of the Wiener kernel $h_m(n_1,...,n_m)$.

By using a Gram-Schmidt orthogonalization procedure, the G-functionals can be shown [10] to be

$$G_m[H_m, X(k)] = \sum_{\Omega_m} H_m(k_1,...,k_m)\delta^k_{k_1+...+k_m} \times \prod_{i=1}^{m} X(k_i) \qquad (4)$$

where the summation must extend over the m-D region Ω_m consisting of various combination $(k_1,...,k_m)$ from integers $\{-N_x,...,-1,1,...,N_x\}$ such that $k_1 > k_2 > ... > k_m$, $k_i \neq -k_j$, and δ^j_i is a Kronecker delta.

The Wiener kernels in the frequency domain for the model (3) can be determined by minimization the mean square error between the DFT $Y(k)$ of the system and $Y_M(k)$ model responses

$$F = E\{\Delta^T \Delta^*\} \to \min$$

where $\Delta^T = [\delta_1,...,\delta_{N_y}]$ is the vector of the complex errors having elements $\delta_k = Y(k) - Y_M(k)$, $E\{\bullet\}$ denotes the average operation, and $*$ represents the complex conjugate.

Minimizing this function, the optimal Wiener kernels become

$$H_m(k_1,...,k_m) = \frac{E\{Y(k_1+...+k_m)\prod_{i=1}^{m} X^*(k_i)\}}{\prod_{i=1}^{m} A^2(k_i)}.$$

In order to construct the estimate of kernel $H_m(k_1,...,k_m)$ which would be suitable in practice let us introduce the periodogram

$$I^l_{yx...x}(k_1,...,k_m) = Y^l(k_1+...+k_m) \times \exp[-j\sum_{i=1}^{m}\varphi_l(k_i)]. \qquad (5)$$

Then as the estimate of kernel $H_m(k_1,...,k_m)$ we can use

$$\overline{H}_m(k_1,...,k_m) = \frac{\sum_{l=1}^{L} I^l_{yx...x}(k_1,...,k_m)}{L\prod_{i=1}^{m} A(k_i)}. \qquad (6)$$

It can be shown [10] that this estimate is unbiased and consistent with variance

$$Var\{\overline{H}_m(k_1,...,k_m)\} = \frac{N^{m-1}}{L\left(C_m^{k_1...k_m}\right)^2 S(k_1)\cdot...\cdot S(k_m)} \times\left(S_y(k_1+...+k_m) - S_{H_m}^{k_1...k_m}(k_1+...+k_m)\right)$$

where $C_m^{k_1...k_m} = |k_1|!\cdots|k_m|!/m!$, $S_y(k)$ denotes the power spectrum of the system output signal $y(n)$, and $S_{H_m}^{k_1...k_m}(k_1+...+k_m)$ is a component of the spectrum $S_y(k)$ caused by a value of the kernel $H_m(k_1,...,k_m)$ at the point $k=k_1+...+k_m$.

4. Identification algorithm

If the random phases $\varphi(k)$ are formed by random sampling from set of numbers $2\pi r/R$, $r=0,...,R-1$, the equation (5) for the periodogram may be rewritten as

$$I^l_{yx...x}(k_1,...,k_m) = Y^l(k_1+...+k_m)] \times\exp\left(-j\frac{2\pi\{s^l_{k_1}+...+s^l_{k_m}\}\bmod R}{R}\right) \qquad (7)$$

where s_k^l is l-th set of random integers, $\{\bullet\}\bmod R$ denotes summation defined modulo R.

The calculation of the Wiener kernels for order $m \geq 2$ may be performed more effectively if it is noted that periodogram (7) may take limited number of values $Y_l(k)\exp[-j(2\pi i/N_x)$, $k=0,...,N_y$, $i=0,...,R-1$. This allows us in advance to form the array of possible products for every DFT $Y_l(k)$. Thus the algorithm of identification consists of the following steps:

1. Generation of the random integers $s^l_1,...,s^l_{N_x}$ and forming the complex Fourier coefficients $X(k)$.
2. Calculation by using of the inverse FFT(IFFT) the l-th block of the input signal

   $x_l(n)=\mathrm{FFT}\{X_l(k)\}$, $n=0,...,N-1$.
3. Stimulation of the system by the input $x_l(n)$ and registration of the response $y_l(n)$.
4. Calculation by use the FFT the complex Fourier coefficients

   $Y_l(k)=\mathrm{FFT}\{y_l(n)\}$, $k=0,...,N_y$.
5. Definition of the array $Z_l(k,i)$ of all possible values of the periodograms

   $Z_l(k,i)=Y_l(k)\exp(-j2\pi i/R)$, $k=0,...,N_y$, $i=0,...,R-1$.
6. Forming the periodograms from the array $Z_l(k,i)$

$$I^l_{yx}(k) = Y^l(k)\exp(-j\frac{2\pi s^l_k}{R}),\ m=1$$

$$I^l_{yx...x}(k_1,...,k_m) = Z_l(k_1+...+k_m,\{s^l_{k_1}+...+s^l_{k_m}\}\bmod R),$$

$$m=2,...,M.$$

7.Calculation of the kernel estimates using eqn.(6).

This algorithm of the kernels estimation bases also on idea of scanning the definition regions Ω_m, m=1,..,M, so that the partial sums $k_1+...+k_m$ and $\{s^l_{k_1}+...+s^l_{k_m}\}\bmod R$ obtained for m-order kernel estimate could be used for calculation the periodogram of the order (m+1).

Since the number C_m combination (k_1,...,k_m) containing in the kernel definition region Ω_m increases rapidly with order m of the kernel, and the number of multiplications, demanding for calculation of the array $Z_l(k,i)$ of all possible values of the periodograms, does not depend on m, the proposed algorithm, as compared with methods [7,8], allows one to decrease the number of multiplications to a marked degree. Actually, the most effective method [8] demands approximately $L(C_1+2C_2+...+mC_m)$ complex multiplication, just as the proposed method requires only $LR(N_y+1)/2$ multiplications. For a comparison, in Table we give a reduction factor of multiplications for N_x=N_y, R=8 and various values m of the kernel order. Thus the computational efficiency rapidly increases with the kernel order.

Kernel order m	The number N_Y of output frequencies			
	16	32	64	128
1	1	1	1	1
2	3	6	12	24
3	26	110	447	1804

Table: Reduction factor of multiplications

The method developed in this paper has been tested by applying two kinds of inputs to the known system: one is a zero-mean Gaussian input, and another is the composite-frequency input signal in the form (2). The kernels calculated by Wiener method are compared to those obtained by the method developed in this paper. Both methods give approximately identical estimates of the kernels in the frequency domain. However, comparison these methods with respect to computation time on IBM PC completely confirms the theoretical calculation mentioned above.

5. Conclusion

A algorithm for identification of discrete nonlinear systems in terms of the orthogonal series was presented. The process generated by inverse FFT algorithm was used as a test signal. For this input the G-functionals and Wiener kernels were defined in the frequency domain. The proposed algorithm offers a significant reduction in computational complexity compared with the known methods since the number of multiplications does not depend on the kernel order.

References

[1] SCHETZEN, M.: The Volterra and Wiener theories of nonlinear systems, (New York: John Wiley, 1980).

[2] RUGH, W.J.: *Nonlinear system theory: The Volterra/Wiener approach*, (Baltimore: The Johns Hopkins University Press, 1981).

[3] BILLINGS, S.A.: Identification of nonlinear systems - A survey, *Proc. IEE*, 127-D(6), 1980, pp. 272-285.

[4] KIM, K.I. & POWERS, E.I.: A digital method of modeling quadratically nonlinear systems with a general random input, *IEEE Tras. on ASSP*, 36(11), 1988, pp.1758-1769.

[5] MERTZIOS, B.G.: Parallel modeling and structure of nonlinear Volterra discrete systems, *IEEE Trans. Circuits & Syst.* 41(5), 1994, pp. 359-371.

[6] MARMARELIS, P.Z. & MARMARELIS, V.Z.: *Analysis of physiological systems: the white-noise approach*, (New York: Plenum Press, 1978).

[7] LEE, Y.W. & SCHETZEN, M.: Measurement of the Wiener kernels of nonlinear system by cross-correlation, *Int. J. Control*, 2, 1965, pp. 237-254.

[8] FRENCH, A.S. & BUTZ, E.G.: Measuring the Wiener kernels of nonlinear system using the fast Fourier algorithm, *Int. J. Control*, 17(3), 1973, pp.529-539.

[9] WIENER, N. : Nonlinear problems in random theory, (MIT Press, 1958).

[10] SHCHERBAKOV, M.A.: Analisys of Wiener Kernels estimation in the frequency domain, *Cybernetics and computer science*, Kiev, no.89, 1991, pp.19-26, (in Russian).

About author

Michael A. SHCHERBAKOV was born in Penza, Russia, on August 24, 1954. He received the Ing. (M.S.) degree in Electrical Engineering from the Polytechnic Institute, Penza, Russia, in 1976, and Csc. (Ph.D) degree in automatic control from the Institute of Cybernetics of Ukrainian Academy of Science, Kiev, the Ukraine, in 1985. The academic year 1989-1990 he was a research fellow in the Department of Control Engineering, Czech Technical University, Prague. Since 1985 he has worked as an Associate Professor with the Department of Information and Computer Systems, State Technical University of Penza, Russia. His research interests are in the nonlinear signal and image processing, system identification and high-order statistic.