UNCONVENTIONAL TOCOGRAPHIC SIGNAL ANALYSIS

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Abstract

Spectrum estimation belongs to the most frequent problems solved by the digital stochastic signal processing.' The application of the unconventional method of spectrum estimation based on maximum entropy approach introduced by J.P.Burg is presented. Drawbacks of classical spectrum estimation methods involving interpretation difficulties are avoided. Basic principles and underlying conceptions yielding the same results as autoregressive modelling and reasonableness of its application are discussed. Stochastic biosignal reflecting fetus movements obtained by the tocography is analyzed by the maximum entropy method and compared to the conventional method of Fourier analysis.

Keywords

tocography, entropy, spectrum estimation, autoregression, prediction error filter

1. Tocographic Signal

Tocography belongs to the methods used for fetus monitoring. Cardiotocography yields simultaneous records of two biosignals: cardiotachographic signal (instantaneous fetal heart rate) and tocographic signal (pressure reflecting uterus changes). The two biosignals provide means to estimate reactions of the fetal heart to pressure changes caused by contractions of the uterus or fetal movements.

Tocographic signal provides information on both regular and irregularly occurring uterus contractions and fetal movements before delivery. The signal can be obtained directly from the uterus by means of a catheter or externally by means of a transducer (tocodynamometer) attached to the intact surface of a maternal abdomen. In the tocographic curve, basal tonus of approximately 1.06 to 1.6 KPa, frequency, duration of contractions and other parameters are observed and diagnostically evaluated. Except gross changes caused by the contractions, also finer alterations caused by the fetus movements or other physiological activities (heart beat, breathing, peristaltics) can be identified in the signal [6]. Typical tocographic signal obtained from an external transducer in the third trimester of pregnancy is shown in Fig. 1.

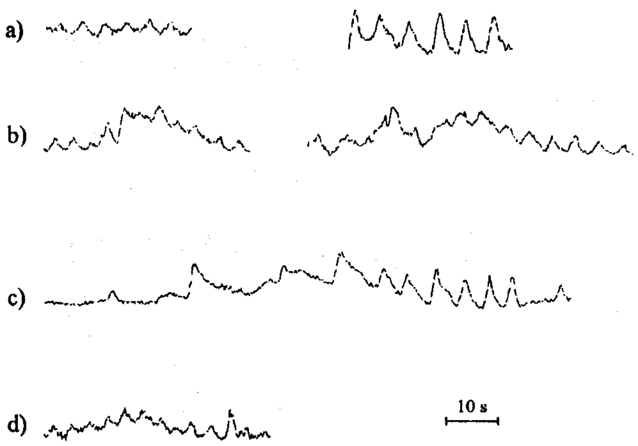

Fig.1 Morphology of the tocographic signal
a) within a quiet period of the fetus sleep b)
within a period of complex body movements c)
within a period of lower extremities movements d)
within a period of upper extremities movements

2. Maximum Entropy Spectrum Analysis (MESA)

2.1 Entropy

From the physical point of view, entropy represents a thermodynamic state variable. In the statistical physics, entropy represents a degree of the system disorder. Entropy in the classical Maxwell-Boltzmann's statistics is defined by means of a number Ni expressing the frequency of the occurrence of phase space elements in the form [4]

$$S = k_b \ln\left(\frac{N!}{\prod_i N_i!}\right) \approx -k_b N \sum_i \frac{N_i}{N} \ln\frac{N_i}{N} \quad (1)$$

where S represents a thermodynamic entropy
k_b represents the Boltzmann's constant
$N = \sum N_i$.

Substituting probabilities p_i for relative frequencies N_i/N, entropy is defined by [3]

$$H(p) = -\sum_i p_i \ln p_i \ . \tag{2}$$

Similarly for the analog random variable given by the density function $p(x)$, entropy is [3]

$$H(p) = -\int_x p(x) \ \ln p(x) \, dx \tag{3}$$

From the mathematical point of view, such an entropy is a measure of uncertainty concerning the results of an experiment before it is performed or a measure of an information gain after the experiment has been performed.

For a digital spectrum estimation, description of entropy of a stationary stochastic Gaussian bounded signal sampled by a sampling period Δt can be expressed by means of a power spectral density $S(f)$ in the following form [1]

$$\int_{\frac{-1}{2\Delta t}}^{\frac{1}{2\Delta t}} \ln S(f) \, df \tag{4}$$

2.2 Maximum Entropy Principle

Maximum entropy principle represents one of the fundamental thermodynamic phenomena. It allows to quantify an inner structure of a system in the state of equilibrium by means of a distribution function with parameters depending only on several macroscopic quantities. In general, an application of the principle on a probability distribution allows to construct a distribution function based on several known statistical moments. That indirectly yields a possibility to determine all statistical moments that would not be available otherwise [3].

A similar situation occurs with the spectral estimation. In the Wiener-Chinchin relation that reflects a power spectrum of a stationary stochastic signal, an infinite number of autocorrelation lags occurs. However, actually there are only several first lags known. All unknown values are with conventional methods of spectral analysis usually considered zero (or, values are directly substituted, e.g. by periodical repetitive values). It has an undesired impact on estimation features, e.g. convolution ripples, resolution in the frequency domain and statistical bias. If the interval for signal processing is principally limited and the compromise between the resolution and the elimination of ripples by means of windowing is not satisfying, entropy maximization by means of indirect substitution of unavailable autocorrelation values can be used. Out of all spectral functions whose inverse transformation corresponds to the known values of autocorrelation, the function that maximizes entropy functional should be chosen. To solve this variotional problem, the linear prediction error filter (PEF) seems to be suitable.

2.3 Prediction Error Filter (PEF)

Search for maximum entropy by means of partial derivations of the stochastic process entropy formula with respect to unknown values of autocorrelation shows that an inverse power spectrum is expressed by only a finite number of autocorrelation values. The autocorrelation of the inverse spectrum can be expressed by an impulse response of an FIR filter. If the FIR filter with the input of the analysed signal is designed so that the output yields a white power spectrum, then the searched spectrum will be determined by an inverse power transfer function of the filter. It is featured by the PEF optimized in the mean square error sense. (The output of a PEF yields a difference between the actual and linearly predicted value of the input signal derived from several previous values and the number of the values depends on the filter order.)

If the autocorrelation values $R_{xx}(m)$ for $|m|<M$ are known, based on optimization conditions for PEF, filter parameters are derived in a matrix form as follows [1]:

$$[R_{xx}]^{(M)}\left(\gamma^{(M)}(0), \gamma^{(M)}(1), \mathrm{K}, \gamma^{(M)}(M)\right)^T = \left(S^{(M)}, 0, \ldots, 0\right)^T \tag{5}$$

where $[R_{xx}]^{(M)}$ is an autocorrelation matrix and

$[R_{xx}]^{(M)}_{ij} = R_{xx}(|i-j|)$ for $i,j = 0,\ldots,M$

$\gamma^{(M)}(m)$ is an impulse response of PEF

$\gamma^{(M)}(0) = 1$ and $\gamma^{(M)}(m)$ for $m = 1,\ldots,M$ are prediction coefficients

$S^{(M)}$ is the mean square error of the prediction

M is a PEF order, maximum discrete lag with an available autocorrelation.

The extension based on maximum entropy is represented in the generalized sense of the abovementioned equation by adding rows $i>M$ to the autocorrelation matrix and further zero values to the column vector on the right side of the equation. Power spectrum of the signal is ultimately defined by means of PEF parameters by the following equation

$$S(f) = \frac{S^{(M)} \Delta t}{\left| \sum_{m=0}^{M} \gamma^{(M)}(m) \, \mathrm{e}^{-j2\pi f \Delta t m} \right|^2} \ . \tag{6}$$

This shows that MESA is equivalent to autoregressive (AR) signal modelling. The spectrum is determined by a transfer function of a hypothetic system (inverse transfer of PEF), with an input signal having white spectrum and an output signal having the same autocorrelation as the analysed signal. Parameters of the AR model are determined by the so called Yule-Walker relations equal to the matrix equation resulting from the PEF optimization [5].

2.4 Autocorrelation Estimation

If the autocorrelation estimation provides an autocorrelation matrix that is positively definite then the mean square error of prediction is positive, the autocorrelation extension is stable, the PEF is a minimum-phase filter and the AR model transfer is stable.

An unconventional simulateneous spectrum-correlation estimation technique using PEF was introduced by J.P. Burg [2] . The algorithm is based on a gradual increase of the PEF order and minimization of the mean square error in each iteration step.

3. Application of the Method

Spectral analysis of a tocographic signal depicted in the Fig. 2 within a 10 second interval of a quiet sleep of fetus was performed. Sampling frequency of 100 Hz was used. In both, Fourier analysis (DTFT = Discrete Time Fourier Transform) and MESA, 500 samples of the signal were processed. In the conventional Fourier analysis, a rectangular window with the ratio of the main and sides lobes of 4.73 ($3/2\pi$) was applied. For the MESA method, the PEF of order 300 was applied. A software program using a recursive algorithm for spectral-correlation estimation was created and used.

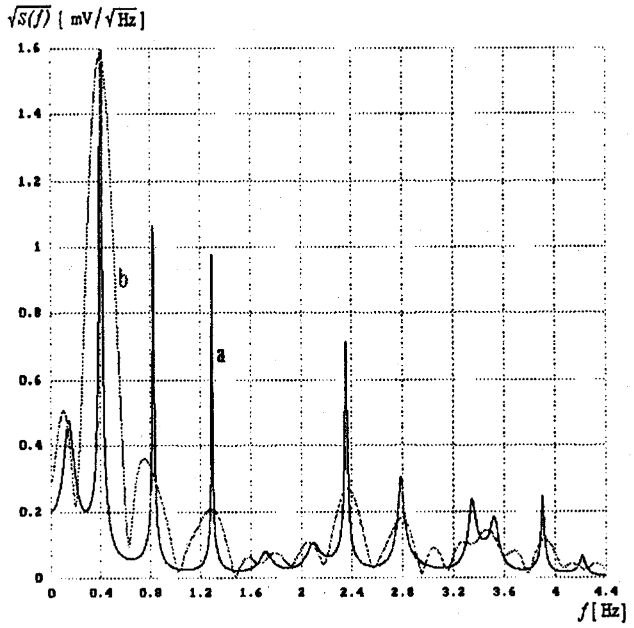

Fig.2 Spectrum of the tocographic signal obtained by
a) the maximum entropy method (N=500, M=300)
b) the classical Fourier analysis (N=500)

The results show a discrepancy between the power of the dominant peak at 0.4 Hz that possibly reflects breathing activity of the mother and the successive contribution at 0.8 Hz that could correspond to the so called "breathing movements" of the fetus. Based on DTFT, it is not clear whether the maximum at 0.8 Hz is caused by the convolution distortion or reflects a single frequency component.

By an DTFT estimation, spectral resolution can be raised only by means of the prolonged interval processed, while by MESA approach it can be reached by an increased PEF order. Spectral resolution controlled by the number of samples N (DTFT) and the PEF order M (MESA) are depicted in Fig.3 and Fig.4 respectively.

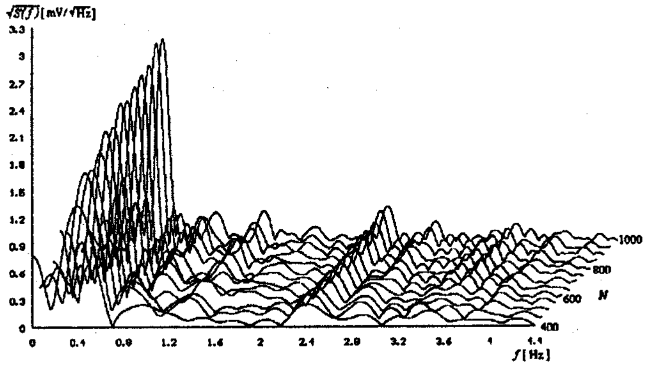

Fig.3 DTFT based spectrum with different processed data lengths (N=400, ... ,1000 with a step of 50)

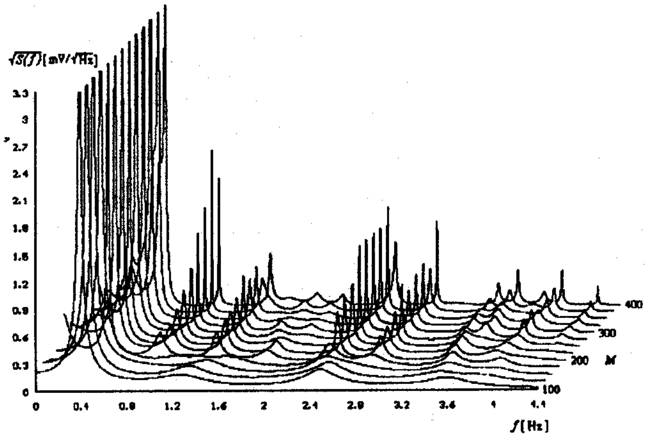

Fig.4 MESA spectrum with different PEF orders (M=100, ... ,400 with a step of 25, N=600)

4. Conclusions

Maximum entropy method of spectral analysis is a very useful tool in digital processing of stationary random processes containing several spectral components that are necessary to distinguish.

The advantage of MESA approach is obvious especially if the frequency components are very close to each other and spectral resolution cannot be enhanced by means of expanding of processed data since one of the reasons is a nonstationary character of the signal. Another application of the method can be shown on a tocographic signal containing an extremely strong spectral component and other frequency components overlapped by successive convolution maxima. This is the case where a nonuniform window would decay spectrum resolution.

In general, MESA approach yields better results also for smaller amounts of analysed data than those used in conventional estimation (Fig.3, Fig.4). However, the application of the method introduces also some other problems, e.g. splitting of the spectral component, peak shifts, emphasis of unrelevant frequency components etc.

References

[1] BURG,J.P.: Maximum Entropy Spectral Analysis. Proceedings of the 37th Meeting of the Society of Exploration Geophysicists, Oklahoma City, October 31, 1967

[2] BURG,J.P.: A New Analysis Technique for Time Series Data. NATO Advanced Study Institute on Signal Processing with Emphasis on Under-Water Acoustics, Enschede, the Netherlands, August 12-23, 1968

[3] GUIASU,S.-SHENITZER,A.: Princip maxima entropie (Maximum Entropy Principle). Pokroky matematiky, fyziky a astronomie, Vol. 31, No.4, pp. 214-224,1986

[4] KREMPASKÝ,J.: Fyzika (Physics). Bratislava, Alfa 1988

[5] MADLOVÁ,A.: Maximum Entropy Spectrum Analysis. Preliminary PhD Report, Bratislava, FEI STU 1995

[6] VIDIŠČÁKOVÁ,J.: Tocographic Signal Processing. Proceedings of the 7th Intl. IMEKO TC-13 Conference on Measurement in Clinical Medicine, Model Based Biomeasurements, September 6-9, 1995

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Jozef PÚČIK was born in Bratislava in 1972. He obtained his first degree in electrical engineering from the Faculty of Electrical Engineering and Information Technology of the Slovak Technical University in Bratislava in 1996. He received Rector's award for excellent academic results during his undergraduate studies and Dean's award for brilliant diploma thesis. After graduation he became a postgraduate student in Electronics at the Department of Radioelectronics. His main field of investigation covers biomedical signal processing.