Several Remarks on Fractal Image Block Coding

Ľubomír DEDERA
Ján CHMÚRNY
Dept. of Computer Science
Military Academy
031 19 Liptovský Mikuláš
Slovakia
e-mail: dedera@valm.sk

Abstract

In this paper the dependence of PSNR on the size of both a codebook pool and range blocks is compared. The statistical properties of the values of transformation coefficients and distances between coded range blocks and optimal domain blocks in the fractal image block coding scheme are discussed.

Keywords

fractals, image coding.

1. Introduction

Fractal image compression, based on the theory of contractive transformations, began with the work of Michael Barnsley [1], [2] and Arnauld Jacquin [3], [4]. Since then a large amount of work on the topics has been undertaken which has given the fractal image coding the power to become a serious competitor of the established compression techniques [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16].

The main aim of this paper is to compare statistical properties of the values of transformation coefficients and distances between coded range blocks and optimal domain blocks in the „traditional" fractal image block coding scheme [5] for various sizes of range blocks and two kinds of images: traditional 512x512x8 „Lena" (Fig.1) and thermovision 272x480x8 „Tatra" (Fig. 2). The model used is described in detail in section 2 and the results are presented in section 3.

2. Used model

Let $\boldsymbol{I}$ be an original image. A set $\boldsymbol{R}$ of non-overlapping partitions of $\boldsymbol{I}$, $\boldsymbol{R} = \{R_1, \dots, R_n\}$ such that $\bigcup_{i=1}^{n} R_i = \boldsymbol{I}$, is called a *range pool*; R_i, $i = 1, \dots, n$ are called *range blocks*. In the experiments described later square range blocks of the same size have been used.

A set $\boldsymbol{D}$ of (overlapping) partitions of $\boldsymbol{I}$, , is called a *domain pool*; D_i, $i = 1, \dots, m$ are called *domain blocks*. In this case square domain blocks of twice the size of range blocks distributed uniformly over the image $\boldsymbol{I}$ have been used.

Each domain block $D_i \in \boldsymbol{D}$ has been scaled down to the size of a range block using pixel averaging operator and then 8 isometries of the square have been applied on it [3], [5]. The resulting pool $\boldsymbol{C}$ of the size $8m$ is called a *codebook pool*.

Next we can consider each range block as a vector $R \in \mathbf{R}^p$ where p is the number of pixels in the range block R. The encoding problem for the range block R is then the least squares problem

$$\min_{x \in \mathbf{R}^2} \|R - \mathbf{A}x\| \tag{1}$$

where $\mathbf{A}$ is a $p \times 2$ matrix with columns C, $(1,\dots,1)^T$ and $x = (a,b)^T \in \mathbf{R}^2$ is a vector of coefficients. If the codebook block $C \in \boldsymbol{C}$ is not in the linear span $\left[(1,\dots,1)^T\right]$, then the minimisation problem (1) has unique solution

$$\bar{x} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T R \tag{2}$$

where the matrix $\mathbf{A}^+ = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T$ is a so called *pseudo-inverse* of $\mathbf{A}$. The approximated „collage block" can be expressed as

$$\mathrm{col}(R) = \mathbf{A}\mathbf{A}^+ R. \tag{3}$$

The aim of encoding is to find the best-matching $C \in \boldsymbol{C}$ to each $R \in \boldsymbol{R}$ and the coefficients a, b. The code consists of the following items:

- index of the optimal domain block
- applied isometry

Fig. 1 Original „Lena" image.

- $a, |a| < 1$
- b.

The coefficient a represents contrast scaling and b luminance shift. Since the method is based on the theory of contractive transformations and Banach's fixed point theorem (the condition $|a| < 1$ assures convergence), decoding can be performed using the following iteration scheme

$$R^{(i+1)} = a\,\mathrm{is}\circ\mathrm{avg}\left(D^{(i)}\right) + b \qquad (4)$$

where is denotes the isometry operator, avg the averaging operator applied on the optimal block D and $^{(\ldots)}$ denotes an iteration step. One iteration cycle consists of the application of eq. 4 on each $R \in \boldsymbol{R}$. There are more sophisticated decoding techniques [11], but this method is sufficient for this purpose.

3. Results

3.1 Dependence of PSNR on the sizes of range blocks and codebook pool

The dependence of PSNR on the size of range blocks for both images and the codebook pool size 1600 is presented in Fig. 3. The dependence on both the range block size and the codebook pool size for the image „Lena" can be observed in Fig. 4. The graph in Fig. 3 shows that the results for the thermovision image „Tatra" are worse (in terms of PSNR) than those for „Lena". It can be seen in Fig. 4 that the larger the size of range blocks the lower the influence of the size of the codebook on the quality of the decoded image. In other words, the computationally demanding part of the algorithm (searching for the best domain block through the codebook pool) does not bring a remarkable improvement in the quality of a decoded image.

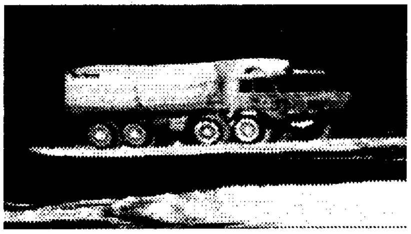

Fig. 2 Original „Tatra" image.

3.2 Distribution of the values of transformation coefficients

The distribution of the coefficient a of the image code in dependence on the size of range blocks for the codebook size 1600 is in Fig. 5 and Fig. 6. For both images, particularly in the case of „Tatra", there is a trend that the larger the size of range blocks the more concentrated the values of the coefficient a around 0; in other words, the resulting decoded images are more influenced by blocks of constant brightness than by domain blocks (see eq. 4).

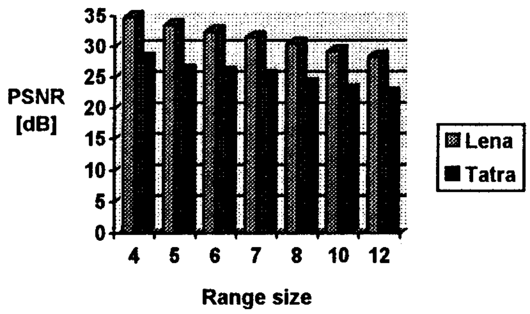

Fig. 3 PSNR of decoded images „Lena" and „Tatra", codebook size 1600, in dependence on the size of range blocks.

3.3 Distribution of the distances between coded range blocks and optimal domain blocks

The distribution of the distances between coded range blocks and corresponding domain blocks for the codebook size 1600 is in Fig. 7 and Fig. 8. In this case there are no significant differences between histograms for various sizes of range blocks; but the results show that it is more likely that the optimal domain block is found among domain blocks that are relatively closer to the encoded range block.

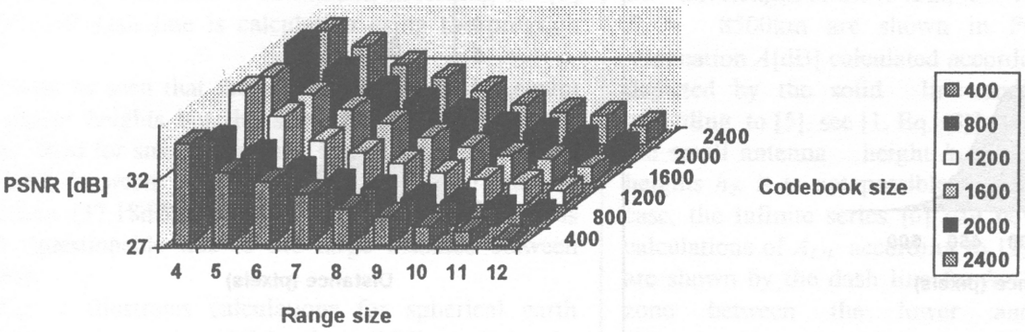

Fig. 4 PSNR of the decoded image „Lena" for various sizes of the codebook in dependence on the size of range blocks

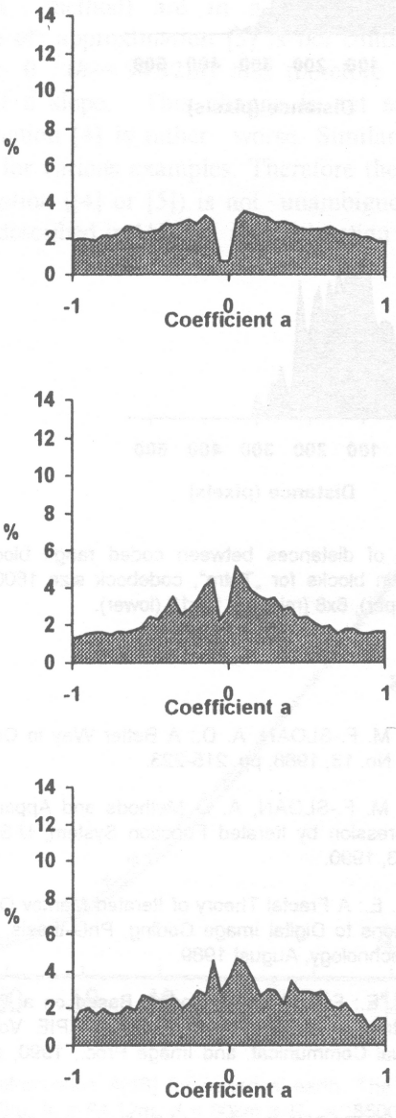

Fig. 5 Histogram of values of transformation coefficients *a* for „Lena", codebook size 1600, range sizes 4x4 (upper), 8x8 (middle), 12x12 (lower).

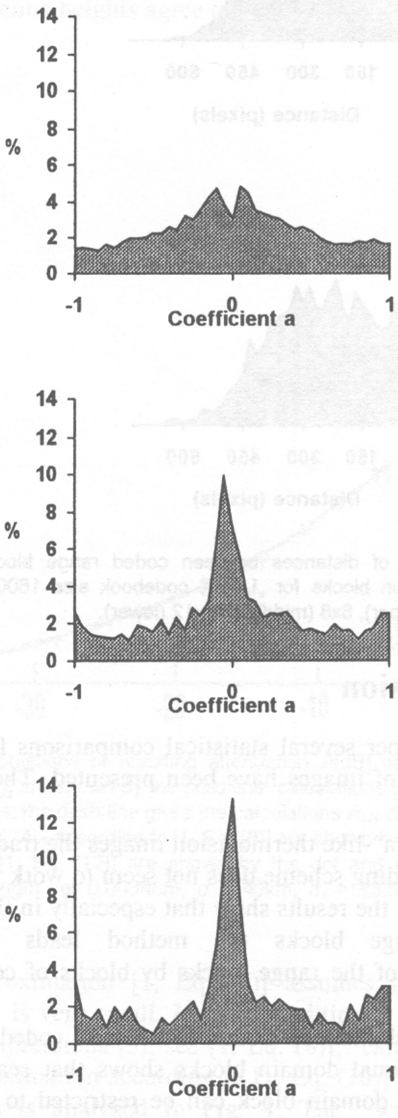

Fig. 6 Histogram of values of transformation coefficients *a* for „Tatra", codebook size 1600, range sizes 4x4 (upper), 8x8 (middle), 12x12 (lower).

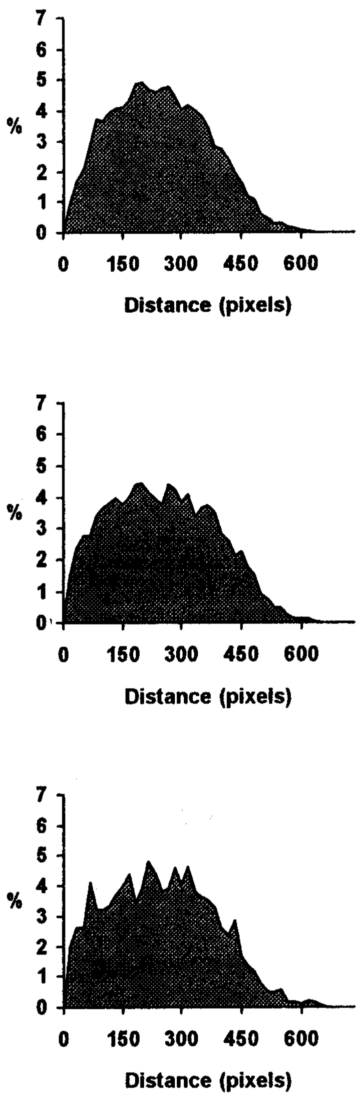

Fig. 7 Histogram of distances between coded range blocks and optimal domain blocks for „Lena", codebook size 1600, range sizes 4x4 (upper), 8x8 (middle), 12x12 (lower).

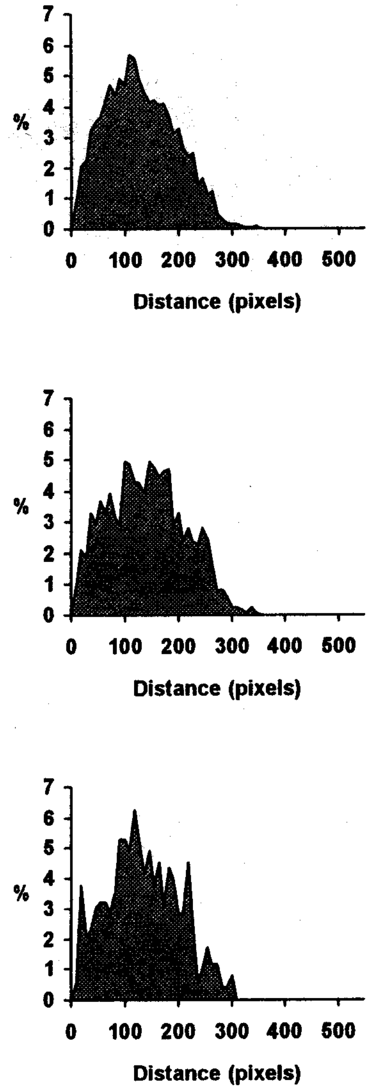

Fig. 8 Histogram of distances between coded range blocks and optimal domain blocks for „Tatra", codebook size 1600, range sizes 4x4 (upper), 8x8 (middle), 12x12 (lower).

4. Conclusion

In this paper several statistical comparisons for two different kinds of images have been presented. The main conclusions are:

• for „Tatra"-like thermovision images the traditional fractal block coding scheme does not seem to work well in terms of PSNR; the results show that especially in the case of larger range blocks the method leads to an approximation of the range blocks by blocks of constant brightness;

•the distribution of distances between coded range blocks and optimal domain blocks shows that searching for a (suitable) domain block can be restricted to blocks that are relatively close to the coded range block. Statistical properties regarding such a modified method are different in several aspects and will be published in the next paper.

References

[1] BARNSLEY, M. F.-SLOAN, A. D.: A Better Way to Compress Images, Byte No. 13, 1988, pp. 215-223.

[2] BARNSLEY, M. F.-SLOAN, A. D.:Methods and Apparatus for Image Compression by Iterated Function System, U.S. Patent No. 4,941,193, 1990.

[3] JACQUIN, A. E.: A Fractal Theory of Iterated Markov Operators with Applications to Digital Image Coding, PhD thesis, Georgia Institute of Technology, August 1989.

[4] JACQUIN, A. E.: Fractal Image Coding Based on a Theory of Iterated Contractive Image Transformations, SPIE Vol. 1360, Proc. of Visual Communicat. and Image Proc., 1990, pp. 227-239.

[5] SAUPE, D.-HAMZAOUI, R.: Complexity Reduction Methods for Fractal Image Compression, in I.M.A. Conf. Proc. on Image Processing; Mathematical Methods and Applications, J. M. Blackledge (ed.), Oxford University Press, 1995.

[6] SAUPE, D.: Breaking the Time Complexity of Fractal Image Compression, Technical Report 53, Institut für Informatik, Freiburg, 1994.

[7] SAUPE, D.:Accelerating Fractal Image Compression by Multi-Dimensional Nearest Neighbour Search, in Proc. DCC'95 Data Compression Conf., J. A. Storer, M. Cohn (eds.), 1995.

[8] ØIEN, G. E., BAHARAV, Z., LEPSØY, S., MALAH, D., KARNIN, E.: A New Improved Collage Theorem with Applications to Multiresolution Fractal Image Coding, in Proc. of ICASSP, 1994.

[9] ØIEN, G. E., SOLLID, A., RAMSTAD, T. A: Hybrid Image Compression Combining Transform Coding and Attractor Coding, in Proc. of NOBIM-92, 1992, pp. 286-293.

[10] LUNDHEIM, L.: Fractal Signal Modelling for Source Coding, PhD thesis, Norwegian Institute of Technology, September 1992.

[11] LEPSØY, S.-ØIEN, G. E.: Attractor Image Compression Featuring Direct Attractor Optimisation and Non-Iterative Decoding, in Proc. of INDUMAT-92, 1992, pp. 182-190.

[12] LEPSØY, S.- ØIEN, G. E.: Fast Attractor Image Encoding by Adaptive Codebook Clustering, in Fractal Image Compression--Theory and Applications to Digital Images, Y. Fisher (ed.), Springer-Verlag, New York, 1994.

[13] FISHER, Y.: Fractal Image Compression with Quadtrees, in Fractal Image Compression--Theory and Applications to Digital Images, Y. Fisher (ed.), Springer-Verlag, New York, 1994.

[14] DUDBRIDGE, F.: Least-squares Block Coding by Fractal Functions, in Fractal Image Compression--Theory and Applications to Digital Images, Y. Fisher (ed.), Springer-Verlag, New York, 1994.

[15] DAVOINE, F.-CHASSERY, J.-M.: Adaptive Delaunay Triangulation for Attractor Image Coding, in 12th International Conference on Pattern Recognition, Jerusalem, October 1994.

[16] BEDFORD, T. J., DEKKING, F. M., KEANE, M. S.: Fractal Image Coding Techniques and Contraction Operators, Technical Report, Delft University of Technology, Delft, 1992.

About authors...

Ľubomír DEDERA was born in Liptovský Mikuláš, Slovakia, on March 1, 1967. He received RNDr. (MS) degree in computer science from the Faculty of Mathematics and Physics, Comenius University, Bratislava in 1990. Now he is a PhD student in the field of digital image processing in the Department of Informatics and Computers at the Military Academy in Liptovský Mikuláš.

Ján CHMÚRNY was born in Martin, Slovakia, on June 7, 1923. He received his undergraduate Diploma degree in electrical engineering from the Slovak Technical University in Bratislava in 1947, the CSc (PhD) degree in radioelectronics from the Moscow Technical University in 1954, and the DrSc degree in radioelectronics from the Technical University of Brno (Czech Republic). He is currently a Professor of electrical engineering at the Military Academy in Liptovský Mikuláš.