STABILITY OF DC OPERATING POINTS

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Abstract

DC operating points of a linearized noninertial network can be, according to the character of their stability, classified into 3 categories: 1. stable, 2. unstable, 3. conditionally stable (conditionally unstable). In the paper it is shown that the process of decision can be based on modified node voltage formulation of network equations. The suggested process consists of formulation of the system matrix, matrix inversion and simple arithmetical manipulations with the elements of the resultant matrix.

Keywords

stability of operating points, input immittances

1. Introduction

Consider a complicated resistive (noninertial) network consisting of linear and nonlinear resistors and controlled sources, and ideal operational amplifiers. All the operating points of the network have already been found and we are interested in their stability [1], [2].

The stability of each operating point will be investigated separately.

The characteristics of nonlinear network elements are linearized at the operating point. The problem of operating point stability is thus transformed into a problem of stability of the corresponding linearized network model.

We are dealing with a noninertial network but we are aware that parasitic (stray) capacitances are present between its nodes and parasitic inductances are in series with its branches.

To start our investigation we first assume the existence of a single stray capacitance $C>0$ between nodes i and j, as shown in Fig.1. This capacitance, together with the input resistance R_{in} of the network (which is under our

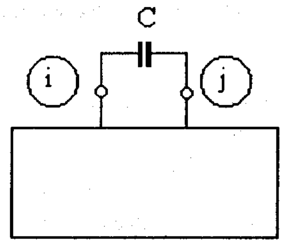

Fig.1. Stray capacitance C between nodes i and j of the linearized noninertial network

assumption noninertial), forms a parallel RC network with a single pole

$$p = -\frac{1}{C.R_{in}} \qquad (1)$$

If $R_{in} > 0$ then $p<0$ and the network is stable. Negative sign of the input resistance measured between the nodes of the capacitance C, results in instability.

When cutting any branch we can observe its input conductance, G_{in}. This conductance, in combination with a stray inductance L, creates a series RL network with

$$p = -\frac{1}{L.G_{in}}. \qquad (2)$$

Again, if $G_{in} > 0$, the network is stable. Negative sign of the input conductance indicates instability.

As it follows from the above considerations, the stability of the operating point in question can be verified very easily. If all input resistances measured between any pair of nodes and all input conductances observed in all branches are positive, the network is stable under any circumstances, i.e. with arbitrary stray capacitances and inductances and any combination thereof.

A more complicated situation arises when some of the input immittances are negative. The network is conditionally stable (or conditionally unstable, as one wishes). The stability depends on the location of the stray elements in the network or on their total number (two or more).

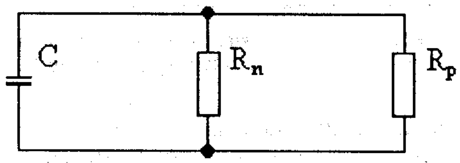

Fig.2a.The case of input resistance consisting of a positive resistance R_p and negative resistance R_n in parallel with a stray capacitance C

We shall illustrate this with a simple network shown in Fig.2a. The network has a negative input resistance $R_{in}<0$ between the nodes i and j. It is unstable with any positive capacitance C. As seen, the network consists of a parallel connection of a positive resistance R_p>0 and a negative resistance $R_n = -|R_n|$<0 of the rest of the network. This negative resistance may be a consequence of internal feedback loops.

According to our previous assumption,

$$R_{in} = \frac{R_p . R_n}{R_p + R_n} = \frac{-R_p |R_n|}{R_p - |R_n|} < 0 \tag{3}$$

denominator of the fraction being positive. Thus

$$R_p > |R_n|. \tag{4}$$

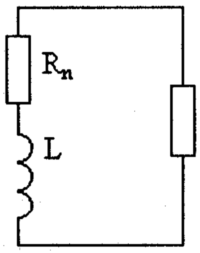

Fig.2b. The case of input resistance consisting of a positive resistance R_p and negative resistance R_n with a stray inductance L

Assume now that instead of the parallel C we connect a series inductance L as in Fig.2b. The input conductance seen by the inductance

$$G_{in} = \frac{1}{R_p + R_n} = \frac{1}{R_p - |R_n|} > 0 \tag{5}$$

so that the network in Fig.2b is stable.

It is therefore obvious that a parallel combination of a positive and a negative resistance is always only conditionally stable. This is true even in the case when $R_p < |R_n|$.

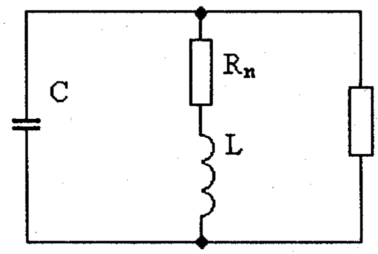

Fig.2c. The case of input resistance consisting of a positive resistance R_p and negative resistance R_n with both C and L

The network in Fig.2c contains simultaneously two stray elements: a parallel C and a series L. Its characteristic equation is

$$\lambda^2 LC + \lambda\left(\frac{L}{R_p} - C|R_n|\right) + 1 - \frac{|R_n|}{R_p} = 0 \tag{6}$$

If

$$|R_n| < R_p$$

and

$$\frac{L}{R_p} - C|R_n| > 0, \text{ i.e. if } \frac{L}{C} > R_p |R_n| \ , \tag{7}$$

the network is stable, otherwise it is unstable.

2. Effective Numerical Calculation of Input Resistances and Conductances

Modified node voltage formulation is used to describe the linearized network. The equations are

$$\begin{bmatrix} \mathbf{Y} & \mathbf{D_1} \\ \mathbf{D_2} & \mathbf{Z} \end{bmatrix} \times \begin{bmatrix} \mathbf{U} \\ \mathbf{I_0} \end{bmatrix} = \begin{bmatrix} \mathbf{J} \\ \mathbf{E} \end{bmatrix}, \tag{8}$$

i.e.

$$\mathbf{Mx} = \mathbf{b} \tag{9}$$

Here

$\mathbf{U} = [U_1, U_2, \dots, U_n]^T$ is vector of n node voltages (the superscript T denotes transposition)

$\mathbf{I_0} = [I_{01}, I_{02}, \dots, I_{0n}]^T$ is vector of m additional currents (output currents of voltage sources and operational amplifiers, controlling currents of current-controlled sources)

Y is an $n\times$ admittance matrix of the regular part of the network

$\mathbf{D_1}$ and $\mathbf{D_2}$ are matrices $n\times m$ and $m\times n$, respectively. They contain dimensionless elements *+1, -1, A* and *B* (amplification factors of voltage-controlled voltage sources and current-controlled current sources)

Z is an $m\times m$ impedance matrix containing internal resistances of voltage sources and transfer resistances W of current-controlled voltage sources

M is the $(n+m)\times(n+m)$ system matrix,

x is a vector of unknown quantities,

b is the right-hand side vector of independent sources.

By inverting the matrix **M** we get

$$\mathbf{R} = \mathbf{M}^{-1} = \begin{bmatrix} R_{11} & R_{12} & \dots & R_{1n} & & & \\ R_{21} & R_{22} & \dots & \dots & & & \\ \dots & \dots & \dots & \dots & & & \\ R_{n1} & R_{n2} & \dots & R_{nn} & & & \\ & & & & G_{11} & & \\ & & & & & \dots & \\ & & & & & & G_{mm} \end{bmatrix} \tag{10}$$

(in (10) we show only elements that will be used in further calculations).

The input resistance between the node i and the datum (reference) node

$$R_{in(i)} = R_{ii} \tag{11}$$

Input resistance between nodes i and j is

$$R_{in(i,j)} = R_{ii} + R_{jj} - R_{ij} - R_{ji} \tag{12}$$

Input conductance measured in series with the resistor R connected (inside the network) between the nodes i and j

$$G_{in(R)} = \frac{1}{R}\left[1 - \frac{R_{in(i,j)}}{R}\right] \tag{13}$$

This conductance is positive if $R_{in(i,j)} < R$, including $R_{in(i,j)} < 0$.

Finally, input conductance measured in series with the branch carrying the current I_{oi}

$$G_{in(I_{oi})} = G_{ii} \tag{14}$$

As a result, calculation of all immittances requires only one matrix inversion plus simple algebraic manipulations with some elements of the resultant matrix.

Example 1

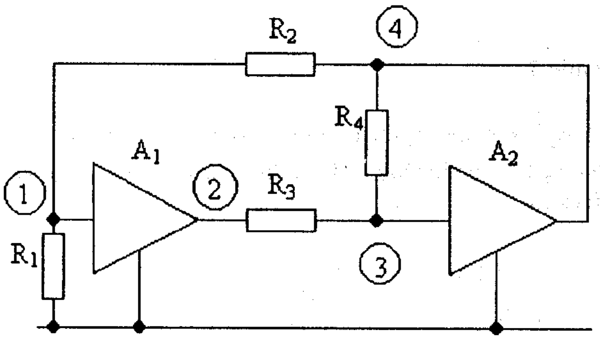

Fig.3. A network with two voltage amplifiers analyzed in Example 1

The network in Fig.3 contains 4 resistors and 2 voltage amplifiers (voltage-controlled voltage sources). If e.g. $R_1 = R_2 = R_3 = R_4 = 1$, the input immittances depend on amplification factors A_1 and A_2.

$$R_{in(1)} = \frac{2 - A_2}{den}, \quad R_{in(2)} = \frac{2}{den}, \tag{15}$$

$$R_{in(1,4)} = \frac{2 - A_2(1 + A_1)}{den}, \quad R_{in(2,3)} = \frac{2 - A_1 A_2}{den}, \tag{16}$$

$$R_{in(3,4)} = \frac{2(1 - A_2)}{den}, \quad R_{in(1,3)} = \frac{4 - 2A_2 - A_1}{den}, \tag{17}$$

$$G_{in(R_1)} = \frac{2 - A_2(1 + A_1)}{den}, \quad G_{in(R_2)} = \frac{2 - A_2}{den}, \tag{18}$$

$$G_{in(R_3)} = \frac{2(1 - A_2)}{den}, \quad G_{in(R_4)} = \frac{2 - A_1 A_2}{den}, \tag{19}$$

$$G_{in(A_2)} = \frac{4 - A_1}{den}, \tag{20}$$

where all denominators are

$$den = 4 - A_2(2 + A_1) \tag{21}$$

The results are graphically shown in the plane (A_1, A_2), see Fig.4.

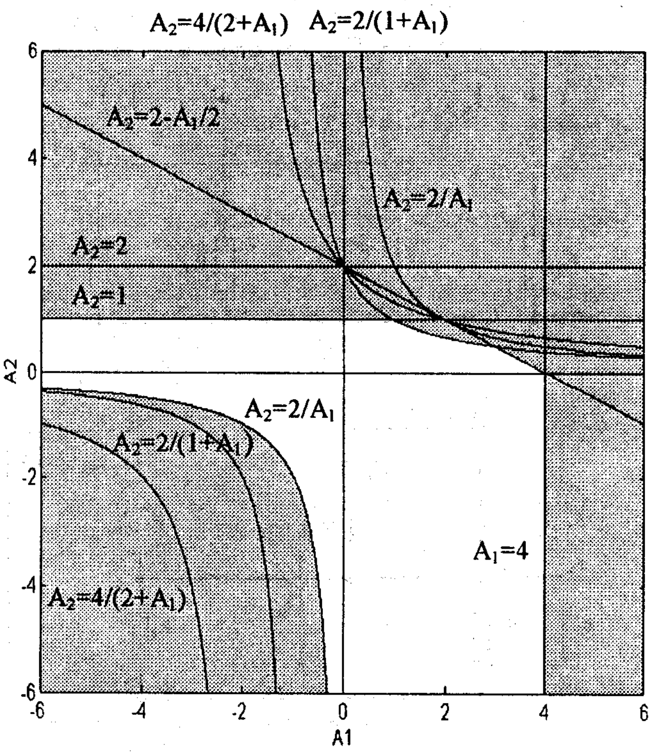

Fig.4. Boundaries of stability regions of the network in Fig.3

The hyperbola

$$den = 0, \text{ i.e. } A_2 = \frac{4}{2 + A_1}$$

determines the boundary where the denominator changes its sign. Other lines represent the loci where the individual numerators change the sign. The hatched area represents conditional instability. The rest of the plane corresponds to the combinations of amplification factors that secure stable operation of the network.

Example 2

For the transistor flip-flop in Fig.5a consider the equivalent linearized scheme in Fig.5b. For simplicity assume infinitely large input resistances of the transistors. The

collector current is proportional to the base-emitter voltage, $i_c = g_m . u_{be}$.
The admittance matrix of the network is

$$\mathbf{Y} = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -gm_1 & -\frac{1}{R_2} \\ 0 & \frac{1}{R_4} + gm_1 + gm_2 & -gm_2 \\ -\frac{1}{R_2} & 0 & \frac{1}{R_2} + \frac{1}{R_3} \end{bmatrix} \tag{22}$$

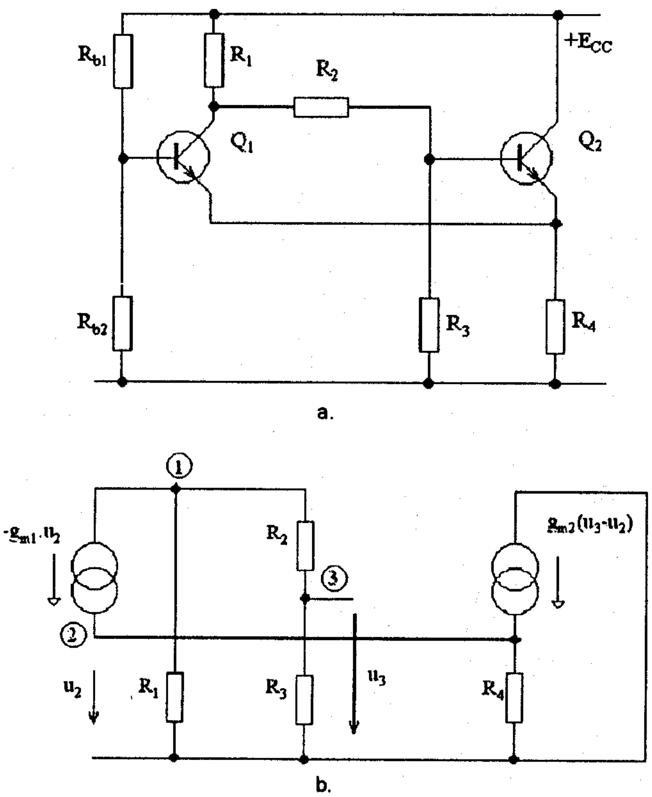

Fig.5. Two-transistor emitter coupled flip-flop network.
a. original scheme, b. linearized equivalent network

The parameters (resistances in kiloohms, transconductances in millisiemens) are:

$R_1 = 2, \ R_3 = 10, \ R_4 = 1, \ gm_1 = gm_2 = 20.$

Assume first $R_2 = 10$. The admittance matrix will be

$$\mathbf{Y} = \begin{bmatrix} 0.6 & -20 & -0.1 \\ 0 & 41 & -20 \\ -0.1 & 0 & 0.2 \end{bmatrix}$$

and the matrix of input resistances

$$\mathbf{R} = \mathbf{Y}^{-1} = \frac{1}{3549} \begin{bmatrix} -820 & -400 & -40410 \\ -200 & -11 & -1200 \\ -410 & -200 & -2460 \end{bmatrix}.$$

Most input resistances are negative due to a strong internal positive feedback in the network:

$$R_{in(1)} = -0.23105, \ R_{in(2)} = -0.0031, \ R_{in(3)} = -0.69315,$$

$$R_{in(1,2)} = -0.65089, \ R_{in(2,3)} = -0.301776.$$

Only the input resistance between nodes 1 and 3 is positive $R_{in(1,3)} = +10.5776$.

With only a single stray capacitance the network is stable only if the capacitance is connected to the nodes 1 and 3 (i.e. in parallel with R_2). If the capacitance appears in any other place, the network is unstable. A question arises whether the network can be stabilized by a simultaneous application of 2 or more capacitances with proper values.

Consider e.g. $C_1 > 0$ connected to the nodes 1-0 and $C_2 > 0$ between 1 and 3. The characteristic equation of the network,

$$-\frac{3549}{100} + \frac{41C_1 - 1877C_2}{5}\lambda + 41C_1C_2\lambda^2 = \\ = a_0 + a_1\lambda + a_2\lambda^2 = 0, \tag{23}$$

indicates that the network is unstable. The coefficients $a_0 = -3549/100$ and $a_2 = +41C_1C_2$ have different signs, independent of the values of C_1 and C_2.

Example 3

When $R_2 \to \infty$ the feedback loop is open and the situation is changed considerably. The admittance and resistance matrices are

$$\mathbf{Y} = \begin{bmatrix} 0.5 & -20 & 0 \\ 0 & 41 & -20 \\ 0 & 0 & 0.1 \end{bmatrix},$$

$$\mathbf{R} = \mathbf{Y}^{-1} = \frac{1}{41} \begin{bmatrix} 82 & 40 & 8000 \\ 0 & 1 & 200 \\ 0 & 0 & 410 \end{bmatrix}.$$

Almost all input resistances are positive

$$R_{in(1)} = 2, \ R_{in(2)} = 0.0244, \ R_{in(3)} = 10,$$

$$R_{in(1,2)} = 1.0488, \ R_{in(2,3)} = 5.146,$$

only $R_{in(1,3)} = -183.122.$

To investigate the character of the stability of the network we use again C_1 from the node 1 to the reference node and C_2 between nodes 1 and 3. The corresponding characteristic equation is in this case

$$\frac{41}{20}+\frac{41C_1-3754C_2}{10}\lambda+41C_1C_2\lambda^2=0. \quad (24)$$

The coefficients $a_0 = 41/20$ and $a_2 = 41C_1C_2$ are both positive. The network is stable if $a_1 > 0$, i.e. if the values of the capacitances are chosen so that their ratio

$$\frac{C_1}{C_2}>\frac{3754}{41}=91.56 \ .$$

3. Effective Determination of the Character of Stability

As shown above, the network under investigation (or the operating point of such a network) may be

1. stable, if all its input resistances and conductances are positive,

2. unstable, if some of the input immittances are negative and the coefficients a_0 and a_k, denoting the lowest and highest power of λ, have different signs

3. conditionally stable (conditionally unstable) if some of the input immittances are negative and the coefficients a_0 and a_k have equal signs.

It is therefore necessary to calculate the coefficients a_0 and a_k. For the coefficient a_0 we obtain

$$a_0 = \det(\mathbf{M}) \quad (25)$$

Its value can be determined easily as a by-product when inverting the matrix $\mathbf{M}$.
The coefficient a_k depends on the number of parasitic reactive elements and on their location in the network. The number k of these elements (must be $k \geq 2$) is proportional to the complexity of the network. Some of these elements are connected to the positive, the rest to the negative immittances. The network matrix $\mathbf{M}$ is transformed to obtain $\mathbf{M_b}$ such that

$$\mathbf{M_b}=\begin{bmatrix}\mathbf{A_0}+p\mathbf{A_1} & \mathbf{B_0}\\ \mathbf{C_0} & \mathbf{D_0}\end{bmatrix}, \quad (26)$$

where

$\mathbf{A_1}$ is a diagonal matrix $k \times k$,

$$\mathbf{A_1} = diag(C_1, C_2, \dots, C_k), \quad C_i, \ i=1,2,\dots,k$$

are parameters of the reactive elements (all positive).

The coefficient a_k is then

$$a_k = \det(\mathbf{A_1})\det(\mathbf{D_0}) = \prod_{i=1}^{k} C_i \det(\mathbf{D_0}), \quad (27)$$

and its sign is the same as the sign of the determinant of the submatrix $\mathbf{D_0}$.
Let us apply the procedure to the network analyzed in Example 2. The original system of node voltages

$$\mathbf{U}=[U_1, \ U_2, \ U_3]^T$$

is replaced with a new system of voltages

$$\mathbf{U}'=[U_1, \ U_1-U_3, \ U_2]^T$$

through the transformation $\mathbf{BU}'=\mathbf{U}$.

The transformation matrix

$$\mathbf{B}=\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 1 & -1 & 0\end{bmatrix}.$$

The resultant admittance matrix of the network in transformed coordinates is then

$$\mathbf{Y_b}=\mathbf{B}^T\mathbf{YB}=\begin{bmatrix}0.6 & -0.1 & -20\\ -0.1 & 0.2 & 0\\ -20 & 20 & 41\end{bmatrix}$$

The submatrix

$$\mathbf{D_0}=[41]$$

and therefore $\det(\mathbf{D_0}) = 41 > 0.$

The determinant of the admittance matrix was not affected by the transformation. Thus

$$a_0=\det(\mathbf{Y_b})=-\frac{3549}{100}<0.$$

The operating point is unstable.

4. Conclusion

The stability of DC operating points of linearized noninertial networks can be evaluated using the same set of basic equations that have been used to calculate the coordinates of the operating points themselves. Each operating point can be classified as 1. stable, 2. unstable or 3. conditionally stable (conditionally unstable). The process of classification described in the paper is quite effective and can be easily incorporated into existing programs for analysis.

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About author ...

Juraj VALSA was born in Zámčl, Czechoslovakia, in 1933. He received the M.E. degree in radioengineering at the Military Technical Academy in Brno in 1956 and the PhD. degree in 1965. He is currently professor at the Department of Theoretical and Experimental Electrical Engineering of the VUT Brno. In pedagogical and research activities he is interested mainly in the methods of CAD of linear and nonlinear electrical and electronic systems and transmission lines. During the academic year 1993-94 he spent 16 months as visiting professor at the University of Waterloo, Ontario, Canada.

DOCTORAL (PHD) THESES

Computer simulation of Switched Networks

author Karel Zaplatílek, MSc

Abstract:

Thesis deals with time domain computer simulation of networks containing externally controlled switches. The mathematical models are based on the modified nodal approach. The circuit equations are assembled in s-domain and numerical inversion Laplace transformation is used for calculation of time response in arbitrary node. Efficient algorithm for periodical steady state simulation was developed within the framework of thesis.

The developed program TSPIN can be used also for simulation of non-switched networks. The practical part of thesis is devoted to the verification of used mathematical models by means of measurement and computer simulation.

This work was conducted and successfully defended at the Brno Military Academy.

Modelling of Piecewise-Linear Systems

author Zdeněk Kolka, MSc

Abstract:

Presented thesis deals with a construction of models of piecewise linear (PL) systems which are suitable both for analysis and synthesis of electrical circuits. The PL approximation can help to find all solutions of a nonlinear system in contrast to generally used Newton's iteration schema. So called implicit state models turned out to be the most universal but they are computation-expensive. One of the goals of presented thesis was to decrease the cost by decreasing of the model order. Two algorithms for one-dimensional model construction have been developed. The first analytical method allows to design the model with no constraint and the second one reduces the model order by means of numerical procedure. The first algorithm was generalized for some two-dimensional relations.

Piecewise linear models can be also used for nonlinear dynamic circuit synthesis. The proposed method is based on well-known decomposition into functional blocks and was successfully demonstrated on realization of nonlinear oscillator of third order.

This work was conducted and successfully defended at the Technical University Brno.