SOME APPLIED ASPECTS OF RATIONAL HIGHER-ORDER S-Z TRANSFORMATIONS

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Abstract

Some contributions of higher-order s-z transforms are exposed related with the conversion of a continuous-time transfer function into its discrete-time counterpart. The developed algorithm of sensitivity analysis with respect to mapping parameters, prototype coefficients and sampling rate combined with numerical experiments can be efficiently use to motivate the selection of s-z transform and thereby to provide a suitable basis for an optimal solution of a general design problem.

Keywords

s-z transformation, mapping, sensitivity matrix, prewarping

1. Introduction

A derivation of a discrete transfer function from its continuous time prototype represents a significant stage in discrete circuit design. In principle, the use of higher-order s-z transforms [1-8] leads to variety of alternative expressions. The choice can be made provided that major properties such as complexity, frequency-frequency linearity, stability, sensitivity etc. [9] are already known. Currently only partial solutions of this problem have been proposed. The method of s.c. sequence of expressions [10] suggests the sensitivity analysis to be implemented at multiple frequency points by a re-evaluation of equations previously generated. Alternatively, the sensitivity functions are developed in terms of a discrete-frequency variable [11]. In both cases the inevitable involvement of the current frequency only complicates the solution.

Here below an attempt is made to consider the impact of the sampling frequency and the basic parameters on the transfer function performance. In result, some stages of the general solution are determined and thereafter combined with numerical experiments to complete the study. The parameter-dependent analysis is developed and sensitivity functions with respect to the sampling rate and sets of mapping parameters and prototype transfer coefficients are found. The prewarping procedure carried out towards the poles and zeros is also included. In effect, all these helpfully assist to implement an extensive comparative analysis and to clarify some major features of discrete system functions obtained via different s-z mappings. The bilinear version is taken throughout the text to serve as a reference and Monte-Carlo simulations are also implemented in parallel to produce alternative assessments.

2. Evaluation procedure of discrete-time transfer functions

Let a discrete circuit has to be designed on the base of a continuous-time transfer function:

$$H(s) = K_o\left[\prod_{k=1}^{M}\left(S - s_{z,k}T\right)\right]\left[\prod_{k=1}^{N}\left(S - s_{p,k}T\right)\right]^{-1} = \\ = K_o\left[\sum_{k=0}^{M} b_k S^k\right]\left[\sum_{k=0}^{N} a_k S^k\right]^{-1}, \quad M \le N. \tag{1}$$

Here the complex frequency variable $s = \sigma \pm j\omega$ is weighted by the sampling period $S = sT$, $T = 1/f_c$. K_o is the gain cofactor $\left(a_0 = b_0 = 1\right)$.

The poles and zeros of the prototype (1) are specified as

$$s_{p,k} = \sigma_{p,k} \pm j\omega_{p,k}, \qquad s_{z,k} = \sigma_{z,k} \pm j\omega_{z,k} \tag{2}$$

The desired discrete-time transfer function is formulated as

$$H(z) = K_o\left[\sum_{k=0}^{KL} B_k z^{-k}\right]\left[\sum_{k=0}^{KL} A_k z^{-k}\right]^{-1}, \\ z = \exp\left(\sigma T \pm j\omega T\right), K = \max\left\{M, N\right\}, L = \max\left\{m, n\right\} \tag{3}$$

in the discrete-time domain after processing the prototype (1) by a certain s-z transform

$$\frac{1}{s^* T} = F(z) = \left[\sum_{k=0}^{m} \beta_k z^{-k}\right]\left[\sum_{k=0}^{n} \alpha_k z^{-k}\right]^{-1} \tag{4}$$

In this expression the mapping parameters $\left\{\alpha_k, \beta_k\right\}$ have been subject to a prior normalization with respect to $\max\left\{\left|\alpha_k\right|, \left|\beta_k\right|\right\}$. The general expression (4) proves to be effective not only for mappings based on the classical numerical integration methods produced by a polynomial approximation:

$$x_{i+1}=\sum_{k=0}^{n}a_k x_{i-k}+h\sum_{k=-1}^{m}\beta_k f\left(x_{i-k},t_{i-k}\right), \tag{5}$$

but also in case of various nonconventional methods. In Table 1 a selection of perspective s-z transforms defined on the ground of an equal maximal sampling rate is presented.

Normally, the prototype function (1) is subject to a prior prewarping procedure. Some algorithms suggest the prewarped transfer function $H^*(s)$ to be obtained preserving at least the shape of the prototype frequency response. Usually, cutoff frequencies or imaginery parts of the poles are involved in the respective re-evaluation [9].

Alternatively, it has been offerred [12] to carried out the prewarping procedure over both parts of poles and zeros uniformly denoted $s_r=\sigma_r\pm j\omega_r$. Substituting all powers of $z=\exp(s_r T)$ in (4) by their Euler equivalents the following relation holds:

$$\sigma_r^* T\pm j\omega_r^* T=$$
$$=\frac{\sum_{k=0}^{n}\sum_{l=0}^{m}\alpha_k\beta_l\exp\left[-(k+l)\sigma_r T\right]\left\{\cos\left[(l-k)\omega_r T\right]\pm j\sin\left[(l-k)\omega_r T\right]\right\}}{\sum_{k=0}^{m}\sum_{l=0}^{m}\beta_k\beta_l\exp\left[-(k+l)\sigma_r T\right]\cos\left[(l-k)\omega_r T\right]} \tag{6}$$

that can be used to evaluate the prewarped poles or zeros $s_r^*=\sigma_r^*\pm j\omega_r^*$. In turn, the prewarped transfer function

$$H^*(s)=K_o\left[\prod_{k=1}^{M}\left(S-s_{z,k}^*T\right)\right]\left[\prod_{k=1}^{N}\left(S-s_{p,k}^*T\right)\right]^{-1}= \tag{7}$$
$$=K_o\left[\sum_{k=0}^{M}b_k^* S^k\right]\left[\sum_{k=0}^{N}a_k^* S^k\right]^{-1},\quad M\le N$$

is obtained replacing all poles and zeros in expression (1) by their predistorted counterparts:

$$s_{p,k}^*=\sigma_{p,k}^*\pm j\omega_{p,k}^*,\quad s_{z,k}^*=\sigma_{z,k}^*\pm j\omega_{z,k}^* \tag{8}$$

Table 1. Some higher-order s-z transforms

Ref.	Mapping function F(z)	Ref.	Mapping function F(z)
	ADAMS MOULTON discrete integration rules		GRAHAM-LINDQUIST discrete integrators [1]
AM2	$\frac{1+z^{-1}}{2\left(1-z^{-1}\right)}$	H021	$\frac{2+4z^{-1}}{5-4z^{-1}-z^{-2}}$
AM3	$\frac{5+8z^{-1}-z^{-2}}{12\left(1-z^{-1}\right)}$	H031	$\frac{6+18z^{-1}}{17-9z^{-1}-9z^{-2}+z^{-3}}$
AM4	$\frac{9+19z^{-1}-5z^{-2}+z^{-3}}{24\left(1-z^{-1}\right)}$	H041	$\frac{12+48z^{-1}}{37-8z^{-1}-36z^{-2}+8z^{-3}-z^{-4}}$
			DOSTAL parametric transformations (a=.2927) [3]
AM5	$\frac{251+64z^{-1}-264z^{-2}+106z^{-3}-19z^{-4}}{720\left(1-z^{-1}\right)}$	TD1	$\frac{1+az^{-1}}{(1+a)\left(1-z^{-1}\right)}$
	MILN-SIMPSON discrete integration rules		(k=0.1) [4]
MS2	$\frac{2z^{-1}}{1-z^{-2}}$	TD4	$\frac{1+16k+4z^{-1}+(22-32k)z^{-2}+4z^{-3}+(1+16k)z^{-4}}{8\left(1+2z^{-1}-2z^{-3}-z^{-4}\right)}$
			TIK discrete integration rule [5]
MS3	$\frac{1+4z^{-1}+z^{-2}}{3\left(1-z^{-2}\right)}$	TIK	$\frac{1+3.5804z^{-1}+z^{-2}}{2.7902\left(1-z^{-2}\right)}$
	HAMMING discrete integration rules		AL-ALAOUI discrete differentiator [6]
HA1/2	$\frac{17+51z^{-1}+3z^{-2}+z^{-3}}{24\left(2-z^{-1}-z^{-2}\right)}$	ALA	$\frac{1+0.5358z^{-1}+0.0718z^{-2}}{0.8039\left(1-z^{-2}\right)}$
			LE BIHAN (X=0.793) discrete integrator [7]
HA2/3	$\frac{25+91z^{-1}+43z^{-2}+9z^{-3}}{24\left(3-2z^{-1}-z^{-2}\right)}$	LEB	$\frac{1-\chi+(1+\chi)z^{-1}}{2\left(1-z^{-1}\right)}$
			GUROVA-GEORGIEV transformation [8]
HA1/3	$\frac{26+73z^{-1}+30z^{-2}+10z^{-3}}{24\left(3-z^{-1}-z^{-2}-z^{-3}\right)}$	NLT	$\frac{1+3.8765z^{-1}+z^{-2}}{2.9382\left(1-z^{-2}\right)}$

The new coefficients $\{a_i^*, b_i^*\}$ are easily computed using the well-known elementary symmetric functions [13]:

$$a_i^* = (-1)^{N-i} \sum_{1 \le l_1 < \ldots < l_i \le N} \left[\prod_{r=1}^{i} s_{l_r,p}^* T \right], \quad (9)$$

$$b_i^* = (-1)^{M-i} \sum_{1 \le l_1 < \ldots < l_i \le M} \left[\prod_{r=1}^{i} s_{l_r,z}^* T \right], \quad (10)$$

Next, the s.c. Plug-in-Expansion method [2] is applied to evaluate the required output coefficients $\{A_k, B_k\}$ in (3):

$$A_k = \sum_{i=0}^{N} \left[a_i^* \sum_{l=0}^{k} \left(c_{i,l} d_{n-i,k-l} \right) \right], \quad k \in (0, NL), \quad (11)$$

$$B_k = \sum_{i=0}^{M} \left[b_i^* \sum_{l=0}^{k} \left(c_{i,l} d_{n-i,k-l} \right) \right], \quad k \in (0, ML), \quad (12)$$

where provisional quantities denoted $c_{i,l}$ and $d_{n-i,k-l}$ are evaluated after polynomials in (4) have been raised to integer power of $i \in (0, K)$:

$$\left(\alpha_0 + \alpha_1 z^{-1} + \ldots + \alpha_n z^{-n} \right)^i = c_{i,0} + c_{i,1} z^{-1} + \ldots + c_{i,ni} z^{-ni}, \quad (13)$$

$$\left(\beta_0 + \beta_1 z^{-1} + \ldots + \beta_n z^{-n} \right)^i = d_{i,0} + d_{i,1} z^{-1} + \ldots + d_{i,ni} z^{-ni}. \quad (14)$$

A simple algorithm to compute coefficients $c_{i,l}$ and $d_{n-i,k-l}$ is shown in the Appendix A.

3. Estimates of frequency response deviations

Some frequency response estimates such as absolute deviation $\delta(\omega)$ between $H(S)$ and $H(z)$ given in (1) and (3)

$$\delta(\omega) = \left| H(S = j\omega T) \right| - \left| H[z = \exp(j\omega T)] \right|, \quad (15)$$

the maximal deviation $\delta_{\max}(\omega_m)$ and the mean value δ_{IAE}

$$\delta_{IAE} = \frac{1}{\omega_2 - \omega_1} \int_{\omega_1}^{\omega_2} |\delta(\omega)| d\omega, \quad \omega \in (\omega_1, \omega_2) \quad (16)$$

are evaluated to illustrate the capacity of the procedure described above.

In Table 2 the results of two numerical design examples are presented. In the first example the bilinear version appears to be superior while in the second one some other mappings indicate superiority. Obviously, the "best" choice is definitely related with the design problem of consideration and looking for an optimal solution a decision have to be made in any particular case.

Table 2. Higher-order s-z transforms vs frequency response estimates

	Numerical example No 1 [11]: 4-th order 40 Hz band-pass filter with central frequency $f_0 = 1\,\text{kHz}$, $f_c = 8\,\text{kHz}$, $f \in (0, 2\,\text{kHz})$			Numerical example No 2 [12]: 6-th order 2 kHz band-pass filter with central frequency $f_0 = 2.4\,\text{kHz}$, $f_c = 32\,\text{kHz}$, $f \in (0, 8\,\text{kHz})$		
gain K_o	-0.078646895			-0.048719272		
poles	-87.766252 ± j6188.2513, -89.742324 ± j6376.2582			-2854.8133 ± j22177.552, -4999.1447 ± j12849.367, -1076.3239 ± j8145.0575		
zeros	± j5754.6882, ± j6847.0533			± j38773.293, ± j4153.8122, 0.0		
rule	δ_{IAE}, dB	$\delta_{\max}$, dB	ω_m, s^{-1}	δ_{IAE}, dB	$\delta_{\max}$, dB	ω_m, s^{-1}
AM2	.0005	-.0016	1003	.5811	-.7289	5369
AM3	.0084	-.0037	1006	1.3029	.5789	5441
AM4	.0045	.0077	989	1.2729	.4864	5436
AM5	.0152	-.0067	1011	.2133	-.2880	5378
MS2	.0161	.0020	940	.2362	.4592	5434
MS3	.0018	.0005	944	.0256	-.0494	5407
HA1/2	.0030	.0003	1051	.3847	.0253	809
HA2/3	.0014	-.0006	1020	.1146	-.0818	5405
HA1/3	.0020	-.0005	1016	.4533	-.1500	5156
H021	.0091	.0026	1061	1.1967	.4492	5434
H031	.0028	.0005	1003	.3723	.0304	5411
H041	.0055	.0010	942	.2742	-.1516	5401
TD1	.0032	.0059	971	4.4943	93.3018	6171
TD4	.0288	-.0021	1062	1.3308	-1.1146	5347
TIK	.0015	.0005	991	.0915	-.1359	5402
ALA	.0019	-.0006	1019	8.0252	101.0826	6171
LEB	.3754	1.0756	1018	10.3201	98.3522	661
NTL	.0018	.0007	945	0.0426	-.0730	5405

4. Sensitivity estimates with respect to the initial parameters

In fact, the required coefficients $\{A_k, B_k\}$ determined by (11) and (12) are functions of (1) the prototype coefficients $\{a_j, b_j\}$ and (2) the mapping coefficients $\{\alpha_r, \beta_r\}$. Without loss of generality the next consideration is done in terms of $\{s_{p,k}, s_{z,k}\}$ and $\{s^*_{p,k}, s^*_{z,k}\}$ assuming that $\{a_j, b_j\}$ and $\{a^*_j, b^*_j\}$ can be always found from (9) and (10).

In what follows a procedure to assess the logarithmic sensitivity of coefficients $\{A_k, B_k\}$ with respect to the both sets of parameters $\{\sigma_r, \omega_r\}$ and $\{\alpha_r, \beta_r\}$ mentioned above is developed. Vectors **x** and **p** are introduced accounting for the parameters involved:

$$\mathbf{x} = \left[\{\sigma_{r,p}\} \vdots \{\omega_{r,p}\} \vdots \{\sigma_{r,z}\} \vdots \{\omega_{r,z}\} \vdots \{\alpha_r\} \vdots \{\beta_r\}\right]^T$$
$$\mathbf{p} = [\mathbf{A} \vdots \mathbf{B}]^T = \left[A_0\ A_1 \cdots A_{KL+1}\ B_0\ B_1 \cdots B_{KL+1}\right]^T \tag{17}$$

Henceforth, boldface letters indicate vectors and matrices and the superscript T denotes transpose.

The sensitivity matrix defined as $\mathbf{S}^p_x \approx \mathbf{x}\partial\mathbf{p} / \mathbf{p}\partial\mathbf{x}$ is:

$$\mathbf{S}^p_x = \begin{bmatrix}\mathbf{S_A} \\ \mathbf{S_B}\end{bmatrix} = \begin{bmatrix}\mathbf{S}^{\mathbf{A,B}}_{\sigma,\omega} \vdots \mathbf{S}^{\mathbf{A,B}}_{\alpha,\beta}\end{bmatrix} = \begin{bmatrix}\mathbf{S}^{\mathbf{A}}_{\sigma,p}\ \mathbf{S}^{\mathbf{A}}_{\omega,p} & \mathbf{0} & \mathbf{0} & \vdots \mathbf{S}^{\mathbf{A}}_{\alpha}\ \mathbf{S}^{\mathbf{A}}_{\beta} \\ \mathbf{0} & \mathbf{0} & \mathbf{S}^{\mathbf{B}}_{\sigma,z}\ \mathbf{S}^{\mathbf{B}}_{\omega,z} & \vdots \mathbf{S}^{\mathbf{B}}_{\alpha}\ \mathbf{S}^{\mathbf{B}}_{\beta}\end{bmatrix},$$
$$\mathbf{S}^p_x \in \Re^{(2KL+2)x(2KL+2L+2)} \tag{18}$$

The block-matrix $\mathbf{S}^{\mathbf{A,B}}_{\sigma,\omega}$ is decomposed into sub-matrices with equal dimensions $\Re^{(KL+1)xK}$, their elements express the individual sensitivities with respect to the original poles and zeros. It can be shown that the following matrix factorisations hold:

$$\mathbf{S}^{\mathbf{A}}_{\sigma,p} = \mathbf{S}^{\mathbf{A}}_{a}\mathbf{P}_a\mathbf{P}_\sigma, \quad \mathbf{S}^{\mathbf{B}}_{\sigma,z} = \mathbf{S}^{\mathbf{B}}_{b}\mathbf{Z}_b\mathbf{Z}_\sigma,$$
$$\mathbf{S}^{\mathbf{A}}_{\omega,p} = \mathbf{S}^{\mathbf{A}}_{a}\mathbf{P}_a\mathbf{P}_\omega, \quad \mathbf{S}^{\mathbf{B}}_{\omega,z} = \mathbf{S}^{\mathbf{B}}_{b}\mathbf{Z}_b\mathbf{Z}_\omega \tag{19}$$

Likewise, the block-matrix $\mathbf{S}^{\mathbf{A,B}}_{\alpha,\beta}$ is decomposed into sub-matrices with equal dimensions $\Re^{(KL+1)x(L+1)}$, their elements determine the individual sensitivities with respect to the mapping coefficients. Similarly,

$$\mathbf{S}^{\mathbf{A}}_{\alpha} = \mathbf{S}^{\mathbf{A}}_{c,\alpha} + \mathbf{S}^{\mathbf{A}}_{a}\mathbf{P}_a\mathbf{P}_\alpha, \quad \mathbf{S}^{\mathbf{B}}_{\alpha} = \mathbf{S}^{\mathbf{B}}_{c,\alpha} + \mathbf{S}^{\mathbf{B}}_{b}\mathbf{Z}_b\mathbf{Z}_\alpha,$$
$$\mathbf{S}^{\mathbf{A}}_{\beta} = \mathbf{S}^{\mathbf{A}}_{d,\beta} + \mathbf{S}^{\mathbf{A}}_{a}\mathbf{P}_a\mathbf{P}_\beta, \quad \mathbf{S}^{\mathbf{B}}_{\beta} = \mathbf{S}^{\mathbf{B}}_{d,\beta} + \mathbf{S}^{\mathbf{B}}_{b}\mathbf{Z}_b\mathbf{Z}_\beta \tag{20}$$

The detailed description of all matrix factors denoted on the right-hand side in (19) and (20) is given in Appendix B.

Looking for a general solution, the well-known matrix norms $\|.\|$ [13] can be introduced to assess the sensitivity. $\|\mathbf{S}\|_F$ gives a global sensitivity estimate. $\|\mathbf{S}^{\mathbf{A,B}}_{\sigma,\omega}\|_F$ offers separate assessments of the prototype coefficient's influence. $\|\mathbf{S}^{\mathbf{A,B}}_{\alpha,\beta}\|_F$ and $\|\mathbf{S}^{\mathbf{A,B}}_{\alpha,\beta}\|_1$ measure the contribution of the mapping parameters, $\|\mathbf{S}\|_\infty$ and $\|\mathbf{S}_{\alpha,\beta}\|_1$ expose the most sensitive output coefficient and the most influential mapping parameter respectively. What is more important, the last two norms assess the precision of the initial coefficient specification required to achieve the prescribed precision of the output coefficients. The most sensitive element $|S_{i,j}|_{\max}$ of the matrix **S** indicates the highest individual sensitivity.

In Table 3 sensitivity estimates obtained from the second example are presented. Data of bilinear version are given in absolute values and included in the first line but the others are presented in a quotient form to their bilinear counterparts. The data in the last column denoted Δ are produced by Monte-Carlo procedure. They measure the area of a tolerance field bounded by the upper and the lower worst-case magnitude values in the frequency range $f \in (f_{\min}, f_{\max})$. During 100 simulations the coefficients $\{A_k, B_k\}$ have been computed in terms of parameters $\{\alpha_k, \beta_k\}$ perturbed randomly by up to $\pm 0.1\%$ and subsequently a frequency response at 500 frequency points has been evaluated.

Comparing the last column with the others one can find out certain correlation between corresponding data. Hence the norms may serve as reliable sensitivity estimates. The contributions specific for each type of mappings and each kind of parameters can be also distinguished.

5. Sensitivity estimates with respect to the sampling rate

The following analysis is done in a classical way in terms of the period T instead of $f_c = 1/T$. Implying the coefficients $\{a^*_j(s^*_r), b^*_j(s^*_r)\}$ are implicit function of T and using (3) one can define $S^H_T \approx T dH/H dT$ as

$$S^H_T = \frac{\sum_{k=0}^{KL} \frac{dB_k}{dT} z^{-k} - \sum_{k=0}^{KL} ksB_k z^{-k+1}}{T^{-1}\sum_{k=0}^{KL} B_k z^{-k}} - \frac{\sum_{k=0}^{KL} \frac{dA_k}{dT} z^{-k} - \sum_{k=0}^{KL} ksA_k z^{-k+1}}{T^{-1}\sum_{k=0}^{KL} A_k z^{-k}} \tag{21}$$

Table 3. Higher-order s-z transforms vs sensitivity matrix estimates

Numerical example No 2 [12]: 6-th order 2 kHz band-pass filter with central frequency $f_0 = 2.4\,\text{kHz}$, $f_c = 32\,\text{kHz}$, $f \in (0, 8\,\text{kHz})$

rule \ estimate	$\lVert S \rVert_F$	$\lVert S^{A,B}_{\sigma,\omega} \rVert_F$	$\lVert S^{A,B}_{\alpha,\beta} \rVert_F$	$\lVert S \rVert$	$\lVert S_{\alpha,\beta} \rVert_1$	$\lvert S_{i,j} \rvert_{max}$	$\Delta_{\alpha,\beta}\,[dBs^{-1}]$
	absolute values						
AM2	427.00	13.98	426.80	313.70	962.80	133.20	183.0
	relative values						
AM3	2.04	1878	1.94	2.73	2.34	2.60	2.39
AM4	1.79	17.94	1.69	1.73	2.54	1.06	3.04
AM5	2.01	25.59	1.90	1.79	2.90	.94	2.62
MS2	.01	.03	.01	.02	.01	.02	1.13
MS3	.30	2.11	.29	.53	.22	.52	1.01
HA1/2	1.47	1.59	1.47	3.56	.81	2.99	2.07
HA2/3	.18	1.23	1.78	.29	.19	.25	1.32
HA1/3	.39	1.55	.38	.71	.39	.61	1.53
H021	2.07	11.04	2.04	5.02	1.44	5.19	2.22
H031	1.84	1.47	1.84	4.18	1.07	2.82	2.29
H041	2.61	16.39	2.55	7.38	1.51	6.70	2.07
TD1	1.28	1.64	1.27	1.17	1.12	1.28	3.37
TD4	1.87	12.34	1.82	2.39	2.33	2.22	1.15
TIK	.54	5.11	.55	1.28	.35	1.16	1.02
ALA	8.24	38.43	8.15	20.68	3.92	22.16	2.64
LEB	104.00	1.44	104.10	212.60	40.59	28.30	4.43
NTL	.34	2.61	.33	.66	.25	.63	1.01

Next, two vectors are introduced denoted $\mathbf{z}, \widetilde{\mathbf{z}} \in \Im^{(KL+1)}$:

$$\mathbf{z} = \left[z^0\ z^{-1}\ z^{-2} \cdots z^{-KL}\right]^T,$$

$$\widetilde{\mathbf{z}} = \frac{d\mathbf{z}}{dT} = \left[\frac{dz^0}{dT}\ \frac{dz^{-1}}{dT}\ \frac{dz^{-2}}{dT} \cdots \frac{dz^{-KL}}{dT}\right]^T, \tag{22}$$

and two singular row-matrices $\mathbf{A}, \mathbf{B} \in \Re^{(KL+1)x(KL+1)}$ are constituted:

$$\mathbf{A} = \{A_k\} = \begin{bmatrix} A_0\ A_1 \cdots A_{KL} \\ \mathbf{0} \end{bmatrix}, \mathbf{B} = \{B_k\} = \begin{bmatrix} B_0\ B_1 \cdots B_{KL} \\ \mathbf{0} \end{bmatrix} \tag{23}$$

Likewise, matrices denoted $\widetilde{\mathbf{A}}, \hat{\mathbf{A}}$ and $\widetilde{\mathbf{B}}, \hat{\mathbf{B}}$ are formulated:

$$\widetilde{\mathbf{A}} = \{dA_k/dT\}, \quad \hat{\mathbf{A}} = \{kA_k\}, \qquad k \in (0, KL), \tag{24}$$

$$\widetilde{\mathbf{B}} = \{dB_k/dT\}, \quad \hat{\mathbf{B}} = \{kB_k\}, \qquad k \in (0, KL) \tag{25}$$

Derivatives denoted above are determined in a product form using

$$\frac{da_i^*}{dT} = \sum_{k=0}^{N} \frac{da_i^*}{dS_{k,p}^*} \frac{d\left(\sigma_{k,p}^* T \pm j\omega_{k,p}^* T\right)}{dT},$$

$$\frac{db_i^*}{dT} = \sum_{k=0}^{M} \frac{db_i^*}{dS_{k,z}^*} \frac{d\left(\sigma_{k,z}^* T \pm j\omega_{k,z}^* T\right)}{dT}, \tag{26}$$

in the products $\dfrac{\partial A_k}{\partial a_i^*}\dfrac{da_i^*}{dT}, \dfrac{\partial B_k}{\partial b_i^*}\dfrac{db_i^*}{dT}$.

The first cofactors in (26) are found from (9) and (10), but expression (6) is used to derive the second cofactors.

It can be shown that the next relations hold:

$$\mathbf{B}\widetilde{\mathbf{z}} = -s\hat{\mathbf{B}}\mathbf{z}, \quad \mathbf{A}\widetilde{\mathbf{z}} = -s\hat{\mathbf{A}}\mathbf{z}, \tag{27}$$

Finally, substituting (22)-(25) and (27) in (21) and assuming $s = j\omega$ one can derive the result given below:

$$S_T^H = S_{\text{Re}} + jS_{\text{Im}} = T\frac{\widetilde{\mathbf{B}}\mathbf{z} - \hat{\mathbf{B}}\mathbf{z}}{\mathbf{B}\mathbf{z}} - T\frac{\widetilde{\mathbf{A}}\mathbf{z} - \hat{\mathbf{A}}\mathbf{z}}{\mathbf{A}\mathbf{z}} = \left(\frac{\widetilde{\mathbf{B}}\mathbf{z}}{\mathbf{B}\mathbf{z}} - \frac{\widetilde{\mathbf{A}}\mathbf{z}}{\mathbf{A}\mathbf{z}}\right) - j\omega T\left(\frac{\hat{\mathbf{B}}\mathbf{z}}{\mathbf{B}\mathbf{z}} - \frac{\hat{\mathbf{A}}\mathbf{z}}{\mathbf{A}\mathbf{z}}\right) \tag{28}$$

In fact, the matrix ratios given above represent real numbers but the coefficient S_T^H can not be evaluated straightforward. The reason is that matrices $\mathbf{A}$ and $\mathbf{B}$ are noninvertible because of matrix singularity. However, S_T^H could be assessed by means of ratios $\mathrm{N}_A/\mathrm{M}_A$ valid for an arbitrary vector $\mathbf{z}$ ([13], ch.6):

$$\frac{\lVert\widetilde{\mathbf{A}}\mathbf{z}\rVert}{\lVert\mathbf{A}\mathbf{z}\rVert} = \frac{\lVert\widetilde{\mathbf{A}}\mathbf{z}\rVert/\lVert\mathbf{z}\rVert}{\lVert\mathbf{A}\mathbf{z}\rVert/\lVert\mathbf{z}\rVert} \le \frac{\mathrm{N}_A}{\mathrm{M}_A} \tag{29}$$

where for $z \neq 0$ the following expressions are valid:

$$\mathrm{N}_A=\sup\frac{\|\tilde{\mathbf{A}}\mathbf{z}\|_E}{\|\mathbf{z}\|_E}=\|\tilde{\mathbf{A}}\|_E,\ \mathrm{M}_A=\inf\frac{\|\mathbf{A}\mathbf{z}\|_E}{\|\mathbf{z}\|_E}=\frac{1}{\|\mathbf{A}^{-1}\|_E}\to 0 \quad (30)$$

respecting the singularity of both matrices in (23). Setting up $\mathrm{M}_A=\mathrm{M}_B=\varepsilon\to 0$, one can write

$$|S_{\mathrm{Re}}|<\Sigma_R=\frac{T}{\varepsilon}\left|\|\tilde{\mathbf{B}}\|_E-\|\tilde{\mathbf{A}}\|_E\right|, |S_{\mathrm{Im}}|<\omega\Sigma_I=\omega\frac{T}{\varepsilon}\left|\|\hat{\mathbf{B}}\|_E-\|\hat{\mathbf{A}}\|_E\right| \quad (31)$$

However, the factor ε will disappear taking estimates in a quotient form as mentioned above.

A couple of parameters are introduced to complete the analysis evaluated for $\omega\in(\omega_{\min},\omega_{\max})$:

$$\Gamma_1=arctg(\Sigma_I/\Sigma_R),\ \Gamma_2=\sqrt{\Sigma_R^2+(\omega\Sigma_I)^2} \quad (32)$$

Unlike all other estimates considered to this point Γ_2 is the only frequency dependent one. Alternatively, the worst case value $\Gamma_2(\omega=\omega_{\max})$ can be used.

In Table 4 sensitivity estimates Σ_R and Σ_I valid for the second example are presented. Data are arranged as in Table 3. Monte-Carlo sensitivity analysis is implemented as described above towards $H[z=\exp(j\omega T)]$. A couple of expressions are defined as

$$\Delta_1=arctg(|H_{\mathrm{Im}}|/\omega|H_{\mathrm{Re}}|),\quad \Delta_2=\left(\sqrt{H_{\mathrm{Re}}^2+H_{\mathrm{Im}}^2}\right) \quad (33)$$

They are evaluated for $\omega\in(\omega_{\min},\omega_{\max})$ to measure the area of tolerance fields. The pseudo-random perturbations within the range $\pm 0.001 f_c$ are generated to determine the sampling rate impact on the output coefficients $\{A_k, B_k\}$. Comparing data in columns 3 and 4 with those in columns 5 and 6 respectively one can establish a certain correlation between data. Hence, they can be used to assess the sensitivity in this case and the contributions specific for each mapping can be also distinguished.

6. Conclusions

A compact procedure to derive a discrete-time transfer function from its continuous-time prototype employing rational higher-order s-z transform functions is presented. An algorithm based on some matrix norms is proposed to assess the output coefficient sensitivity with respect to the prototype coefficients, the mapping parameters and the sampling rate. All operations are completely formalised. The estimate values appears to be specific for each s-z transform and represent a reliable basis to discuss the mapping applied properties. The developed analysis combined with numerical experiments can be efficiently used to motivate the selection of s-z transformation and thereby to provide a suitable base for an optimal design solution.

Table 4. Higher-order s-z transforms vs sensitivity estimates with respect to the sampling rate

Numerical example No 2 [12]:
6-th order 2 kHz band-pass filter with central frequency $f_0=2.4$ kHz, $f_c=32$ kHz, $\omega T\in(0,\pi/2)$

	Σ_R	Σ_I	Γ_1	$\Gamma_2\ (\omega T=\pi/2)$	Δ_1	Δ_2
	absolute values					
AM2	8.1661	7.2746	.51589	10.936	38.800	118.0
	relative values					
AM3	.84	.99	1.14	.91	.99	.82
AM4	.59	1.00	1.47	.80	.95	1.24
AM5	.16	1.01	2.51	.68	.91	1.83
MS2	.40	3.00	2.59	2.02	1.96	2.42
MS3	.57	2.28	2.23	1.57	2.16	1.32
HA1/2	.51	1.22	1.79	.90	1.50	1.29
HA2/3	.63	2.08	2.09	1.46	2.55	1.41
HA1/3	.65	1.39	1.70	1.04	1.95	1.32
H021	.61	.60	.99	.60	1.74	.90
H031	.60	.56	1.78	1.04	1.51	1.28
H041	.96	1.41	1.98	1.99	1.94	1.38
TD1	.79	.99	1.20	.88	1.34	.92
TD4	.81	4.72	2.47	3.20	3.94	4.88
TIK	.56	2.25	2.23	1.55	2.17	1.27
ALA	.29	2.01	2.55	1.36	2.25	.97
LEB	278.23	23.44	.09	208.35	1.88	5.93
NTL	.57	2.27	2.23	1.57	2.16	1.31

Appendix A

Suppose coefficients $c_{i,l}$ and $d_{n-i,k-l}$ in (11)-(14) are elements of two sets of vectors denoted $\mathbf{c}_i$, $i \in (1, NL+1)$ and $\mathbf{d}_i$, $i \in (1, ML+1)$. Thus, two quasi-triangular matrices are defined:

$$\mathbf{C} = \begin{bmatrix} \mathbf{c}_0^T & \mathbf{0} \\ \mathbf{c}_1^T & \\ \cdots & \\ \mathbf{c}_{NL+1}^T & \end{bmatrix} \in \Re^{(KL+1)\times(KL+1)}, \mathbf{D} = \begin{bmatrix} \mathbf{d}_0^T & \mathbf{0} \\ \mathbf{d}_1^T & \\ \cdots & \\ \mathbf{d}_{NL+1}^T & \end{bmatrix} \in \Re^{(KL+1)\times(KL+1)} \quad (A1)$$

Obviously, their first two lines look as follows:

$$\begin{aligned} \mathbf{c}_0^T &= [1\ 0\ 0\ \dots\ 0],\ \mathbf{d}_0^T = [1\ 0\ 0\ \dots\ 0], \\ \mathbf{c}_1^T &= [\alpha_0\ \alpha_1\ \alpha_2\ \dots\ 0],\ \mathbf{d}_1^T = [\beta_0\ \beta_1\ \beta_2\ \dots\ 0] \end{aligned} \quad (A2)$$

A simple procedure can be offered to find the others:

1.Define a circulant matrix $\mathbf{P}$ [13] in a form

$$\mathbf{P} = \begin{bmatrix} 0 & 1 & & & \mathbf{0} \\ & 0 & 1 & & \\ & & \ddots & \ddots & \\ & & & 0 & 1 \\ \mathbf{0} & & & & 0 \end{bmatrix} \in \Re^{(K+1)\times(K+1)}, K = \max(M, N) \quad (A3)$$

2. Compute a couple of auxiliary band matrices $\mathbf{Q}_c \in \Re^{KxK}$ and $\mathbf{Q}_d \in \Re^{KxK}$ in a row-by-row sequence with an initial step

$$\mathbf{Q}_{c,1} = \mathbf{Q}_{c,0}\mathbf{P} = \begin{bmatrix} \mathbf{c}_1^T \\ 0 \\ \cdots \\ 0 \end{bmatrix}\mathbf{P}, \quad \mathbf{Q}_{d,1} = \mathbf{Q}_{d,0}\mathbf{P} = \begin{bmatrix} \mathbf{d}_1^T \\ 0 \\ \cdots \\ 0 \end{bmatrix}\mathbf{P}, \quad (A4)$$

using the recurrent relations $\mathbf{Q}_{c,i+1} = \mathbf{Q}_{c,i}\mathbf{P}$, $\mathbf{Q}_{d,i+1} = \mathbf{Q}_{d,i}\mathbf{P}$. Hence, any element belonging to the $i+1-$ st row can be defined as

$$q(i+1, l) = \sum_{i=1}^{K}\sum_{l=1}^{K} q(i,\cdot)p(\cdot,l) \quad (A5)$$

3.Finally, compute both required matrices $\mathbf{C} = \mathbf{C}_K$ and $\mathbf{D} = \mathbf{D}_K$ employing the same scheme and starting with

$$\mathbf{C}_1 = \mathbf{C}_0\mathbf{Q}_c = \begin{bmatrix} \mathbf{c}_0^T \\ 0 \\ \cdots \\ 0 \end{bmatrix}\mathbf{Q}_c, \quad \mathbf{D}_1 = \mathbf{D}_0\mathbf{Q}_d = \begin{bmatrix} \mathbf{d}_0^T \\ 0 \\ \cdots \\ 0 \end{bmatrix}\mathbf{Q}_d \quad (A6)$$

Appendix B

The elements of sub-matrices $\mathbf{S}_a^{\mathbf{A}} \in \Re^{(KL+1)x(K+1)}$ and $\mathbf{S}_b^{\mathbf{B}} \in \Re^{(KL+1)x(K+1)}$ in (20) are found from (11) and (12) as

$$\begin{aligned} &\left\{\frac{1}{A_k}\frac{\partial A_k}{\partial a_i^*}\right\},\ i \in (0, N),\ k \in (0, NL), \\ &\left\{\frac{1}{B_k}\frac{\partial B_k}{\partial b_i^*}\right\},\ i \in (0, M),\ k \in (0, ML) \end{aligned} \quad (B1)$$

The elements of $\mathbf{S}_{c,\alpha}^{\mathbf{A}}$, $\mathbf{S}_{d,\beta}^{\mathbf{A}}$, $\mathbf{S}_{c,\alpha}^{\mathbf{B}}$, $\mathbf{S}_{d,\beta}^{\mathbf{B}} \in \Re^{(KL+1)x(L+1)}$ in (20) are determined from (11) and (12) in a product form as

$$\begin{aligned} &\left\{\frac{\partial A_k}{\partial c_{i,l}}\frac{\partial c_{i,l}}{\partial \alpha_r}\frac{\alpha_r}{A_k}\right\}, \quad \left\{\frac{\partial A_k}{\partial d_{n-i,k-l}}\frac{\partial d_{n-i,k-l}}{\partial \beta_r}\frac{\beta_r}{A_k}\right\} \\ &\left\{\frac{\partial B_k}{\partial c_{i,l}}\frac{\partial c_{i,l}}{\partial \alpha_r}\frac{\alpha_r}{B_k}\right\}, \quad \left\{\frac{\partial B_k}{\partial d_{n-i,k-l}}\frac{\partial d_{n-i,k-l}}{\partial \beta_r}\frac{\beta_r}{B_k}\right\} \end{aligned} \quad (B2)$$

The elements

$$\left\{\frac{\partial \sigma_r^*}{\partial \sigma_r}\sigma_r\right\}, \left\{\frac{\partial \sigma_r^*}{\partial \omega_r}\omega_r\right\}, \left\{\frac{\partial \omega_r^*}{\partial \sigma_r}\sigma_r\right\}, \left\{\frac{\partial \omega_r^*}{\partial \omega_r}\omega_r\right\} \quad (B3)$$

of sub-matrices denoted in (19)

$$\begin{aligned} &\mathbf{P}_\sigma = [\mathbf{P}_{\sigma\sigma} \vdots \mathbf{P}_{\omega\sigma}]^T \in \Re^{2KxK}, \quad \mathbf{P}_\omega = [\mathbf{P}_{\sigma\omega} \vdots \mathbf{P}_{\omega\omega}]^T \in \Re^{2KxK} \\ &\mathbf{Z}_\sigma = [\mathbf{Z}_{\sigma\sigma} \vdots \mathbf{Z}_{\omega\sigma}]^T \in \Re^{2KxK}, \mathbf{Z}_\omega = [\mathbf{Z}_{\sigma\omega} \vdots \mathbf{Z}_{\omega\omega}]^T \in \Re^{2KxK} \end{aligned} \quad (B4)$$

are found from (6) with respect to original poles and zeros.

The elements

$$\begin{aligned} &\left\{\frac{\partial \sigma_r^*}{\partial \alpha_i}\alpha_i\right\}, \quad \left\{\frac{\partial \omega_r^*}{\partial \alpha_i}\alpha_i\right\}, \quad r \in (1, N), \\ &\left\{\frac{\partial \sigma_r^*}{\partial \beta_i}\beta_i\right\}, \quad \left\{\frac{\partial \omega_r^*}{\partial \beta_i}\beta_i\right\}, \quad r \in (1, M) \end{aligned} \quad (B5)$$

of the sub-matrices denoted in (20)

$$\begin{aligned} &\mathbf{P}_\alpha = [\mathbf{P}_{\sigma\alpha} \vdots \mathbf{P}_{\omega\alpha}]^T \in \Re^{2Kx(L+1)}, \quad \mathbf{P}_\beta = [\mathbf{P}_{\sigma\beta} \vdots \mathbf{P}_{\omega\beta}]^T \in \Re^{2Kx(L+1)}, \\ &\mathbf{Z}_\alpha = [\mathbf{Z}_{\sigma\alpha} \vdots \mathbf{Z}_{\omega\alpha}]^T \in \Re^{2Kx(L+1)}, \mathbf{Z}_\beta = [\mathbf{Z}_{\sigma\beta} \vdots \mathbf{Z}_{\omega\beta}]^T \in \Re^{2Kx(L+1)}, \end{aligned} \quad (B6)$$

are found differentiating (6) with respect to the mapping parameters α_i and β_i respectively.

The elements

$$\left\{\frac{\partial a_i^*}{\partial \sigma_{r,p}^*}\right\}, \left\{\frac{\partial a_i^*}{\partial \omega_{r,p}^*}\right\}, \left\{\frac{\partial b_i^*}{\partial \sigma_{r,z}^*}\right\}, \left\{\frac{\partial b_i^*}{\partial \omega_{r,z}^*}\right\}, \quad (B7)$$

in sub-matrices $\mathbf{P}_a \in \Re^{(K+1)x2K}$ and $\mathbf{Z}_b \in \Re^{(K+1)x2K}$ in (19) are evaluated straightway from expressions (9) and (10). The derivatives denoted $\partial c_{i,l}/\partial \alpha_r$ and

$\partial d_{n-i,k-l}/\partial \beta_r$ in (B2) can be considered as elements of two 3-dimensional matrices respectively:

$$\frac{\partial \mathbf{C}}{\partial \alpha} \in \Re^{(KL+1)x(KL+1)x(L+1)}, \frac{\partial \mathbf{D}}{\partial \beta} \in \Re^{(KL+1)x(KL+1)x(L+1)} \quad \text{(B8)}$$

Their elements subscribed i,l,r constitute sub-matrices $\partial \mathbf{C}/\partial \alpha_r$ and $\partial \mathbf{D}/\partial \beta_r$. They can be determined performing an additional step as follows. Let a differentiation with respect to α_r and β_r is carried out in left-hand sides of (13) and (14). The result is

$$\frac{\partial}{\partial \alpha_r}\left(\alpha_0+\alpha_1 z^{-1}+\cdots \alpha_n z^{-n}\right)^k = k\left(\alpha_0+\alpha_1 z^{-1}+\cdots \alpha_n z^{-n}\right)^{k-1} z^{-r}$$

$$\frac{\partial}{\partial \beta_r}\left(\beta_0+\beta_1 z^{-1}+\cdots \beta_n z^{-n}\right)^k = k\left(\beta_0+\beta_1 z^{-1}+\cdots \beta_n z^{-n}\right)^{k-1} z^{-r} \quad \text{(B9)}$$

The quantities on right-hand sides could be considered as elements of two sets of vectors denoted $\partial \mathbf{c}_k^T/\partial \alpha_r$ and $\partial \mathbf{d}_k^T/\partial \beta_r$. Each quantity associated with the factor z^{-p} is embedded into $p+1-st$ successive position. Both sets constitute respectively successive rows of both 3D matrices $\partial \mathbf{C}/\partial \alpha$ and $\partial \mathbf{D}/\partial \beta$. Thereby each layer of them represents itself a matrix constituted in a way similar to this proposed in Appendix A to form matrices $\mathbf{C}$ and $\mathbf{D}$.

Taking into account expressions in (B9) one can found simultaneously the elements of vectors $\partial \mathbf{c}_k^T/\partial \alpha_r$ and $\partial \mathbf{d}_k^T/\partial \beta_r$ if $\mathbf{c}_{k-1}^T$ and $\mathbf{d}_{k-1}^T$ are multiplied by k and shifted r positions right respecting the factor z^{-r}. Implementing such row-by-row computations any element embedded in $i+1-st$ row of layers $\partial \mathrm{C}/\partial \alpha_r, (r=0,\ldots,n)$ and $\partial \mathrm{D}/\partial \beta_r, (r=0,\ldots,m)$ is obtained in terms of a certain previous-row element according to the following rule:

$$q(i+1,\ j+r) = iq(i,j) \quad \text{(B10)}$$

In result, the required matrices are constituted and their $r-th$ layers have the following form:

$$\frac{\partial \mathbf{C}}{\partial \alpha_r} = \begin{bmatrix} 0\ldots 0\,0 \\ 0\ldots 0\,\partial\,\mathbf{c}_0^T/\partial \alpha_r \\ 0\ldots 0\,\partial\,\mathbf{c}_1^T/\partial \alpha_r \\ \ldots\ldots\ldots\ldots \\ 0\ldots 0\,\partial\,\mathbf{c}_{NL+1}^T/\partial \alpha_r \end{bmatrix}, \frac{\partial \mathbf{D}}{\partial \beta_r} = \begin{bmatrix} 0\ldots 0\,0 \\ 0\ldots 0\,\partial\,\mathbf{d}_0^T/\partial \beta_r \\ 0\ldots 0\,\partial\,\mathbf{d}_1^T/\partial \beta_r \\ \ldots\ldots\ldots\ldots \\ 0\ldots 0\,\partial\,\mathbf{d}_{NL+1}^T/\partial \beta_r \end{bmatrix} \quad \text{(B11)}$$

$$|\leftarrow k \rightarrow|\leftarrow N\,L{+}1{-}r \rightarrow| \qquad |\leftarrow k \rightarrow|\leftarrow M\,L{+}1{-}r \rightarrow|$$

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