POLYSPECTRAL ANALYSIS OF SIGNALS: AN INTRODUCTION

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Abstract

In this paper, basic terms of the higher order spectrum theory as moments and cumulants of random variables and stationary random processes as well as cumulant and moment spectra are introduced. A part of this paper is devoted to bispectrum estimators. The objective of this paper is also description of bispectrum application for quadratic phase coupling detection.

Keywords

moments, cumulants, cumulant spectrum, moment spectrum, bispectrum, quadratic phase coupling

1. Introduction

One frequently used digital signal processing technique has been the estimation of the power spectrum of discrete-time deterministic or stochastic signals. In power spectrum estimation, the process under consideration is treated as a superposition of uncorrelated harmonic components. The distribution of power among these frequency components is then estimated. As such, phase relations between frequency components are suppressed. The information contained in the power spectrum is essentially presented in autocorrelation sequence; this would suffice for the complete statistical description of a Gaussian process of known mean.

However, there are practical situations where we have to look beyond the power spectrum or autocorrelation domain to obtain information regarding deviation from Gaussianess and presence of nonlinearities. The **higher-order spectra (HOS)** (greater than two), also known as polyspectra [1,2] defined in terms of higher-order statistics, do contain such information. The well-known example of the HOS is the third-order spectrum, called a **bispectrum**, which is defined to be the Fourier transform of the third-order cumulant sequence of a stationary random process. The **power spectrum** is the second-order spectrum. There are four motivations behind the use of polyspectral analysis in signal processing [1,2]:

- Suppressing Gaussian noise of unknown spectral characteristics in detection, parameter estimation, analysis and classification problems; bispectrum also suppresses non-Gaussian noise with symmetric probability density function.
- Reconstructing the phase and magnitude response of signals or systems.
- Detecting and characterizing nonlinearities in time series.
- The components of the HOS can be used as advanced features based on which a classification of states of analysed system can be carried out. This approach can be used in the field of quality control and diagnostics (see e.g. [3]).

The facts confirming the above mentioned statements as well as a comprehensive review of basic applications of higher-order statistics and HOS are discussed in detail in [1,2].

The objective of this paper is to introduce basic terms of HOS theory as well as to point out to possible application of polyspectral analysis in the field of quadratic phase coupling detection. The particular topics of the paper are illustrated by using conveniently selected examples.

2. Moments and Cumulants of Random Variables

Given a set of n real random variables $\{x_1, x_2, \ldots, x_n\}$ their joint **moments** of order $r=k_1+k_2+\ldots+k_n$ are given by:

$$Mom\left[x_1^{k_1}, x_2^{k_2}, \ldots, x_n^{k_n}\right] = E\left\{x_1^{k_1}.x_2^{k_2}.\ldots.x_n^{k_n}\right\} = = (-j)^r \left.\frac{\partial^r \Phi(\omega_1, \omega_2, \ldots, \omega_n)}{\partial\omega_1^{k_1}\partial\omega_2^{k_2}\ldots\partial\omega_n^{k_n}}\right|_{\omega_1=\omega_2=\ldots=\omega_n=0} \quad (1)$$

where

$$\Phi(\omega_1,\omega_2,\ldots,\omega_n) = E\{\exp[j(\omega_1 x_1 + \omega_2 x_2 + \ldots + \omega_n x_n)]\} \quad (2)$$

is the first joint characteristic function of the random variables $\{x_1, x_2, \ldots, x_n\}$. $E\{.\}$ denotes the expectation operation. For two random variables $\{x_1, x_2\}$ we have e.g. the following second-order moments:

$$Mom[x_1, x_2] = E\{x_1.x_2\}, \quad Mom\left[x_1^2\right] = E\left\{x_1^2\right\},$$

$$Mom\left[x_2^2\right] = E\left\{x_2^2\right\} \quad (3)$$

The second joint characteristic function $\tilde{\Psi}(\omega_1,\omega_2,\ldots,\omega_n)$ is defined as the natural logarithm of $\Phi(\omega_1,\omega_2,\ldots,\omega_n)$; i.e.,

$$\Psi(\omega_1,\omega_2,\ldots,\omega_n) = \ln\Phi(\omega_1,\omega_2,\ldots,\omega_n). \quad (4)$$

The joint **cumulants** of the r-th order $Cum\left[x_1^{k_1}, x_2^{k_2}, \ldots, x_n^{k_n}\right]$ are defined as the coefficients in the Taylor expansion of the second characteristic function about zero, i.e.,

$$Cum\left[x_1^{k_1}, x_2^{k_2}, \ldots, x_n^{k_n}\right] = \\ = (-j)^r \left.\frac{\partial^r \tilde{\Psi}(\omega_1, \omega_2, \ldots, \omega_n)}{\partial \omega_1^{k_1} \partial \omega_2^{k_2} \ldots \partial \omega_n^{k_n}}\right|_{\omega_1=\omega_2=\ldots=\omega_n=0} \quad (5)$$

Thus, the joint cumulants can be expressed in terms of the joint moments of a set of random variables. E.g. the moments

$$\begin{aligned} m_1 &= Mom[x_1] = E\{x_1\}, \\ m_2 &= Mom[x_1, x_1] = E\{x_1.x_1\}, \\ m_3 &= Mom[x_1, x_1, x_1] = E\{x_1.x_1.x_1\}, \\ m_4 &= Mom[x_1, x_1, x_1, x_1] = E\{x_1.x_1.x_1.x_1\}, \end{aligned} \quad (6)$$

of random variable $\{x_1\}$ are related to its cumulants by

$$\begin{aligned} c_1 &= Cum[x_1] = m_1, \\ c_2 &= Cum[x_1, x_1] = m_2 - m_1^2, \\ c_3 &= Cum[x_1, x_1, x_1] = m_3 - 3m_2 m_1 + 2m_1^3, \\ c_4 &= Cum[x_1, x_1, x_1, x_1] = \\ &= m_4 - 4m_3 m_1 - 3m_2^2 + 12 m_2 m_1^2 - 6m_1^4. \end{aligned} \quad (7)$$

3. Moments and Cumulants of Stationary Processes

If $\{X(k)\}$, $k = 0, \pm 1, \pm 2, \pm 3, \ldots$ is a real stationary random process and its moments up to order n exist, then

$$Mom[X(k), X(k+\tau_1), \ldots, X(k+\tau_{n-1})] = \\ = E\{X(k).X(k+\tau_1).....X(k+\tau_{n-1})\} \quad (8)$$

will depend only on the time differences $\tau_1, \tau_2, \ldots, \tau_{n-1}$ where $\tau_i = 0, \pm 1, \pm 2, \pm 3, \ldots$ for all i. Now, we can write the moments of $\{X(k)\}$ as:

$$m_n^x(\tau_1, \tau_2, \ldots, \tau_{n-1}) = E\{X(k).X(k+\tau_1)\ldots X(k+\tau_{n-1})\}. \quad (9)$$

Similarly, the n-th order cumulants of $\{X(k)\}$ are $(n-1)$ dimensional functions which we can write in the form:

$$c_n^x(\tau_1, \tau_2, \ldots, \tau_{n-1}) = Cum[X(k), X(k+\tau_1), \ldots, \\ , X(k+\tau_{n-1})]. \quad (10)$$

Then, we can obtain the following relationships between moments and cumulant sequences of $X(k)$:

the 1-st order cumulant (mean value):

$$c_1^x = m_1^x = E\{X(k)\}, \quad (11)$$

the 2-nd order cumulant (covariance sequence):

$$c_2^x(\tau_1) = m_2^x(\tau_1) - (m_1^x)^2 = m_2^x(-\tau_1) - (m_1^x)^2 = c_2^x(-\tau_1), \quad (12)$$

the 3-rd order cumulant:

$$c_3^x(\tau_1, \tau_2) = m_3^x(\tau_1, \tau_2) - m_1^x[m_2^x(\tau_1) + m_2^x(\tau_2) + \\ + m_2^x(\tau_2 - \tau_1)] + 2(m_1^x)^3. \quad (13)$$

By putting $\tau_i = 0$ in (12) and (13) and assuming $m_1^x = 0$ we get

$$\gamma_2^x = E\{X(k)^2\} = c_2^x(0), \quad \gamma_3^x = E\{X(k)^3\} = c_3^x(0,0), \\ \gamma_4^x = E\{X(k)^4\} - 3[\gamma_2^x]^2 = c_4^x(0,0,0). \quad (14)$$

The quantities γ_2^x, γ_3^x and γ_4^x are called **variance, skewness** and **kurtosis**, respectively. The above given equations gives the variance, skewness and kurtosis measures in terms of cumulant lags.

4. Cumulant Spectra

Let us suppose that the process $\{X(k)\}$, $k = 0, \pm 1, \pm 2, \pm 3, \ldots$ is real, strictly stationary with n-th order cumulant sequences $c_n^x(\tau_1, \tau_2, \ldots, \tau_{n-1})$ defined by (10). Then, assuming that cumulant sequence satisfies the condition of absolute summation the n-th order cumulant spectrum $C_n^x(\omega_1, \omega_2, \ldots, \omega_{n-1})$ of $\{X(k)\}$ exists, is continuous, and is defined as the $(n-1)$ dimensional discrete Fourier transform of the n-th order cumulant sequence. The n-th order **cumulant spectrum** is thus defined:

$$C_n^x(\omega_1, \omega_2, \ldots, \omega_{n-1}) = \sum_{\tau_1=-\infty}^{\infty} \ldots \sum_{\tau_{n-1}=-\infty}^{\infty} c_n^x(\tau_1, \tau_2, \ldots, \tau_{n-1}) \\ .\exp\{-j(\omega_1\tau_1 + \ldots + \omega_{n-1}\tau_{n-1})\}$$

for $|\omega_i| \le \pi$; $i = 1, 2, \ldots, n-1$, $|\omega_1 + \omega_2 + \ldots + \omega_{n-1}| \le \pi$ (15)

The power spectrum, bispectrum and trispectrum are special cases of the n-th order cumulants. They are defined by (15) for $n=2, 3, 4$ and can be expressed as follows:

Power spectrum: n=2

$$C_2^x(\omega) = \sum_{\tau=-\infty}^{\infty} c_2^x(\tau)\exp\{-j(\omega\tau)\} \quad \text{for } |\omega| \le \pi \quad (16)$$

Bispectrum: n=3

$$C_3^x(\omega_1, \omega_2) = \sum_{\tau_1=-\infty}^{\infty} \sum_{\tau_2=-\infty}^{\infty} c_3^x(\tau_1, \tau_2)\exp\{-j(\omega_1\tau_1 + \omega_2\tau_2)\} \quad (17)$$

for $|\omega_1| \le \pi, |\omega_2| \le \pi, |\omega_1 + \omega_2| \le \pi$.

Trispectrum: n=4

$$C_4^x(\omega_1, \omega_2, \omega_3) = \sum_{\tau_1=-\infty}^{\infty} \sum_{\tau_2=-\infty}^{\infty} \sum_{\tau_3=-\infty}^{\infty} c_4^x(\tau_1, \tau_2, \tau_3) \\ .\exp\{-j(\omega_1\tau_1 + \omega_2\tau_2 + \omega_3\tau_3)\} \quad (18)$$

for $|\omega_1| \le \pi$, $|\omega_2| \le \pi$, $|\omega_3| \le \pi$, $|\omega_1 + \omega_2 + \omega_3| \le \pi$, where $c_i^x(\tau_1,\tau_2,\ldots,\tau_{i-1})$ is the i-th order cumulant sequence of $\{X(k)\}$.

The definitions of the **moment spectra** $M_n^x(\omega_1,\omega_2,\ldots,\omega_{n-1})$ are similar to those given for the cumulant spectra (15), where the cumulants $c_n^x(\tau_1,\tau_2,\ldots,\tau_{n-1})$ are substituted by the moments $m_n^x(\tau_1,\tau_2,\ldots,\tau_{n-1})$. It has been shown that polyspectral analysis based on cumulant spectra can find more applications than that of moment spectra. The motivation for using cumulants rather than moments is discussed e.g. in [1]. A comprehensive review of basic properties of cumulants, moments as well as cumulant and moment spectra can be found e.g. in [1,2,4].

5. Bispectrum

In this tutorial review the emphasis is put on the third-order spectrum, also called the bispectrum, its properties and several application problems that can directly benefit from it.

Let $\{X(k)\}$ be a real, discrete, zero-mean stationary process.

If $m_3^x(\tau_1,\tau_2)$ denotes the third moment sequence of $\{X(k)\}$, i.e.,

$$m_3^x(\tau_1,\tau_2) = E\{X(k)X(k+\tau_1)X(k+\tau_2)\} \tag{19}$$

then the bispectrum of $\{X(k)\}$ is defined as

$$B(\omega_1,\omega_2) = \sum_{\tau_1=-\infty}^{\infty}\sum_{\tau_2=-\infty}^{\infty} m_3^x(\tau_1,\tau_2)\exp\{-j(\omega_1\tau_1+\omega_2\tau_2)\} \tag{20}$$

Since the third-order moments and cumulants are identical, the bispectrum is the third-order cumulant spectrum [1,2].

It follows from (19) that the third moments obey the following symmetry properties [1, 2]:

$$\begin{aligned} m_3^x(\tau_1,\tau_2) &= m_3^x(\tau_2,\tau_1) = m_3^x(-\tau_2,\tau_1-\tau_2) = \\ &= m_3^x(\tau_2-\tau_1,-\tau_1) = m_3^x(\tau_1-\tau_2,-\tau_2) = \\ &= m_3^x(-\tau_1,\tau_2-\tau_1) \end{aligned} \tag{21}$$

As consequence, knowing the third moments in any one of the six sectors, I through VI, shown in Fig. 1(a), would enable us to find the entire third moment sequence. These sectors include their boundaries so that, for example, sector I is an infinite wedge bounded by the lines $\tau_1 = 0$, and $\tau_1 = \tau_2$; $\tau_1,\tau_2 \geq 0$.

From the bispectrum definition and the properties of the third-order moments, it follows :

- $B(\omega_1,\omega_2)$ is generally complex, i.e., it has magnitude $|B(\omega_1,\omega_2)|$ and phase $\Psi_B(\omega_1,\omega_2)$

$$B(\omega_1,\omega_2) = |B(\omega_1,\omega_2)|\exp\{j\Psi_B(\omega_1,\omega_2)\}, \tag{22}$$

- $B(\omega_1,\omega_2)$ is doubly periodic with period 2π, i.e.

$$B(\omega_1,\omega_2) = B(\omega_1+2\pi,\omega_2+2\pi). \tag{23}$$

- Symmetry regions in bispectrum domain :

$$\begin{aligned} B(\omega_1,\omega_2) &= B(\omega_2,\omega_1) = B^*(-\omega_2,-\omega_1) \\ &= B^*(-\omega_1,-\omega_2) = B(-\omega_1-\omega_2,\omega_2) \\ &= B(\omega_1,-\omega_1-\omega_2) = B(-\omega_1-\omega_2,\omega_1) \\ &= B(\omega_2,-\omega_1-\omega_2). \end{aligned} \tag{24}$$

Thus knowledge of the bispectral in the triangular region $\omega_2 \geq 0, \omega_1 \geq \omega_2, \omega_1+\omega_2 \leq \pi$ shown in Fig. 1(b) is enough for a complete description of the bispectrum. It is worth noting that the computation of $B(\omega_1,\omega_2)$ in (20) is done over one of the twelve sectors shown in Fig. 1(b) and the symmetries (24) are then utilized.

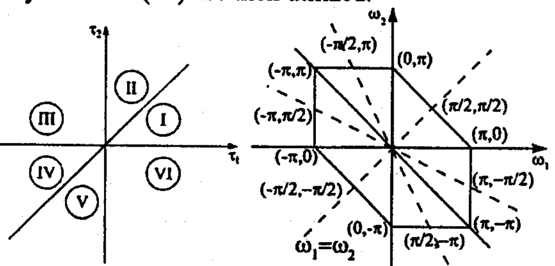

Fig. 1. (a) Symmetry regions of the third-order moments (21)
(b) Symmetry regions of the bispectrum (24)

6. Bispectrum Estimators

The problem met within practice is one of estimating the bispectrum of a process when a finite set of observation measurements is given. There are two chief approaches that have been used to estimate the bispectrum, namely, the conventional (Fourier type) and the parametric approach which is based on autoregresive (AR), moving average (MA), and ARMA models. The conventional methods for bispectrum estimation and their properties is the subject of discussion in this section. They may be classified into the following two classes:

- indirect class of techniques which are approximations of the definition of the bispectrum given by (21) and (22),
- direct class of techniques.

While these approximations are straightforward, limitations on statistical variance of the estimates, computer time, and memory impose severe problems on their implementation. In fact, the computations may be surprisingly expensive despite the use of fast Fourier transform (FFT) algorithms.

6.1 Indirect Class of Conventional Bispectrum Estimators

Let $\{X(1), X(2), \ldots, X(N)\}$ be the given data set. Then in order to estimate bispectrum the following steps should be done:

1) Segment the data into K records of M samples each, i.e., $N=KM$.
2) Subtract the average value of each record.

3) Assuming that $\{x^{(i)}(k), k=0, 1, \dots, M-1\}$ is the data set per segment $i=1, 2, \dots, K$, obtain an estimate of the third-moment sequence:

$$m_3^{x(i)}(\tau_1,\tau_2)=\frac{1}{M}\sum_{l=s_1}^{s_2}x^{(i)}(l)x^{(i)}(l+\tau_1)x^{(i)}(l+\tau_2) \quad (25)$$

where

i $=1, 2, \dots, K$

s_1 $=\max(0, -\tau_1, -\tau_2)$

s_2 $=\min(M-1, M-1-\tau_1, M-1-\tau_2)$.

4) Average $m_3^{x(i)}(\tau_1,\tau_2)$ over all segments:

$$\hat{m}_3^x(\tau_1,\tau_2)=\frac{1}{K}\sum_{i=1}^{K}m_3^{x(i)}(\tau_1,\tau_2). \quad (26)$$

5) Generate the bispectrum estimate:

$$\hat{B}_{IN}(\omega_1,\omega_2)=\sum_{\tau_1=-L}^{L}\sum_{\tau_2=-L}^{L}\hat{M}_3^x(\tau_1,\tau_2)W(\tau_1,\tau_2) . \exp\{-j(\omega_1\tau_1+\omega_2\tau_2)\} \quad (27)$$

where $L<M-1$ and $W(\tau_1,\tau_2)$ is two-dimensional window function. Let us note that the computations of the bispectrum estimate in (27) may be substantially reduced if symmetry properties of third moments (21) are taken into account for the calculation of $m_3^{x(i)}(\tau_1,\tau_2)$ in (25) and if the symmetry properties of the bispectrum show in (24) are incorporated in the computations of (27).

As in the case of conventional power spectrum estimation, to get better estimates, suitable windows should be used. The windows function should satisfy the following constraints:

a) $W(\tau_1,\tau_2)=W(\tau_2,\tau_1)=W(-\tau_1,\tau_2-\tau_1)=$
$=W(\tau_1-\tau_2,-\tau_2)$ (28)
(symmetry properties of third moments);

b) $W(\tau_1,\tau_2)=0$ outside the region of support of $\hat{M}_3^x(\tau_1,\tau_2)$;

c) $W(0,0)=1$ (normalizing condition);

d) $W(\omega_1,\omega_2)\geq 0$, for all (ω_1,ω_2). (29)

A class of functions which satisfies constraints (30), for $W(\tau_1,\tau_2)$, is the following:

$$W(\tau_1,\tau_2)=d(\tau_1)d(\tau_2)d(\tau_1-\tau_2), \quad (30)$$

where

$$d(\tau_1)=d(-\tau_1), \quad (31)$$

$$d(\tau_1)=0, \quad \tau_1>L \quad (32)$$

$$d(0)=1, \quad (33)$$

$$D(\omega)\geq 0, \quad \text{for all } \omega \quad (34)$$

Equations (30) and (31-34) allow a reconstruction of two-dimensional window functions for bispectrum estimation using standard one-dimensional lag windows. However, not all conventional power spectrum windows satisfy constrain (34) (for the details see e.g. [1, 2]).

6.2 Direct Class of Conventional Bispectrum Estimators

Let $\{X(1), X(2), \dots, X(N)\}$ be the available set of observations for bispectrum estimation. Let us assume that f_s is the sampling frequency and $\Delta_0=f_s/N_0$ is the required spacing between frequency samples in the bispectrum domain along horizontal or vertical direction. Thus N_0 is the total number of frequency samples.

1) Segment the data into K segments of M samples each, i.e., $N=KM$, and subtract the average value of each segment. If necessary, add zeros at each segment to obtain a convenient length M for the FFT.

2) Assuming that $\{X^{(i)}(k), k=0, 1, \dots, M-1\}$ are the data of segment $\{i\}$, generate the DFT coefficients:

$$Y^{(i)}(\lambda)=\frac{1}{M}\sum_{k=0}^{M-1}X^{(i)}(k)\exp(-j2\pi k\lambda/M), \quad (35)$$

$\lambda=1, 2, \dots, M/2$ $\quad i=1, 2, \dots, K$.

3) In general, $M=M_1\times N_0$, where M_1 is a positive integer (assumed odd number), i.e., $M_1=2L_1+1$. Since M is even and M_1 is odd, we compromise on the value of N_0 (closest integer). Estimate the bispectrum by frequency-domain averaging:

$$\hat{b}_i(\lambda_1,\lambda_2)=\frac{1}{\Delta_0^2}\sum_{k_1=-L_1}^{L_1}\sum_{k_2=-L_1}^{L_1}Y^{(i)}(\lambda_1+k_1)Y^{(i)}(\lambda_2+k_2) . Y^{(i)*}(\lambda_1+\lambda_2+k_1+k_2) \quad (36)$$

over the triangular region:
$0\leq\lambda_2\leq\lambda_1$ and $\lambda_1+\lambda_2\leq f_s/2$ (Fig.1(b)).
For the special case where no averaging is performed in the bispectrum domain $M_1=1$, $L_1=0$ and therefore :

$$\hat{b}_i(\lambda_1,\lambda_2)=\frac{1}{\Delta_0^2}Y^{(i)}(\lambda_1)Y^{(i)}(\lambda_2)Y^{(i)*}(\lambda_1+\lambda_2) \quad (37)$$

4) The bispectrum estimate of the given data is the average over the K pieces:

$$\hat{B}_D(\omega_1,\omega_2)=\frac{1}{K}\sum_{i=1}^{K}\hat{b}_i(\omega_1,\omega_2) \quad (38)$$

where

$$\omega_1=\left(\frac{2\pi f_s}{N_0}\right)\lambda_1 \quad \omega_2=\left(\frac{2\pi f_s}{N_0}\right)\lambda_2. \quad (39)$$

7. Quadratic Phase Coupling

The bispectrum can find many applications [1, 2]. However, in this paper we will study just one. There are situations in practice where because of interaction between two harmonic components of a process there is contribution to the power at their sum and/or difference frequencies. Such a phenomenon which could be due to quadratic nonlinearities gives rise to certain phase relations called **quadratic phase coupling**. In certain application it is necessary to find out if peaks at harmonically related position in the power spectrum are, in fact, coupled. Since the power spectrum suppressed all phase relations it cannot provide the answer. The bispectrum, however, is capable of detecting and quantifying phase coupling [1, 2].

A possible application of the quadratic phase detection in the field of automatic diagnosis was applied e.g. in [3].

We will illustrate this phenomenon by the following simple example. Let us consider the processes:

$$X_I(k) = \cos(\lambda_1 k + \varphi_1) + \cos(\lambda_2 k + \varphi_2) + \cos(\lambda_3 k + \varphi_3) \quad (40)$$

and

$$X_{II}(k) = \cos(\lambda_1 k + \varphi_1) + \cos(\lambda_2 k + \varphi_2) + \cos(\lambda_3 k + \varphi_1 + \varphi_2) \quad (41)$$

where $\lambda_3 = \lambda_1 + \lambda_2$, i.e., $(\lambda_1, \lambda_2, \lambda_3)$ are harmonically related and $\varphi_1, \varphi_2, \varphi_3$ are independent random variables uniformly distributed between $[0, 2\pi]$.

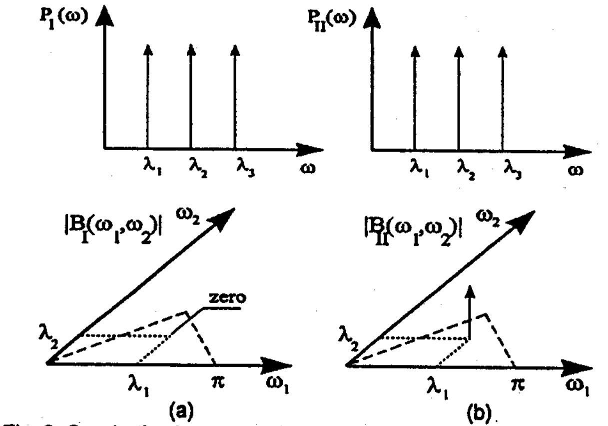

Fig. 2. Quadratic phase coupling.
a) Power spectrum and bispectrum of $X_I(k)$ described by (40).
b) Power spectrum and bispectrum of $X_{II}(k)$ described by (41).

From (40), it is apparent that λ_3 is an independent random-phase variable. On the other hand, λ_3 of $X_{II}(k)$ in (41) is a result of phase coupling between λ_1 and λ_2. One can easily verify that $X_I(k)$ and $X_{II}(k)$ have identical power spectra $(P_I(\omega) = P_{II}(\omega))$ consisting of impulses at λ_1, λ_2, and λ_3. However, the bispectrum of $X_I(k)$ is identically zero whereas the bispectrum of $X_{II}(k)$ shows an impulse in the triangular region $\omega_2 \geq 0, \omega_1 \geq \omega_2, \omega_1 + \omega_2 \leq \pi$. The impulse is located at $\omega_1 = \lambda_1, \omega_2 = \lambda_2$ if $\lambda_1 \geq \lambda_2$. The power spectrum and the magnitude bispectrum illustrating the above presented considerations concerning quadratic phase coupling phenomenon for the discussed example are given in the Fig. 2.

8. Examples

Phenomenon of quadratic phase coupling and ability of the conventional methods to resolve harmonic components in the bispectrum domain will be illustrated in this section. The objective of this section is also an illustration of characteristic properties of bispectrum of Gaussian signal.

Example 1: Consider the real discrete process

$$X(n) = \sum_i \cos(\omega_i n + \Phi_i) + W(n), \quad i=a, \ldots, f \quad (42)$$

where

$$\omega_a = 2\pi(0.076125), \quad \omega_b = 2\pi(0.09375),$$
$$\omega_c = 2\pi(0.288875), \quad \omega_d = 2\pi(0.3045),$$
$$\omega_e = \omega_a + \omega_c, \quad \omega_f = \omega_b + \omega_d$$
$$\Phi_e = \Phi_a + \Phi_c, \quad \Phi_f = \Phi_b + \Phi_d,$$

where $\Phi_a, \ldots, \Phi_d$ are independent and uniformly distributed on $(0, 2\pi)$.

The true bispectrum magnitude has impulses at (ω_a, ω_c) and (ω_b, ω_d). The level of the additive white Gaussian noise (*W(n)*) was set at -40dB. Fig. 3(a), shows the bispectrum magnitude estimate by using contour plot provided by the conventional indirect method when 64 records of 128 samples each are taken. We can see that method is successful in resolving the two peaks in this case. Fig. 3(b), shows the corresponding estimates when there are just 16 records with only 40 samples each. From the Fig. 3, it can be seen that magnitude bispectrum could be used for the quadratic phase coupling phenomenon detection. The results of the computer experiment also confirms that for successful quadratic phase coupling detection it is neccessary to use a corresponding method of bispectrum estimation. The conventional approach have failed to resolve the two peaks.

Example 2: Consider zero-mean Gaussian white noise with unit standard deviation. Let us assume that we have available 100000 samples of process. Figure 4 shows the averaged bispectrum for segment lengts M=100 and number of averaging K=10 (Fig. 4.(a)), K=100 (Fig. 4.(b)), K=1000 (Fig.4.(c)). Here, conventional indirect method with optimal window was used for computation. The results of the computer experiment have shown that the mean and variance of the bispectrum magnitude of the tested Gaussian signal converge asymptotically to zero (Tab. 1.). Therefore we can say that these result indicate that Gaussian signal bispectrum obtained by indirect method is asymptotically zero.

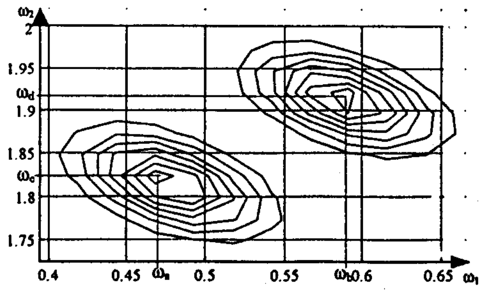

Fig. 3(a). Magnitude bispectrum estimates for "long data"

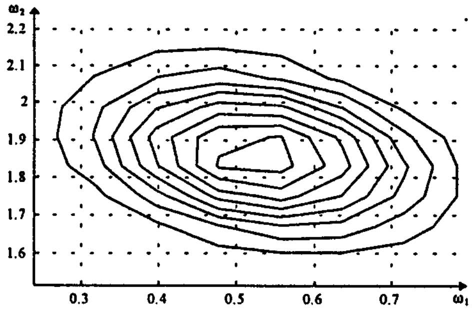

Fig. 3(b). Magnitude bispectrum estimates for "short data"

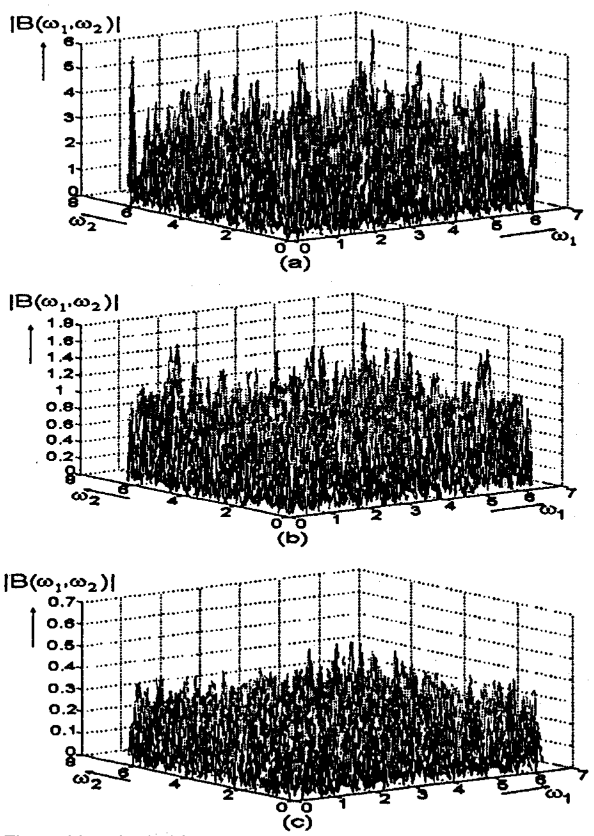

Fig. 4. Magnitude bispectrum estimates for Gaussain noise

M=100		
K	Mean value	Variance
10	1.12778	0.50238
100	0.46408	0.06241
1000	0.14379	0.00591

Tab. 1. Mean values and variance of Gaussian signal bispectrum magnitude.

9. Conclusion

In this paper, basic terms of the HOS theory as moments and cumulants of random variables and stationary random processes as well as definitions of cumulant and moment spectra have been introduced. The possibilities of polyspectral analysis especially bispectral analysis to resolve harmonic components for " long data" and "short data" in the bispectrum domain have been illustrated by using conveniently selected example.

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