AN APPROACH TO 2D WAVELET TRANSFORM AND ITS USE FOR IMAGE COMPRESSION

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Abstract

In this paper is constructed a new type of two-dimensional wavelet transform. Construction is based on lifting scheme. We transform 1D wavelets with symmetrical factorisation to their 2D counterparts. Comparison to existing similar 2D wavelet constructions is given. Application for image compression is given using progressive (SPIHT) and classical type transform coder.

Keywords

wavelet, lifting scheme, image compression

1. Introduction

In recent years wavelets showed their potential for lossy/losless image compression and belongs to the most successful, e.g. FBI Fingerprint compression, progressive coders [3]. This paper is focused on the transform part of such image coders. New means to perform Wavelet transform (WT) offers Lifting scheme [1]. Novel type of 2D WT created from 1D prototype using Lifting and recursive quincunx subsampling, is showed in Section 3. Its base functions have more isotrophy than tensor products of analogous separable transform. Transform is then compared to similar construction [5, 6] and used for image compression with SPIHT [3] and classical type transform coder [2].

2. Lifting Scheme

Originally Lifting scheme was developed to improve properties of biorthogonal wavelets [4]. Later was proved that every wavelet can be factorised into lifting steps [7]. This realisation has many advantages such: speed up, in place calculation, simple treatment of boundaries , possibility of nonlinear operations and irregular lattices, invertibility. Classical implementation of WT uses two band filter bank (FB) with recursion on its low pass (LP) [1, 2]. Equivalent polyphase representaion is depicted in Fig.1.

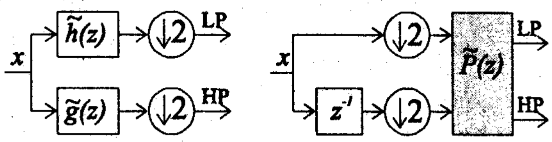


Fig.1 Analysys part of the 2 band filter bank, i.e forward WT a)classical case b)polyphase representation with delay

Polyphase matrix $\tilde{P}(z)$ is assembled from even and odd filter components. Output of the FB analysis part is then:

$$\begin{bmatrix} LP \\ HP \end{bmatrix} = \tilde{P}(z) \begin{bmatrix} x_{even} \\ z^{-1} x_{odd} \end{bmatrix} \tag{1}$$

$$\tilde{P}(z) = \begin{bmatrix} \tilde{h}_e(z) & \tilde{h}_o(z) \\ \tilde{g}_e(z) & \tilde{g}_o(z) \end{bmatrix} \tag{2}$$

For any filter pair (h, g) with $\det[P(z)] = 1$, always exist factorisation of $P(z)$ [7]:

$$P(z) = \begin{bmatrix} K & 0 \\ 0 & \frac{1}{K} \end{bmatrix} \prod_{i=m}^{1} \left\{ \begin{bmatrix} 1 & s_i(z) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ t_i(z) & 1 \end{bmatrix} \right\} \tag{3}$$

Equation (3) allows ladder realization of $\tilde{P}(z)$ by reversible "lifting" steps followed with normalisation as shown in Fig.2.

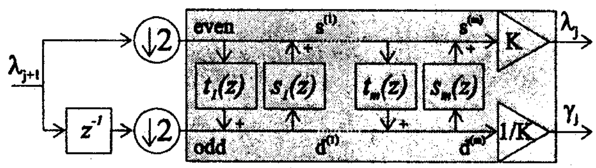


Fig.2 Ladder structure of Lifting steps in forward wavelet transform

Signal is partitioned into even and odd components that are then mutually predicted by t_i (to zero signal in HP part) and updated by s_i (to retain in LP part signal moments). After normalisation is algorithm recursively applied to LP part. Backward transforms simply undone all ladder steps from right to left using reversed operators as in Tab.1.

Lemma 1: Time reversing of t_i and s_i is equivalent to time reversing of analysis filters and switching to polyphase decomposition with advance.

Use of Lemma is advantageous because of time reversed wavelet analysis filters (equal in biorthogonal case) and more natural even/odd signal partitioning. Example of lifting steps for CDF2.2 wavelet are in Tab.1. Here we see how invertibility is assured even possible non-linearity of {.} operation e.g. rounding (we only add/subtract it). In Fig.3 is detailed forward transform part with symmetrical extension at boundaries. Result of lifted WT can be reordered to classical Mallat's form as in Fig.3 using partial bit reversal algorithm shown in [4].

Analysis filters	$\tilde{h}(z)=\frac{\sqrt{2}}{8}\left(-1z^{-2}+2z^{-1}+6+2z-1z^{2}\right)$ $\tilde{g}(z)=\frac{\sqrt{2}}{4}\left(-z^{-2}+2z^{-1}-1\right)$
Normalisation coefficient: $K=\sqrt{2}$	
Forward transform	Backward transform
$s_i^{(0)}=\lambda_{j+1,2i}$	$s_i^{(1)}=1/K\,\lambda_{j,i}$
$d_i^{(0)}=\lambda_{j+1,2i+1}$	$d_i^{(1)}=K\,\gamma_{j,i}$
$d_i^{(1)}=d_i^{(0)}+\{-\frac{1}{2}s_i^{(0)}-\frac{1}{2}s_{i+1}^{(0)}\}$	$s_i^{(0)}=s_i^{(1)}+\{-\frac{1}{4}d_{i-1}^{(1)}-\frac{1}{4}d_i^{(1)}\}$
$s_i^{(1)}=s_i^{(0)}+\{\frac{1}{4}d_{i-1}^{(0)}+\frac{1}{4}d_i^{(0)}\}$	$d_i^{(0)}=d_i^{(1)}+\{\frac{1}{2}s_i^{(1)}+\frac{1}{2}s_{i+1}^{(1)}\}$
$\gamma_{j,i}=1/K\,d_i^{(1)}$	$\lambda_{j+1,2i+1}=d_i^{(0)}$
$\lambda_{j,i}=K\,s_i^{(1)}$	$\lambda_{j+1,2i}=s_i^{(0)}$

Tab.1 One level of lifted WT for CDF(2,2) wavelet

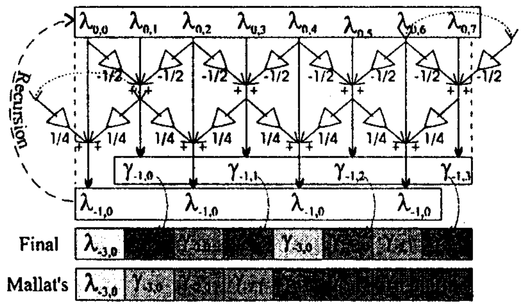


Fig.3 Forward lifting steps for CDF(2,2) with symmetrical extensions at signal boundaries and following coefficient reordering

3. Wavelets in two dimensions

Effective and natural way to realise WT in two dimensions is to apply 1D transform to rows and columns of signal.

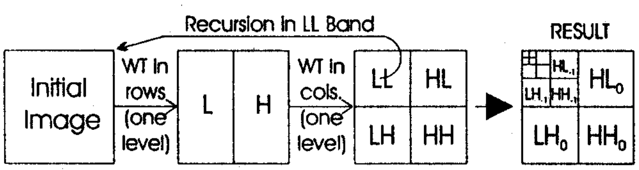


Fig.4 Nonstandard decomposition in 2D wavelet transform

Most used is non-standard decomposition (Fig.4), where basis functions are then tensor products of 1D bases [2]. Realising 2D WT via lifting, coefficients can be reordered analogically to the 1D case.

Another approach is to direct design two dimensional wavelet filters as in [5]. On rectangular lattice this correspond to 4 band filter bank. Our approach uses Quincunx sampling lattice [2] and complete one level decomposition in two step as shown in Fig.5. Similar construction was independently presented also in [5, 6].

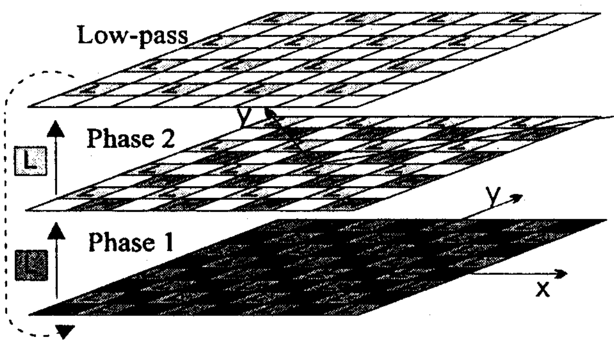
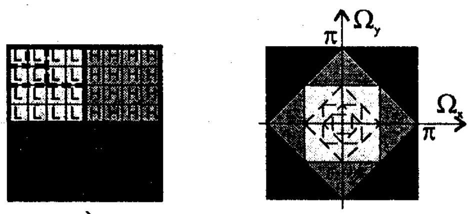


Fig.5 Lattices of the 2D WT using Quincunx and their recursion

To obtain LL band as in Fig.4 using Quincunx we must perform two steps. In Phase1 we predict/update coefficients on Quincunx lattice with the rest of them. In Step 2 we use Low pass output ■ of Phase 1 and similarly predict/update but with lattice rotated by 45°. LL band consisting low pass coefficients L is further recursively decomposed. Corresponding signal partitioning to frequency bands when used ideal half band filters is in Fig.6b. After transform we can reorder coefficients according to Fig.6a, which is representation more suitable for compression methods.

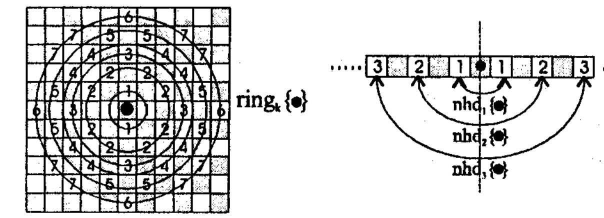


a) b)

Fig.6 Partitioning of 2D WT coefficients when used Quincunx a) Coefficient reordering b) Distribution of frequency bands

In predict/update phases can be used 2D neighbourhood instead of 1D ones. Natural way is to use ring neighbourhoods as depicted in Fig.7.

In [5] authors computed 2D Neville filters (filters that implement polynomial interpolation) of various order on Quincunx lattice. In this article are used their normalized versions (K=1.41421). Their factorisation consist of one predict and update step. Predict coefficients are in Tab.2. Update coefficients are created from predict ones, multiplying them by -1/2. Basic wavelets of Neville Filters are depicted in Fig.9.



2D neighbourhoods on Quincunx 1D lifting neigbourhoods

Fig.7 Predict/update neighbourhoods in 1D and 2D cases

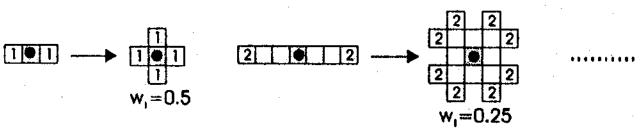


Fig.8 Transformation of neighbourhoods from 1D to 2D rings

O	Numerators for different rings {ring(pixels in ring)}							D
	1(4)	2(8)	3(4)	4(8)	5(8)	6(4)	7(8)	
2	1							2^2
4	10	-1						2^5
6	174	-27	2	3				2^9
8	23300	-4470	625	850	-75	9	-80	2^{16}

Tab.2 Ring coefficients for 2D Neville filters of various order (O) in predict phase. Coefficient = numerator / D .

3.1 One to two-dimensional transform

To design 2D wavelets there are many approaches how do construct them via transform from 1D case. Construction such as McClellan transformation or separable polyphase components are described in [2].

Proposed transform is also of such type. Because of its properties I name it TORING. TORING is generalisation of approach in [6]. Builds 2D lifting steps that can be used for predict/update steps on lattices in Fig.5. Uses weighted coefficients of lifting factorisation of 1D prototype transform but instead of 1D neighbourhoods works with 2D rings as depicted in Fig.7 and Fig.8. Weight w_i for lifting coefficient depends on number of pixels in actual ring:

$$w_i = \frac{2}{\textit{number of pixels in ring}} \tag{4}$$

Because of symmetry of the rings, correctly can be transformed only filters with symmetrical factorisation in each lifting step, i.e. filters where forward predict/update steps can be expressed as:

$$\begin{aligned} d_i^{(st)} &= d_i^{(st)} + \sum_k \alpha_k^{(st)} \sum_j nhd_{k,j}\left\{d_i^{(st)}\right\} \\ s_i^{(st)} &= s_i^{(st)} + \sum_k \beta_k^{(st)} \sum_j nhd_{k,j}\left\{s_i^{(st)}\right\} \end{aligned} \tag{5}$$

where st = 1…m (m is number of predict/update steps), $nhd_{k,j}\{center\}$ is operator which returns value of j-th point in k-th neighbourhood of $center$, $\alpha_k^{(st)}$ and $\beta_k^{(st)}$ are lifting coefficients associated with actual predict/update step and k-th neighbourhood of $center$.

New two-dimensional version of forward predict/update steps can be expressed as follows:

$$\begin{aligned} d_{x,y}^{(st)} &= d_{x,y}^{(st)} + \sum_k \alpha_k^{(st)} w_k \sum_j ring_{k,j}\left\{d_{x,y}^{(st)}\right\} \\ s_{x,y}^{(st)} &= s_{x,y}^{(st)} + \sum_k \beta_k^{(st)} w_k \sum_j ring_{k,j}\left\{s_{x,y}^{(st)}\right\} \end{aligned} \tag{6}$$

where $ring_{k,j}\{center\}$ is 2D neighbourhood operator as in Equation (5), w_k is weight for k-th ring.

Thus can be constructed 2D versions of many biorthogonal filters. To implement them we need to know $\alpha_k^{(st)}$ and $\beta_k^{(st)}$. Tab.3 includes in addition to CDF(2,2) also examples for Deslauries-Dubuc wavelet with 4 vanishing moments DD(4,4) and very known 9-7 filter also with 4 vanishing moments in analysis and synthesis high pass filter.

Type	Lifting coefficients	K
CDF(2,2)	$\alpha_1^{(1)} = -\frac{1}{2}$, $\beta_1^{(1)} = \frac{1}{4}$	$\sqrt{2}$
DD(4,4)	$\alpha_1^{(1)} = -\frac{9}{16}$, $\alpha_2^{(1)} = -\frac{1}{16}$ $\beta_1^{(1)} = \frac{9}{32}$, $\beta_2^{(1)} = -\frac{1}{32}$	$\sqrt{2}$
FBI 9-7	$\alpha_1^{(1)} = -1.586134342$ $\beta_1^{(1)} = -0.05298011854$ $\alpha_1^{(2)} = 0.8829110762$ $\beta_1^{(2)} = 0.4425068522$	1.149604398

Tab.3 2D Lifting coefficients and normalisation for various filters

As neighbourhood operators can be used every meaningful operator, e.g. slanted abscissa to focus on directional information in image. Then one must be careful because of rotating sampling lattice in Phase 2 of decomposition (Fig.5).

For correct treatment of signal at boundaries we can build modified boundary predictors as it was in 1D case.

4. Properties of TORING transform

One of primary motivations for developing this 2D lifted non-separable transform was to create transform, that can use all pixels in neighbourhood for prediction (one predict step in 2D Neville filters uses only half of them). Such transformation should better capture energy concentration. Further, true 2D transform can have more anisotropy and thus distribute error (when using in coding application) equally in all directions. This transform should be based on lifting scheme which has many useful properties [4]. TORING satisfies these requirements.

Operation count is of the same order as in separable case. When using lifting only with predicting ring1, we need 1.5 times more operations (when using higher order rings the ratio is higher). Comparing TORING to Neville filters shows computational advantage, see Tab.5 .

Many useful properties are implicitly assured because of relying on lifting scheme: in place calculation, boundary solutions, possible use of non-linear predictors ...

The TORING-type transforms are biorthogonal, because lifting preserves biorthogonality and recursive Quincunx create trivial biorthogonal basis.

Using higher dimension rings we can extend this transformation to arbitrary dimension.

Wavelet basis function for all used transforms are depicted in Fig.9. Here we can observe changing smoothness and shape. Can be observed, that when using TORING algorithm, increasing order of filter does not increase smoothing significantly. Only decorrelation and predict ability are increased.

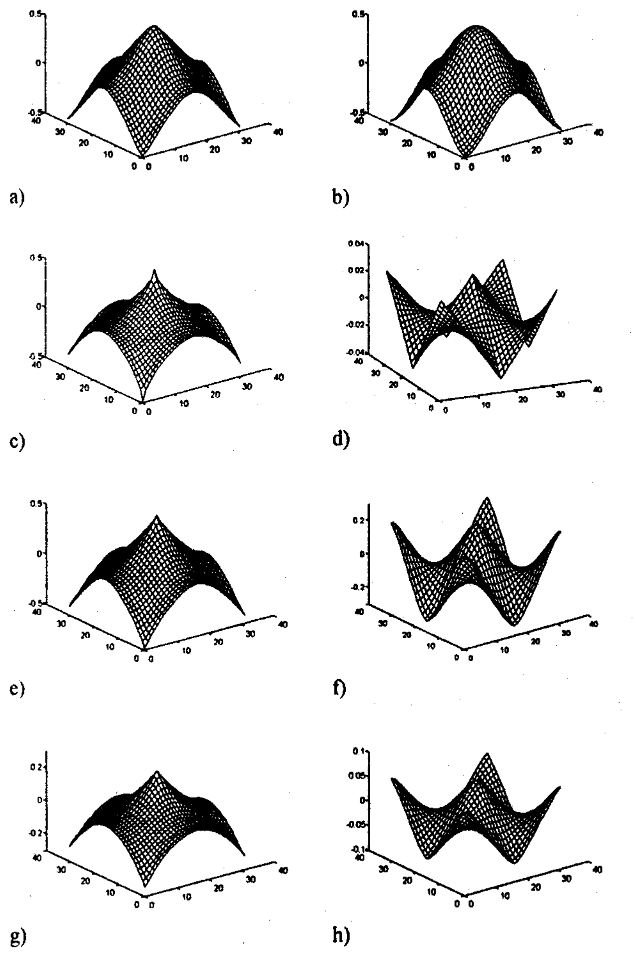


Fig.9 Basic wavelets for various 2D wavelet transforms: a) Neville order 4 b) Neville order 8 c) TORING CDF(2,2) d) CDF(2,2) e)TORING DD(4,4) f) DD(4,4) g)TORING FBI 9-7 h) FBI 9-7

For image compression purposes are in addition smoothness also important decorrelation and energy compaction properties of wavelets. This we can objectively measured with Gain of Transform Coding $\left(G_{TC}\right)$ [1]:

$$G_{TC} = \frac{\frac{1}{M}\sum_{j=0}^{M-1}\sigma_j^2}{\left(\prod_{j=0}^{M-1}\sigma_j^2\right)^{1/M}} \qquad (7)$$

Where M is number of bands and σ_j^2 is variance of coefficients in j-th band. Results for all used transforms on test images are showed in Tab.4. As we can see, TORING 9-7 filter has highest gain among all them.

Approximation property of transforms is also important and is showed in Tab.6. There we can observe that Neville filters are the best (they were designed to interpolate polynomials).

2D Transform (non-separable)	G_{TC}	
	Lena 512x512	Goldhill 512x512
TORING CDF(2,2)	8.99	7.83
Neville4	8.95	8.87
Neville6	9.39	9.55
Neville8	9.24	9.53
TORING DD(4,4)	8.55	8.21
TORING FBI 9-7	12.95	12.58

Tab.4 Comparison of G_{TC} of 2D lifted non-separable transforms

2D Transform	multiplications	additions
TORING CDF(2,2)	1	4
TORING DD(4,4)	2	4
TORING FBI 9-7	2	8
Neville 4	2	14
Neville 6	4	27
Neville 8	7	51

Tab.5 Number of operations to transform one image point in phase 1 for 2D non-separable Wavelet transforms (without energy normalisation).

2D transform	Signal to Noise Ratio (PSNR) in [dB]			
	ZL=1	ZL=2	ZL=3	ZL=4
Neville 8	35.58	29.68	25.67	22.63
Neville 6	35.46	28.62	25.61	22.59
DD(4,4)	35.33	29.61	25.59	22.56
FBI 9-7	35.27	29.58	25.57	22.55
Neville 4	35.14	29.47	25.48	22.49
TORING FBI 9-7	34.96	29.42	25.46	22.49
TORING DD(4,4)	34.71	29.26	25.31	22.35
CDF(2,2)	34.52	29.21	25.31	22.33
TORING CDF(2,2)	34.08	28.96	25.09	22.15

Tab.6 Approximation error in [dB] of 2D wavelets (ZL is number of levels where coefficients were zeroed) using LENA image

Transform Type	Signal to Noise Ratio (PSNR) in [dB]						
	bpp = 2	bpp = 1	bpp = 0.5	bpp = 0.25	bpp = 0.1	bpp = 0.05	bpp = 0.01
TORING CDF(2,2)	43.72	38.9	35.45	32.51	28.88	26.48	21.77
NEVILLE 4	44.21	39.62	36.26	33.15	29.27	26.66	21.94
NEVILLE 6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	26.70	22.14
NEVILLE 8	44.20	39.71	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
TORING FBI 9-7	[illegible]	[illegible]	36.3	33.16	29.25	[illegible]	[illegible]
TORING DD(4,4)	44.1	39.36	35.9	32.89	29.14	26.58	21.83
separable CDF(2,2)	44.29	39.74	36.56	33.38	29.75	27.12	22.28
separable DD(4,4)	44.75	40.24	37.13	33.98	30.14	27.42	22.49
separable FBI 9-7	45.06	40.4	37.21	34.1	30.32	27.49	22.6

Tab.7 PSNR for LENA image (512x512 pixels, 8 bpp) of different 2D WT using SPIHT (bpp is compression ratio in bits per pixel). Best cases are shaded, best is the darkest.

4.1 Application in image compression

With proper coefficient reordering we can substitute transforms in lossy/lossless compression algorithm with non-separable transforms. I used state-of-the-art embedded wavelet image coder SPIHT [3] modified to incorporate with 2D non-separable lifted transforms. Embedded means, that that all codings of the same image at lower bit rates are embedded in the beginning of the output bit stream for the target bit rate. Coding is accomplished in 3 basic steps:

1) discrete wavelet transform
2) coefficient bit plane coding
3) adaptive arithmetic coding

Bit plane coder expects coefficients structured in spatial orientation trees as shown in Fig.9, which represents parent-offspring dependencies of wavelet coefficients. Effectivity depends on ability to quantize trees (or its parts) to zero, according to their significance.

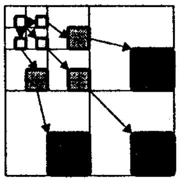


Fig.10 Expected parent-offspring dependencies of WT coefficients

The optimal solution I found, how to adapt coefficients of presented non-separable transforms is to reorder them according to Fig.6a. Compression results for test image are given in Tab.6. TORING 9-7 performs among the best non-separable wavelets used. When looking back at Tab.4 we could expect that 9-7 filter will be clearly the best. But in lossy compression we must take in account also approximation properties (Tab.6) where Neville filters are the best. This in turn is related to the wavelet smoothness (Fig.9).

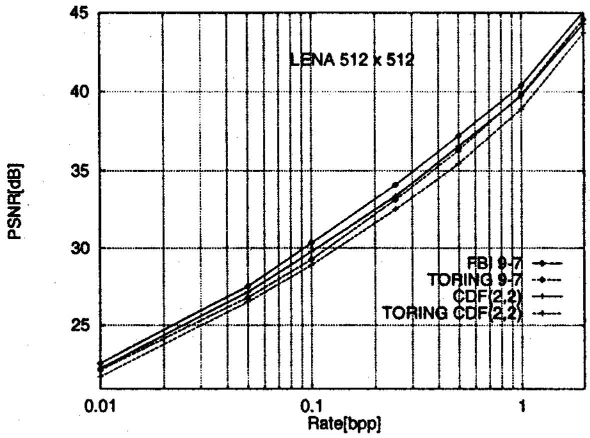


Fig.11 PSNR for TORING wavelet and their separable versions when used in SPIHT algorithm

Used TORING-type wavelets exhibit PSNR by 0.2 - 1dB worse than their separable prototype depending mostly on the test image used (see Fig.11). This can be caused by SPIHT algorithm, which was designed for use with separable transform.

To find out if this is true, and to obtain results also from another type of wavelet coder I test all mentioned wavelet transforms in classical type of transform coder: Transformation - uniform quantization - bit allocation - entropic coding of coefficients (Huffman coder). The differences of PSNR were similar to ones in Fig.11, i.e. this drawback is property of the presented transform scheme.

Objective performance measure such as PSNR is not the best judge in image compression. It captures only average properties but not shapes and artefacts in image. In Fig.12 is depicted part of LENA image degraded by compression algorithm (SPIHT). High compression ratio is used to cause artefacts be more visible. One can see how separable transform spread energy along x and y directions that image shapes are less recognisable. On the other side non-separable transform introduce peaks, depending on smoothness of basis functions. But image is still more readable than in the separable case.

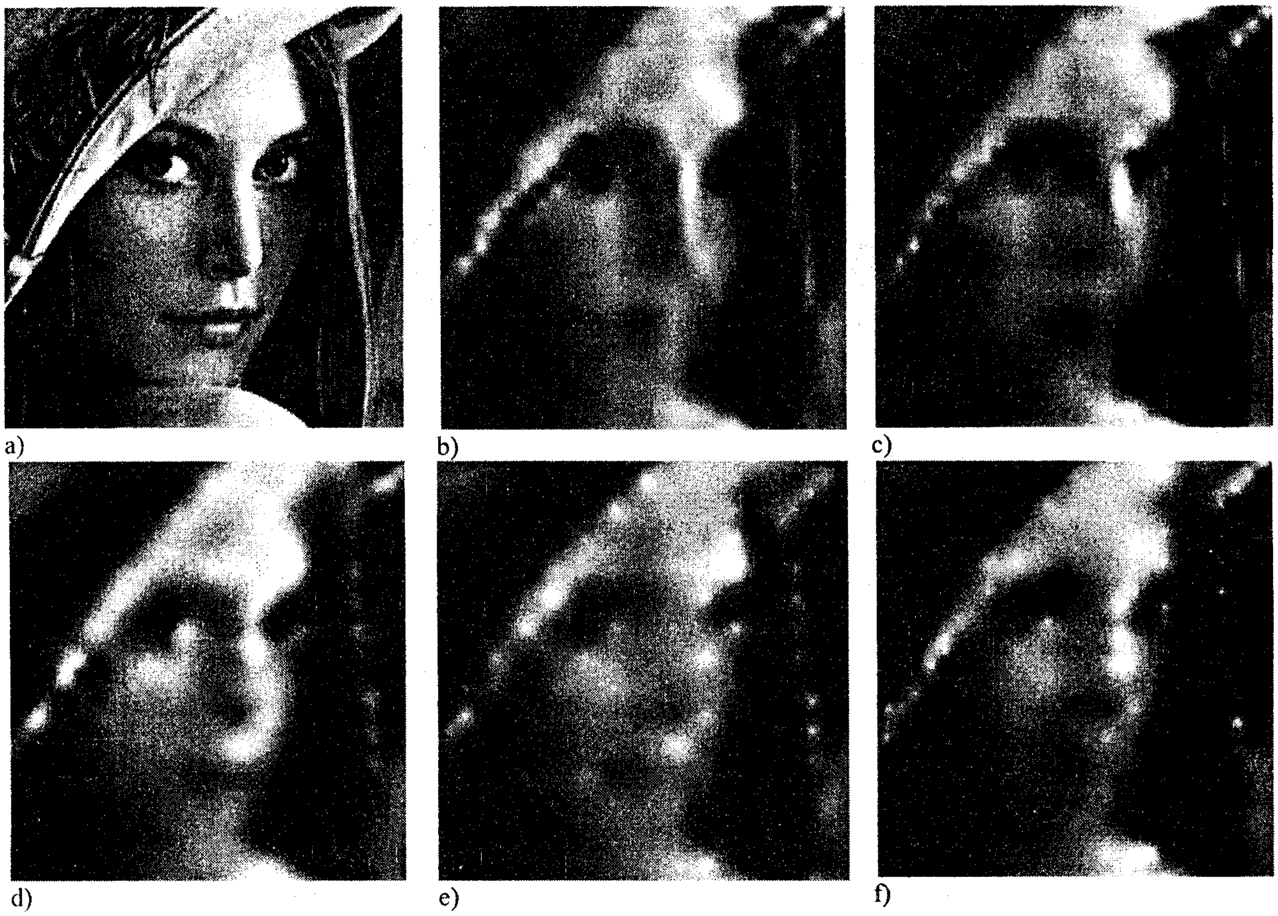


Fig.12 Central part of LENA Image compressed at 0.01 bpp (compression ratio 1: 800) using SPIHT: a) Original b) FBI 9-7 with standard decomposition c) CDF(2,2) with standard decomposition d) Neville 8 e)TORING 9-7 f)TORING CDF(2,2)

5. Conclusion

In this article was developed a new type of non-separable biorthogonal wavelet transform TORING based on lifting scheme. Kernel is created from given 1D prototype that satisfies certain conditions. This is generalisation of approach in [6]. In transform algorithm we use recursive two phase quincunx subsampling. To the same transform algorithm were adapted true two-dimensional Neville filters from [7]. Both (Neville and TORING) have more anisotrophy and capture energy in image better that comparable separable filters. This causes that even having worse PSNR of compressed images, images have visual improvements. In addition TORING has considerable computational advantage over Neville filters (depending on prototype filter used).

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