# Study of 2-input 2-output Blind Signal Separation by Output Decorrelation.

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## Abstract

*The simulations and experiments representing the initial study of the output decorrelation approach to blind signal separation are presented in this paper. The definition of performance indexes for the evaluation and comparison of different algorithms are proposed. Two algorithms are compared. Some first results of real experiments are discussed.*

## Keywords

Blind Signal Separation, Output Decorrelation, Symmetric Adaptive Decorrelator.

## 1. Introduction

The signal separation problem is commonly found in several audio processing applications. This problem has diverse applications in such fields as Telecommunications and Speech Enhancement and it has been extensively studied. The Blind Signal Separation (BSS) problem concerns the situation in which multiple signal sources are simultaneously realised and being observed by a group of sensors. Each one of the sensors collects a different mixture of the source signals. The objective is to recover the original source signals while only the mixed observations of them are available (multiple-input multiple-output). The key assumption for Blind Signal Separation is that the source signals are mutually statistically independent.

Initially, the situation faced by the BSS problem can be represented by the mixing equation

$$x(n) = S(n)\ \mathbf{H}\ , \qquad (1)$$

where $S(n)$= [ $s_1(n)$, $s_2(n)$, ...., $s_m(n)$ ] is the vector formed by concatenation of $m$ vectors $s_i(n)$ = [ $s_i(n)$, $s_i(n-1)$, ...., $s_i(n-P+1)$ ] containing each the last $P-1$ samples of signal $s_i$. **H** is the $m\times m$ mixing matrix (for the case where the number of observations is equal to the number of original signals) with elements $\mathbf{h}_{ji}$ = [ $h_{ji}(0),...,h_{ji}(P-1)$ ]$^T$ representing the coefficients of the transfer function modelling the transmission space between the source $j$ and the sensor $i$. The vector $\mathbf{x}(n)$ = [ $x_1(n)$, $x_2(n)$, ...., $x_m(n)$ ] is thus the vector collecting the $m$ observed signals at the time instant $n$. From expression (1), the signal separation problem itself can be formulated as the computation of another $m\times m$ matrix **W** called "separation matrix" having an output $\mathbf{v}(n)$

$$\mathbf{v}(n) = \mathbf{X}(n)\ \mathbf{W}\ , \qquad (2)$$

where $\mathbf{X}(n)$ is constructed in a similar way as $S(n)$, such that $\mathbf{v}(n)$ approximates $\mathbf{s}(n)$.

## 2. State of the art

The equations (1) and (2) describe the BSS problem for an $m$-input $m$-output case. It is useful to notice that the solution of this problem can be divided in two parts. The first one consists in the choice of a cost function whose minimisation reflects somehow the source separation. The second part will be the choice of an algorithm to minimise the given cost function. Concerning the first part of the solution, there exist various cost functions that have been investigated. Cardoso [4] and Lambert [16] present an overview of cost functions related to several approaches to the problem. R. Lambert also presented an overview of various algorithms for minimising the costs functions for the general $m$-input $m$-output case [16]. In [7] Chan introduced two algorithms for Output Decorrelation of the $m$-input $m$-output case.

This paper will refer to the 2-input 2-output source separation case. The importance of this case relies on the fact that it reflects the basics of BSS, furthermore, the $m$-input $m$-output case could be viewed as an extension of it. For this 2-input 2-output case van Compernolle [9,10,11] presented a system based on Output Signal Decorrelation known as the Symmetric Adaptive Decorrelator (SAD). This system represents a practical, hardware inexpensive and relatively easy to implement solution. It is for these reasons that the study of the SAD system was undertaken and is now presented in this paper.

## 3. SAD using the Steepest-Descent algorithm

In [9,10,11] a detailed description of the Symmetric Adaptive Decorrelator (SAD) depicted in Fig. 2 in both time and frequency domain is made. In this paper the focus will be made on the algorithms used for the adaptive filters and on the effect of these algorithms on the performance of the system. It can be infered from Fig. 2 that the signals $u_1(n)$ and $u_2(n)$ are used as the "error" signals for the filters $W_{21}$ and $W_{12}$ respectively. Similarly, $u_2(n)$ and $u_1(n)$ as the "reference" signals.

In the SAD system using the steepest-descent algorithm the equation for filter coefficient update is [17,20,21]

$$\mathbf{w}_{ji}^{n+1} = \mathbf{w}_{ji}^{n} - 2\,a\,\mu_i(n)\,[\mathbf{R}_j\ \mathbf{w}_{ji}^{n}\ - r_{x_i u_j}\ ]\ , \qquad (3)$$

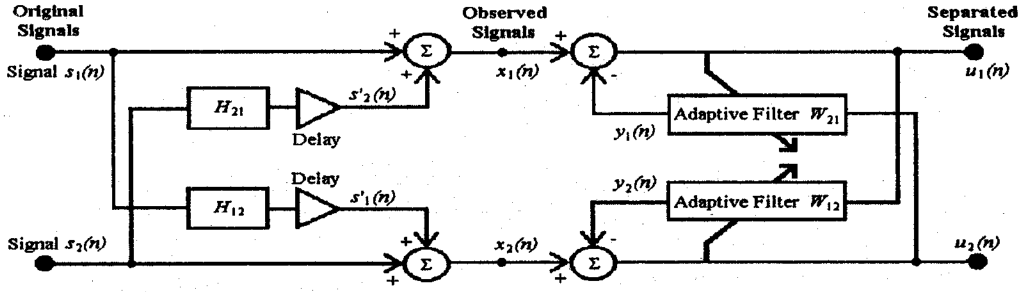


**Fig2. Symmetric Adaptive Decorrelator**

where $\mathbf{w}^n_{ji}$ is the filter coefficient vector at the time instant $n$ (after the $n$-th iteration); $\mu_i(n)$ is called the adaptation step (or step size) at the time instant $n$; $\mathbf{R}_j$ is the autocorrelation matrix of signal $u_j(n)$; $r_{x_i u_j}(k)$ is the cross-correlation of the sequences $x_i(n)$ and $u_j(n)$; $a$ is an arbitrary real factor $a$ such that $0 < a < 1$ (refer to [17] for details). It is clear that the effect of the inclusion of the factor $a$ corresponds to the use of a fraction of the maximal permitted adaptation step, defined [20,21,1,14] by $\mu_i(n) = 1/\lambda_{j\,max}$, where $\lambda_{j\,max}$ is the largest eigenvalue of the matrix $\mathbf{R}_j$.

A computer simulation of the SAD system described was realised using MATLAB. The intention of this simulation was to observe the overall behaviour of the SAD using SD adaptive filters and to observe as well the degradation of the separation process as the correlation of the original signals is increased.

The signals and filters used in the simulation were the following:

- signal $s_1(n)$: Word "Sedum" ("Seven"),
- signal $s_2(n)$: White random noise with zero mean and unitary covariance (The power of both $s_1(n)$ and $s_1(n)$ were equal),
- for sake of simplicity, the mixing filters $H_{ji}$ were chosen as the models of a non-dispersive path,
  $\mathbf{h}_{ji} = [h_{ji}(0), \ldots, h_{ji}(P-1)] = [\,1/\sqrt{2} \;\; 0 \;\; 0\,]$ $(P=3)$, (4)
- the delay was taken of one sampling period.

The performance indexes used for the evaluation of results were the following:

- the Signal to Signal Ratio at the system input channel $i$ ($SSRi_{input}$), which is defined as the ratio of the power of the original signal $s_i(n)$ and the power of the filtered signal $s'_j(n) = s_j(n) * H_{ji}(n)$, $i = 1,2$ ; $j = 2,1$ ( Fig. 1),
- the Signal to Signal Ratio at the system output channel $i$ ($SSRi_{output}$), defined as the power of $s_i(n)$ divided by the power of the difference $[u_i(n) - s_i(n)]$.

The normalised value of the cross-correlation ($NCC$) of the resulting signals $s_1(n)$ and $s_2(n)$ [17] had values ranging from almost 0 to 1, in 20 steps.

Fig. 3 depicts the values of $SSR_{1\,output}$ versus factor $a$ and of the $NCC$ of the original signals. It can be observed that when the $NCC$ of the original signals is close to zero, a proper choice of the value of factor $a$ has to be done. As the cross-correlation of the original signals increases, $SSR\,i_{output}$ decreases to levels even lower than $SSR\,i_{input}$ (even for the best possible choices of the factor $a$). It can be concluded thus, that for values of the $NCC$ of the original signals higher than 0.7 the separation by means of the SAD using the SD algorithm cannot be successfully achieved.

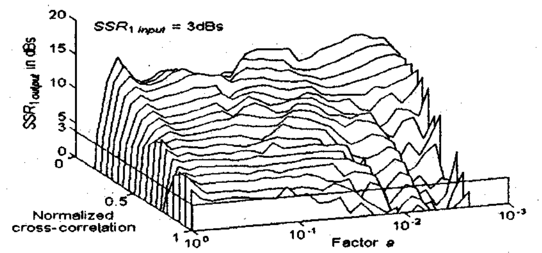


**Fig. 3 $SSR_{1\,output}$ vs. Normalised cross-correlation of $s_1(n)$ and $s_2(n)$ vs. Factor $a$**

## 4. SAD using the LMS algorithm

The SAD system using the LMS algorithm is described by [17]

$$\mathbf{w}^{n+1}_{ji} = \mathbf{w}^n_{ji} + 2\,a\,r_{x_j u_j}(0)\,u_i(n)\,\mathbf{U}^n_j\,, \qquad (5)$$

where $\mathbf{U}^{n\,*}_j$ is the complex conjugate of the vector $\mathbf{U}^n_j = [u_j(n), u_j(n-1), \ldots u_j(n-P+1)]$, consisting of the $P$ last output samples of the signal $u_j(n)$. In this algorithm, the adaptation step $\mu$ was defined [21] by $\mu = r_{x_j u_j}(0)$.

A computer simulation of the SAD/LMS system was also realised. The goals of these simulations were the same as those of the simulations in section 3. The performance indexes used were also the same.

Fig. 4 depicts the values of $SSR_{1\,output}$ for different values of the factor $a$ and of the $NCC$ of the original signals. In a similar way as in section 3, it can be deduced from Fig. 4, that a proper choice of $a$ is necessary for the correct performance of the system. In this case, however, this choice has to be done more carefully. It is well known [22,1,14] that the LMS algorithm has slower convergence than the SD. Regarding this speed of convergence, for the LMS algorithm and the signals used in the simulation, the choice of $a$ can be done such that the adaptation time turns out to be longer than the signal itself. In this situation, the $SSR_{1\,output}$ will not be a very reliable measure of the performance of the system. It is thus possible to take into consideration the $SSR_{1\,output}$ of only those cases for which the adaptation time is lower than a given value. By doing this, the possible choices for the value of factor $a$ will be limited. From Fig. 4 it can be deduced again that as the cross-correlation of the original signals increases, $SSR_{1\ output}$ decreases to levels lower than $SSR_{1\ input}$. It should be noticed that in this case the highest value of the *NCC* for which the system performance is greater than 3dB is just of around 0.3.

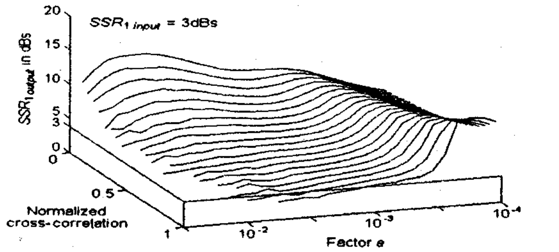


**Fig. 4** $SSR_{1\ output}$ **vs.** *NCC* **of** $s_1(n)$ **and** $s_2(n)$ **vs. Factor** $a$

# 5. Comparison of the SAD performance using the SD and LMS algorithms

Regarding the maximal levels of $SSR_{1\ output}$ reached with each algorithm, the difference between them is not significantly important (around only 1dB). This, on the conditions that the best possible choice of $a$ is made for each one of the algorithms separately and that the value of the *NCC* of the original signals is minimal.

The degradation of the separation process as the previously described conditions change is much faster for the SAD system using the LMS algorithm. While with the SD algorithm, the $SSR_{1\ output}$ can be obtained to be 3dB higher than the $SSR_{1\ input}$ for values of the *NCC* of the original signals going as far as 0.7; with the LMS algorithm this same difference will be only reached for cases where the *NCC* is lower that 0.3.

It should be noticed as well that the performance of the SAD/SD system is less sensible to the choice of $a$ than that of the SAD/LMS. The desired adaptation time is also related with the choice of $a$. It was mentioned in section 4 that the LMS algorithm could lead to long adaptation times for which the $SSR_{1\ output}$ looses value as a proper performance index. For a given adaptation time, the choice of $a$ is limited.

The one advantage of the SAD/LMS system over the SAD/SD is that its implementation requires a smaller number of operations which could be of big importance for real-time applications.

In Fig. 10 several examples of the performance of the SAD system are shown. Fig 10 a) shows the original signal $s_1(n)$ corresponding to the word "sedum"; b) the observed signal $x_1(n)$ obtained in the simulations of sections 4 and 5;c) and d) the separated signal $v_1(n)$ obtained with the SAD system using the SD algorithm for values of $a$ of 0.5 and 0.02 repectively; and e) and f) the separated signal $s_1(n)$ obtained with the SAD system using the LMS algorithm for $a$ equal to 0.025 and 0.00063.

# 6. Experiments with real signals

The goal of the initial experiments realised was to investigate the performance of the SAD/LMS system in a real setting. The signals used for were the words "čtyři" and "dva" ($s_1(n)$ and $s_2(n)$ respectively). The value of the factor $a$ was taken as 0.041.The order of filters $W_{ji}$ was given a value of 20. As it can be appreciated from Fig. 6, the influence of the signal $s_2(n)$ over the observation $x_1(n)$ is more noticeable in the shaded zone denoted A1, and it is greater than the influence of the signal $s_2(n)$ over the separated signal $u_1(n)$ in the corresponding zone A2. Similarly the influence of $s_1(n)$ over $x_2(n)$ in B1 is greater than that over $u_2(n)$ in B2. Another interesting zone to be considered is the one denoted by C, from $u_2(n)$, in which it is possible to observe a distortion in the system output $u_2(n)$ corresponding the presence of a sudden increase of the amplitude of the interfering original signal $s_1(n)$. It should be noticed that in this experiment, the value of the normalised cross-correlation of the original signals used was equal to 0.56, which explains partially the poor improvement observed in the resulting signals.

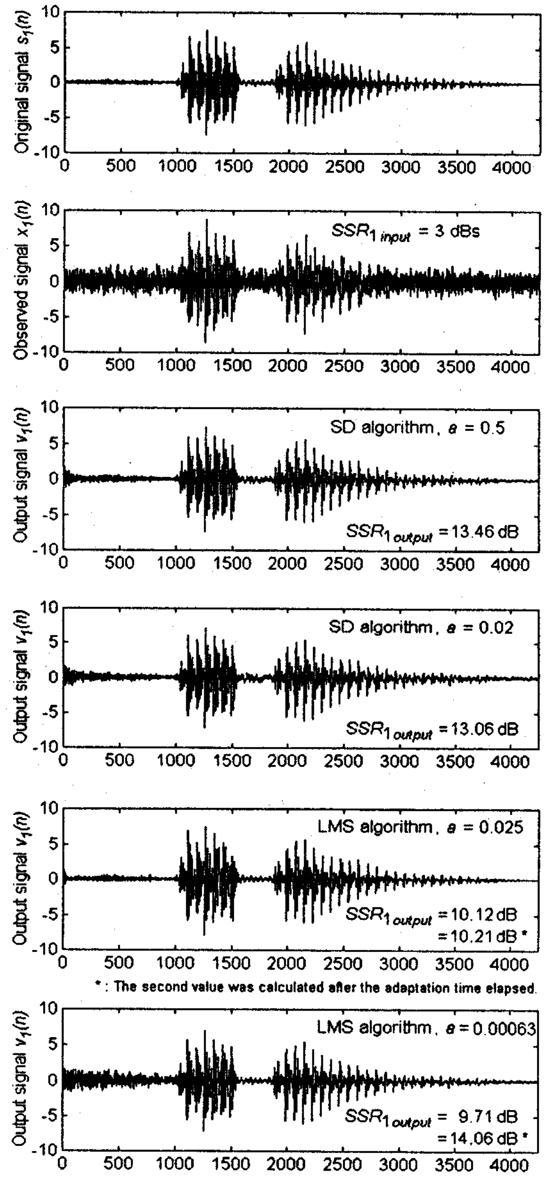


**Fig. 5. Comparison of the signals at different points of the SAD/SD and SAD/LMS systems.**

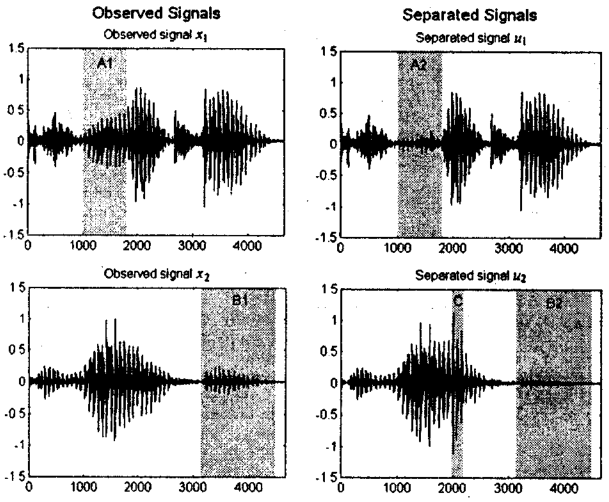


**Figure 6. SAD performance with real signals**

## 7. Acknowledgements

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## 8. Future work

For further study of BSS by Output Decorrelation two branches should be followed. The first one concerns the study of the $m$-input $m$-output case and its comparison with the basic, simpler 2-input 2-output case.

The second branch refers to the application of BSS through Output Decorrelation to real signals in real time. In the first place, further investigation of the algorithms minimising the cost function is still to be done in order to find faster, optimal solutions. The study of the relation between Blind Deconvolution and BSS will also be useful for the application of BSS to real settings.

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