TIME-FREQUENCY ANALYSIS OF SIGNALS GENERATED BY ROTATING MACHINES

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Abstract

This contribution is devoted to the higher order time-frequency analyses of signals. Firstly, time-frequency representations of higher order (TFRHO) are defined. Then L-Wigner distribution (LWD) is given as a special case of TFRHO. Basic properties of LWD are illustrated based on the analysis of mono-component and multi-component synthetic signals and acoustical signals generated by rotating machine. The obtained results confirm usefulness of LWD application for the purpose of rotating machine condition monitoring.

Keywords

Time-frequency representations of higher order, Wigner distribution, L-Wigner distribution.

1. Introduction

In the last decades, great attention is dedicated to the field of fault prevention of mechanical systems. It is because of the fact that the price paid for a fault of some mechanical system part can be very high, especially when we consider secondary loses caused by a small part of bigger mechanical system, possible loss of the whole system or possible injury of a staff. With regards to these facts, mechanical system condition monitoring based on their vibration and noise analysis is necessary and belongs to the most sophisticated method of quality assurance.

The most frequently used digital signal analysis technique applied in the field of machine diagnostics has been the estimation of the power spectrum of vibrations or noise generated by a tested machine. In power spectrum estimation, the process under estimation is treated as a superposition of statistically uncorrelated harmonic components. The distribution of power among these frequency components is then estimated.

For the reliable analyses of rotating machines, it is necessary to measure vibration and sound signals at various speeds of revolution. This is given by the fact that rotating mechanical system is non-linear one where various interactions between its parts are arisen. These interactions depend on the speed of revolution. With regard to these facts, vibration and acoustical signals generated by rotating machine are highly non-stationary. As a result of this, conventional time-invariant spectral analyses are not suitable ones for analysis of a vibration or acoustical signals generated by rotating machines.

One of the possibilities how to remove this drawback is to apply synchronous frequency analyses such as order spectrum [1-4], or order bispectrum [5, 6]. These methods are based on spectrum or bispectrum analysis by using the signal uniformly sampled according to revolution angle of the main shaft of rotating machine. This signal can be obtained by digitally resampling of signals generated by rotating machines and sampled in time domain.

Another approach how to solve the above described problem is to analyse non-stationary noise and vibration signals by means of time-frequency signal analyses. Time-frequency representations of signals (TFRs) are two-dimensional functions of time and frequency [7, 8]. The main problem concerning TFRs of mono-component and multi-component signals are cross-terms. The cross-terms make interpretation of TFRs very difficult. Therefore the great attention in the field of TFR theory is dedicated to the cross-terms reduction.

Recently, higher order time-varying spectra have been defined [9]. They feature advantages and disadvantages of TFRs and higher order spectra. The main advantage of the TFRHO is their ability to reduce cross-terms in a meaningful way.

The basic representation of higher order time-varying spectra is Wigner higher order spectrum (WHOS) [10]. L-Wigner distribution (LWD) is a special case of WHOS, which computes the values of WHOS only along the symmetral of multi-dimensional frequency space.

This contribution deals with the time-frequency analyses of signals generated by rotating machine. These signals are highly non-stationary. Therefore TFRs are convenient for their analyses. The outline of the paper is as follows: Firstly WHOS and LWD are defined. Then, their features are illustrated by simple examples of analysis of mono- and multi-component signals created synthetically. The advantages of LWD are also demonstrated on the analysis of acoustical signals generated by a car engine. In the last section, conclusions and final remarks are presented.

2. Time-Frequency Representations of Higher Order

The basic representation of higher order time-varying spectra is the Wigner higher order spectrum [10, 11] defined in time domain as

$$W_k(t,\omega_1,..,\omega_k)=\int_{\tau_1}..\int_{\tau_k}R_{tk}(\tau_1,..,\tau_k)\prod_{i=1}^{k}e^{-j\omega_i\tau_i}d\tau_i\,, \quad (1)$$

where R_{tk} is the local k-dimensional moment function given by

$$R_{tk}(\tau_1,\tau_2,...\tau_k)=x^*(t-\alpha)\prod_{i=1}^{k}x(t-\alpha+\tau_i) \quad (2)$$

and

$$\alpha=\frac{1}{k+1}\sum_{i=1}^{k}\tau_i\,.$$

Let $X(\omega)$ be the spectrum of analysed signal $x(t)$. Then WHOS can be defined as follows [10]

$$W_k(t,\omega_1,..,\omega_k)=\frac{1}{2\pi}\int_{\theta}R_{\theta k}(\omega_1,..,\omega_k)e^{-jt\theta}d\theta\,, \quad (3)$$

where

$$R_{\theta k}(\omega_1,..,\omega_k)=X^*\left(\sum_{i=1}^{k}\omega_i+\frac{\theta}{k+1}\right)\prod_{i=1}^{k}X\left(\omega_i-\frac{\theta}{k+1}\right) \quad (4)$$

A special case of WHOS (for k=1) is well-known Wigner distribution (WD) [7, 8] defined as

$$WD(t,\omega)=\int_{\tau}x\left(t+\frac{\tau}{2}\right)x^*\left(t-\frac{\tau}{2}\right)e^{-j\omega\tau}d\tau\,, \quad (5)$$

or in terms of signal spectrum $X(\omega)$ as

$$WD(t,\omega)=\frac{1}{2\pi}\int_{\theta}X\left(\omega+\frac{\theta}{2}\right)X^*\left(\omega-\frac{\theta}{2}\right)e^{j\theta t}d\theta\,. \quad (6)$$

It follows from (1) and (4) for multi-component signals that the non-linearity implied by the definition of WHOS gives rise to the heavier presence of cross-terms than that of WD. The cross-terms of WHOS, defined by (1) - (4), can not be removed. However, it is possible to reduce (or completely remove) cross-terms in the case of modified WHOS defined by (1) but only for modified local function R_{tk} given by the equation

$$R_{tk}(\tau_1,..\tau_k)=x^*(t-\alpha)\prod_{i=1}^{l-1}x^*(t-\alpha+\tau_i)\prod_{i=l}^{k}x(t-\alpha+\tau_i)\,, \quad (7)$$

for $l=(k+1)/2$. In this case, auto-terms are located along the symmetral of the multi-dimensional frequency space [12]. The equation for computation of the WHOS along the symmetral of the multi-dimensional frequency space can be obtained from equation (7) in this formula [10-12]

$$W_k(t,\omega)=\frac{1}{2\pi}\int_{\theta}X^{*L}\left(\omega+\frac{\theta}{2L}\right)X^{L}\left(\omega-\frac{\theta}{2L}\right)e^{-jt\theta}P(\theta)d\theta\,, \quad (8)$$

where X(ω) is spectrum of x(t) and P(θ) is window function in frequency domain.

Dual version of equation (5) is known as L-Wigner distribution (LWD) [10-13] defined as

$$LWD(t,\omega)=\int_{\tau}x^{L}\left(t+\frac{\tau}{2L}\right)x^{*L}\left(t-\frac{\tau}{2L}\right)e^{-j\omega\tau}w(\tau)d\tau\,, \quad (9)$$

where $w(\tau)$ is window function in the time domain.

2.1 Mono-Component Signal Analysis

Let us consider a mono-component signal in the next form

$$x(t)=e^{j\phi(t)} \quad (10)$$

The ideal TFR (ITFR) of the signal given by the equation (10) should produce Dirac pulse at the instantaneous frequency $\omega_i(t)=\phi(t)'=d\phi(t)/dt$ [12]

$$ITFR(t,\omega)=\delta(\omega-\phi'(t)) \quad (11)$$

The result of the analysis of $x(t)$ by means of LWD can be derived from equations (9) and (10) as [12]

$$LWD(t,\omega)=\delta(\omega-\phi'(t))*FT\left[e^{j2\sum_{n=3,5...}^{\infty}\phi^{(n)}(t)\frac{\tau^n}{2^nL^{n-1}n!}}\right], \quad (12)$$

where FT is Fourier transformation and $\phi^{(n)}(t)$ is n-th order derivation of phase $\phi(t)$. It can be observed from (12) that LWD of the mono-component signal features the cross-terms caused by the third and higher order derivations (the odd ones) of the signal phase $\phi(t)$, which are divided by factor L^{n-1}. It means that with increasing factor L, the cross terms are significantly reduced comparing with WD (i.e. LWD for L=1).

Above mentioned facts can be illustrated by the next example. Let us consider quadratic frequency modulated signal in the form

$$x(t)=e^{j\pi 1300(t-0.5)^3}\,. \quad (13)$$

WD of this signal is presented in Fig. 1. It is possible to observe the presence of cross-terms, which makes the interpretation of the TFR more difficult. LWD of considered signal $x(t)$ is illustrated by Fig.2. In this case, L=4 is used. From the Fig. 1 and Fig. 2, it is easy to see meaningful reduction of cross-terms in the case of LWD application in comparison with that of provided by the WD.

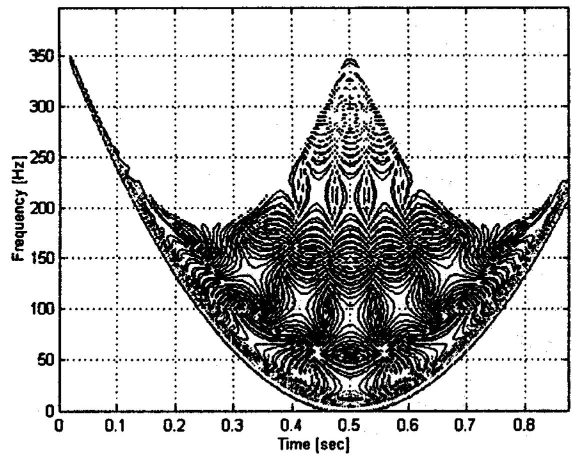

Fig. 1 WD of mono-component signal $x(t)$

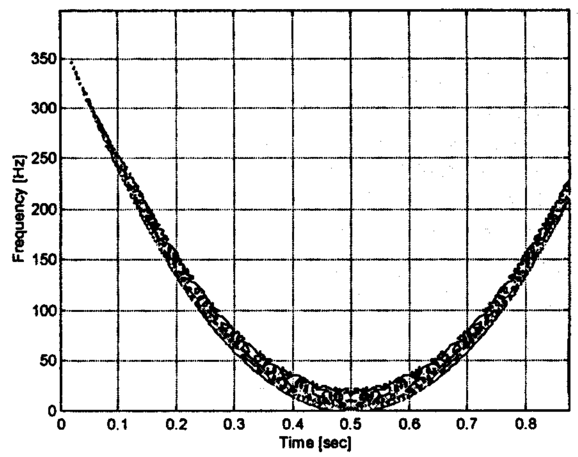

Fig. 2 LWD of mono-component signal $x(t)$ (L=4)

2.2 Multi-Component Signal Analysis

Let us consider a multi-component signal in the form

$$y(t)=\sum_{i=1}^{N} e^{j\phi_i(t)}. \tag{14}$$

For multi-component signals, the non-linearities implied by the definition of LWD give rise the presence of the cross-terms. It is possible to remove the cross-terms of LWD by means of recursive formula [10-13]

$$MLWD_{2L}(t,\omega)=\frac{1}{\pi}\int_{\theta} P(\theta)LWD_L(t,\omega+\theta)LWD_L(t,\omega-\theta)d\theta, \tag{15}$$

where $P(\theta)$ is window function. The starting iteration of equation (15) for $2L$=1 is:

$$MWD(t,\omega)=\frac{1}{\pi}\int_{\theta} P(\theta)STFT(t,\omega+\theta)STFT(t,\omega-\theta)d\theta, \tag{16}$$

where $STFT$ is the Short time Fourier transform defined as

$$STFT(t,\omega)=\int_{-\infty}^{\infty} x(\tau)w^*(\tau-t)e^{-j\omega\tau}d\tau=\int_{-\infty}^{\infty} x(t+\tau)w^*(\tau)e^{-j\omega\tau}d\tau, \tag{17}$$

where $w(t)$ is window function.

By implying appropriate window function $P(\theta)$, it is possible to compute TFR of the multi-component signals without cross terms (or reduced cross-terms).

The above mentioned facts can be illustrated by the next example. Let us consider two-component signal in the form

$$y(t)=e^{j\pi 1500(t-0.5)^3}+e^{j\pi 1500(t-0.5)^3+j2\pi 100t}. \tag{18}$$

The WD of this signal is presented in Fig. 3. It is possible to observe the presence of cross-terms, which makes the interpretation of TFR very difficult. On the other hand, the LWD of $y(t)$ evaluated by (9) for L=2 is given in the Fig 4. It can be seen from this figure, that there are heavy cross-terms in this LWD. The LWD computed by means of the recursive formula (15) (for L=2 and $P(\theta)$ corresponding to Blackman window function) is presented in the Fig. 5. This figure demonstrates the ability of the recursive LWD to reduce cross-terms.

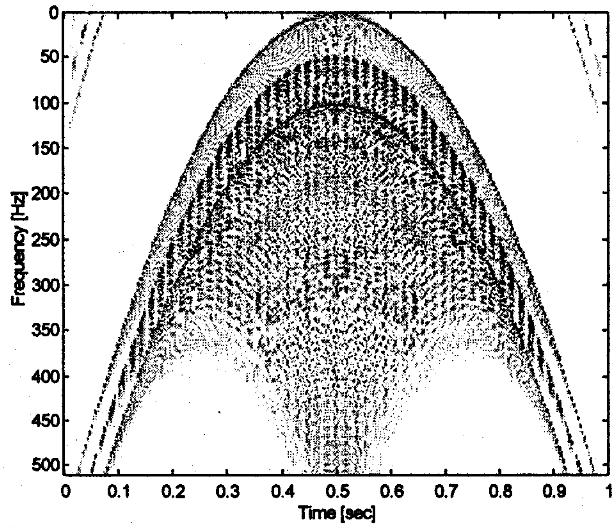

Fig. 3 WD of multi-component signal $y(t)$

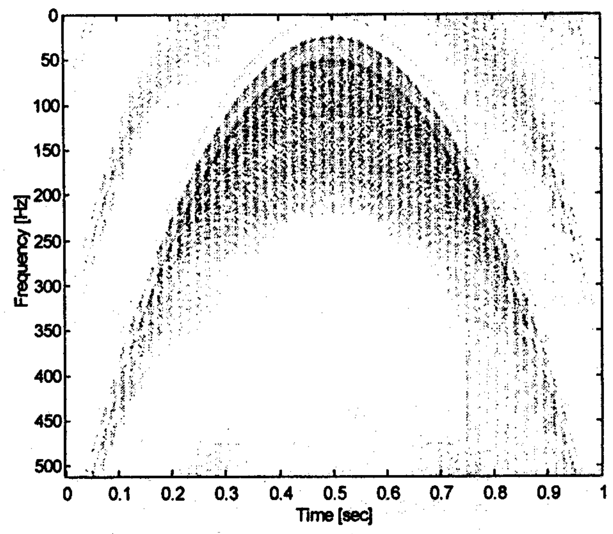

Fig. 4 LWD of multi-component signal *y(t)*, evaluated by(9) for *L*=2

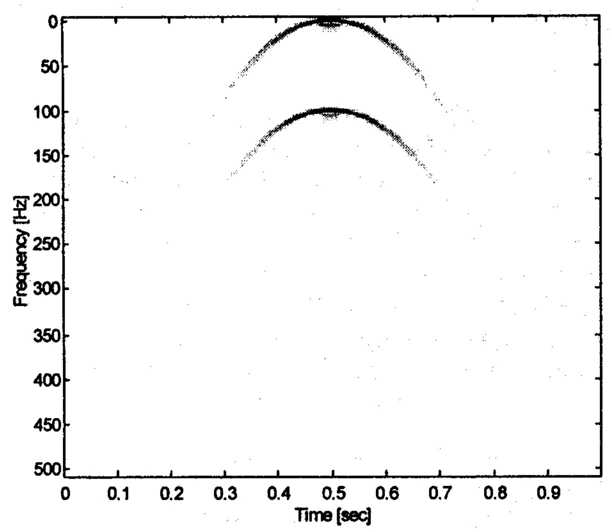

Fig. 5 LWD of multi-component signal *y(t)*, evaluated by(15) for*L*=2

3. Analyses of Signals Generated by Rotating Machines

Rotating machines such as engines or gear boxes are very complex systems from noise and vibration analyses point of view. There are many different sources of noise and vibration in these complex systems. So interactions between these sources can lead to nonlineary coupling of measured signals.

The waveforms of sound or vibration signals generated by rotating machines are dependent on the rotation speed of the main-shaft of machine given by the number of revolutions per minutes (RPM). This fact results in appearing of a smearing effect and effect of frequency modulation of signals. It also means that in this case signals scanned on rotating machines are non-stationary ones. In order to eliminate these effects one can use TFRs which describe spectral content of analysed signal with regard to the time.

In this section, the TFR of the sound signals generated by a car engine will be presented. The acoustical signal to be analysed was measured by microphone placed above the car engine. The sampling frequency was 24kHz. The speed of engine revolution was changing from 2000 RPM to 4000 RPM.

Firstly, only a part of measured sound signal was analysed. This part of a signal was obtained by time and frequency constriction in such a way in order to receive mono-component signal. It was done in order to show advantages of the LWD over the WD analysing mono-component signals. The WD of mono-component signal measured from the car engine is given in Fig. 6. It is possible to recognise that instantaneous frequency is changing in the time. These changes are related to the changes of revolution speed. WD features intensive cross-terms. On the other hand, the cross-terms are suppressed in the case of analysing signal by LWD (Fig. 7). The presented LWD was computed for *L*=2 and rectangular window function. It is also possible to recognise that LWD not only suppresses the cross-terms but it also preserves good time-frequency resolution of WD.

Another situation can be observed in the case of analysis of multi-component signal generated by the car engine. Its WD is given in Fig. 8. WD of the analysed signal contains signal-terms and cross-terms. The intensive cross-terms makes the interpretation of TFR very complicated. LWD of multi-component signal for *L*=2 and rectangular window function is presented in Fig. 9. The LWD features very intensive cross-terms which make the interpretation of TFR impossible.

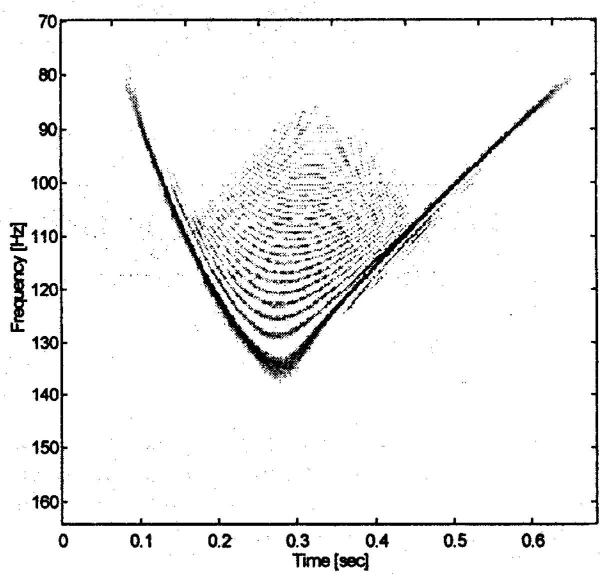

Fig. 6 WD of mono-component car engine signal

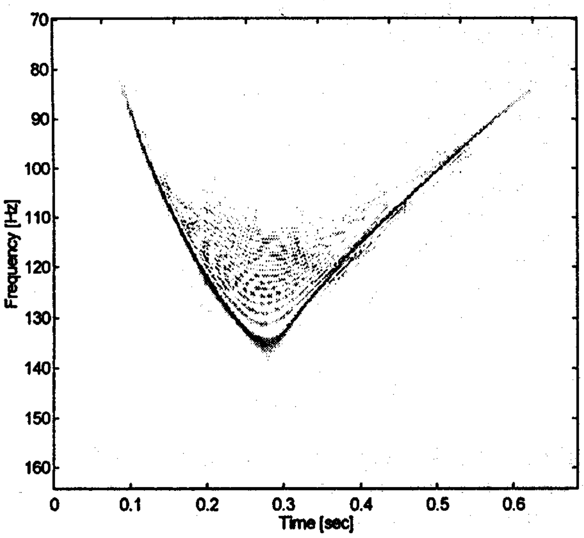

Fig. 7 LWD of mono-component car engine signal computed by recursive formula (15) for L=2

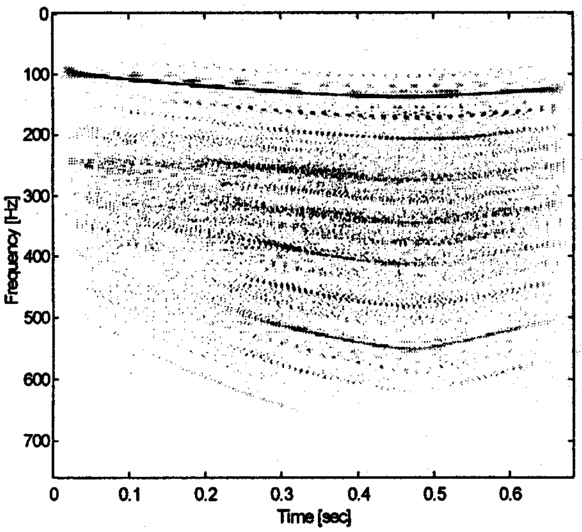

Fig. 8 WD of multi-component car engine signal

The LWD computed by means of the recursive formula (15) of multi-component signal measured at car engine is given in Fig. 10. We can see from this figure the excellent cross-term reduction and preservation of good time-frequency resolution.

4. Conclusion

In this paper, TFRHO and LWD have been defined. Their properties were discussed considering mono-component and multi-component signals.

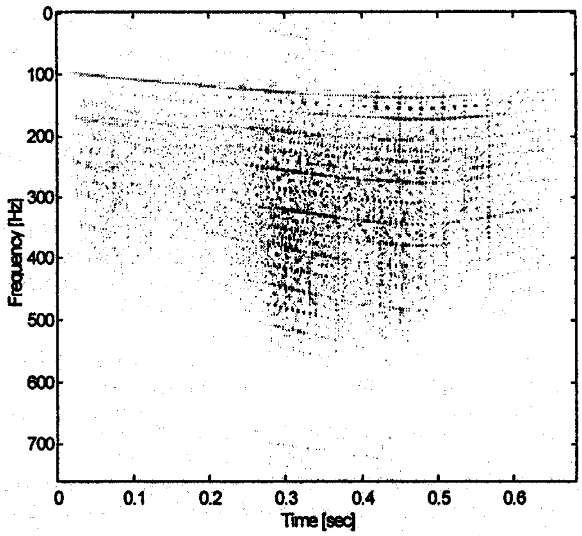

Fig. 9 LWD of multi-component car engine signal computed by (9)

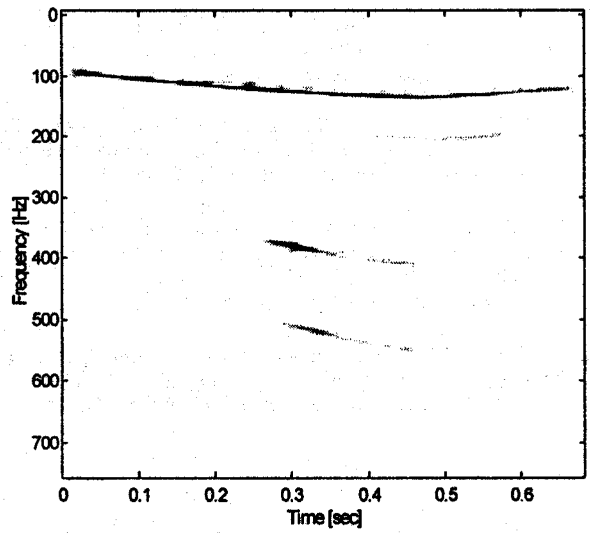

Fig. 10 LWD of multi-component car engine signal computed by recursive formula (15) for L=2

The described TFR analyses were applied on signal generated by the car engine. In the case of mono-component signal, it is obvious that LWD has better performance then that of WD. LWD features good time-frequency resolution and moreover can be cross-terms free. However, in the case of multi-component signal, LWD contains intensive cross-terms. This cross-terms can be removed or reduced by means of the LWD computed by recursive formula.

The obtained results indicate that the LWD evaluated by the recursive formula could be the meaningful tool for the rotating machine condition monitoring.

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