POLYPHASE ORDER ANALYSIS BASED ON CONVOLUTIONAL APPROACH

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Abstract

The condition of rotating machines can be determined by measuring of periodic frequency components in the vibration signal which are directly related to the (typically changing) rotational speed. Classical spectrum analysis with a constant sampling frequency is not an appropriate analysis method because of spectral smearing. Spectral analysis of vibration signal sampled synchronously with the angle of rotation, known as order analysis, suppress spectral smearing even with variable rotational speed. The paper presents optimised algorithm for polyphase order analysis based on non power of two DFT algorithm efficiently implemented by chirp FFT algorithm. Proposed algorithm decreases complexity of digital resampling algorithm, which is the most complex part of complete spectral order algorithm.

Keywords

Chirp FFT, order analysis, polyphase windowing, fixed point DSP, state space digital oscillator

1. Introduction

Rotating machines such as engines or gear boxes are complex mechanical systems. The condition or quality of rotating machines can be determined by measuring of their vibration signals [1]. The waveforms of vibration signals generated by reciprocating machinery depends on rotational speed given by the number of revolutions per minutes (rpm). Typically, reciprocating machinery are tested or working under continuously changing rotational speed. This significantly complicates classical spectrum analysis of vibration signals sampled with constant sampling frequency. This is caused by the fact that the signal frequency components, which are proportional to the rotational speed, change their position in the spectrum and cause spectral smearing.

These problems can be eliminated by signal sampling synchronous with the angle of rotation [2]. Spectral components proportional to the rotational speed do not change their position in the spectrum even if the rotational speed is changed. This type of analysis is known as **order analysis**. The analysable order number is directly proportional to the number of sampling points synchronous with the angle of rotation per revolution.

2. Signal Acquisition Synchronous with Angle of Rotation

The choice of a suitable method for signal sampling synchronous with the angle of rotation depends on the requirements, since the different methods each have their advantages and disadvantages.

2.1 Acquisition with Variable Sampling Frequency

With this method, sampling time points of the AD converter are controlled directly by an external clock pulses synchronous with the angle of rotation, which are typically produced by rotation transmitter or phase-locked loop (PLL). Producing sampling time points using angle of rotation transmitters is in principle the most accurate method but this form of sampling generally requires a relatively complex hardware, such as tunable analogue anti-aliasing filter coupled with the actual rotational speed.

2.2 Acquisition with Constant Sampling Frequency

This method uses digital resampling of time-equidistant samples of vibration signal [2,3], which means that analogue modules are replaced by equivalent digital signal processing operations typically implemented by high performance digital signal processor (DSP).

Signal acquisition with a constant sampling frequency makes it possible to analyse signals synchronised to both time and angle of rotation. The original vibration signal, for example, is available for acoustic or psychoacoustic evaluation with full acquisition bandwidth. Unlike the version with variable sampling frequency, just a simple analogue low-pass filter with fixed cut-off frequency is required as anti-aliasing filter, since signals are preferably acquired with ΣΔ–AD converters. They have high amplitude resolution (16 bits and over) so that high dynamic range is guaranteed.

The vibration signal is sampled at equidistant time intervals and temporarily stored in a buffer. At the same time, the time points of occurrence of synchro pulses are recorded and stored in the relation to the sampled vibration values. Block diagram of this method is depicted on Fig.1 [2].

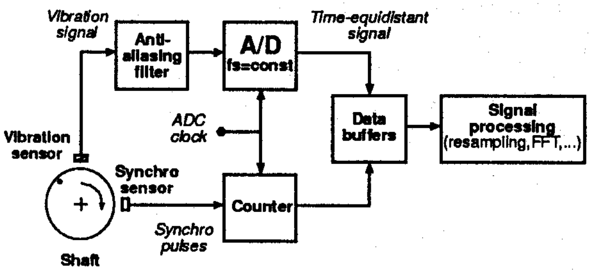

Fig.1 Principle of signal acquisition with fixed sampling frequency

Using a preselected model, the actual angle progression of rotation can be approximated between the synchro pulses. Typical model used in real-time implementations uses constant rotational speed approximation between two synchro pulses given by linear equation:

$$\phi(t) = \omega t + \phi_0 \qquad (1)$$

where ω is angular velocity and ϕ_0 is initial angle (although more complex quadratic model with constant acceleration is sometimes considered, its computational complexity is too large for real-time performance, moreover accuracy of linear model can be improved by increasing number of synchro pulses per revolution). Then it is possible to determine the sampling points corresponding to a constant change in angle (interpolation on the time axis).

Resampling of temporarily stored time-equidistant sampling values is then done at the desired time points synchronous with the angle of rotation and the amplitude of the vibration signal (tracking anti-aliasing filter) is reconstructed by using appropriate interpolation procedure (interpolation on the amplitude axis). Such interpolation is theoretically possible for any signal whose bandwidth has been limited before sampling to the Nyquist frequency. Signal flow of complete digital resampling is depicted on Fig.2.

For real-time performance a FIR filter with variable filter window length is very efficient solution. The measured vibration equidistant values $v(nT)$ adjacent to the required sampling time point t_K are convoluted with the pulse response of the FIR filter $g(t)$ [2]:

$$v_{syn}(t_K) = D \sum_{n \in FIR\ window} v(nT) g(t_K - nT) . \qquad (2)$$

Depending on the actual rotational speed, a variable number of adjacent interpolation samples is used for filtering and appropriate decimation constant D is used [2]. In the following text digital version of (2) is referred as $v_{syn}(n)$.

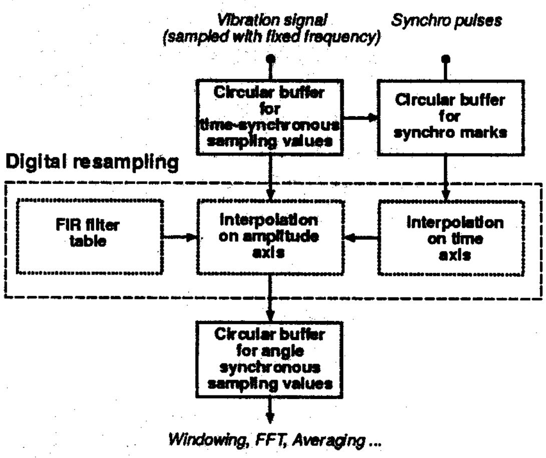

Fig.2 Signal flow of complete digital resampling algorithm

Digital resampling is the most computationally intensive part of described order analysis approach. Its complexity is directly proportional to the number of interpolated (resampled) angle equidistant points. The next chapters describe method which allows to decrease number of resampling points to the minimal possible value (for given order analysis resolution).

3. Order Selective Analysis

For certain tasks, such as gear box diagnostics [2-4], order spectra with high resolution as well as high order are required. This generally results in long DFT length (16384 and over computed by FFT). Provided that the construction of the rotating machinery is known (e.g. for gear box it is the number of teeth at any gear), it is possible to use this a priori information and to compute only the required spectral components. Thus, the FFT arithmetic complexity can be significantly reduced by using a special polyphase window [2,5]. This method uses the fact, that typically only certain harmonics (e.g. multiples of the meshing frequencies) are important for analysis. Proposed algorithm [6] further decreases arithmetic complexity of orthogonal spectral analysis [2,4] by decreasing the number of resampling points and will be demonstrated on the two shaft gear box [4].

3.1 Orthogonal Order Analysis

Orthogonal order spectral analysis computes the DFT coefficients of the vibration signal with respect to the different rpm without any spectral leakage between different components. That requires simultaneous processing of an integer number of shafts revolutions. For the two shaft gear box the signal length depends on the transmission ratio

$$t_r = \frac{z_2}{z_1} \tag{3}$$

between the shafts (z_1, z_2 are corresponding numbers of teeth of input shaft1 and output shaft2). As an example we suppose a gear pair with transmission ratio $t_r = 44/23$ [4]. Since there is no common integer divider (except 1) angle synchronous sampling of both shafts requires acquisition of 23 resp. 44 revolutions of each shaft. If we intend to analyse up to the 3rd gear mesh frequency of the shaft1 ($z_1 = 23$), at least 69 shaft order spectrum with suborder resolution of 1/44 order have to be calculated [4]. That requires an order spectrum with more than 3036 frequency bins so DFT with the size at least 6072 must be used. Moreover, if a Hann window $h(n)$ is applied, the size of power of two real input FFT algorithm, which is typically used for efficient DFT computation, must be $N_{ord} = 16384$. The FFT is applied on the real input signal

$$x(n) = h(n) v_{syn}(n), \qquad n = 0,1,\ldots,N-1 \tag{4}$$

where $x(n)$ is windowed version of revolutions synchronous vibration signal $v_{syn}(n)$ and value N depends on used algorithm. The block diagram of orthogonal order analysis algorithm based on power of two FFT algorithm is depicted on Fig.3.

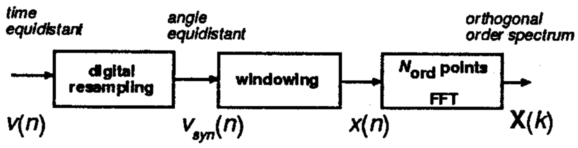

Fig.3 Orthogonal order analysis based on power of two FFT algorithm

Main disadvantage of this algorithm is generally very large (for typical fixed point DSP) FFT size with large memory requirements. This is consequence of requirement to compute all spectral components without any spectral leakage.

3.2 Order Analysis based on Decimated power of Two FFT Algorithm

This method of the order analysis uses polyphase windowing [2,5], which is based on modulo-M reduction or wrapping of a signal [7]. It is defined by dividing the signal $x(n)$ into B continuous non-overlapping blocks of length M, wrapping the blocks around to be time-aligned with the first block, and adding them up:

$$x_{dec}(n) = \sum_{i=0}^{B-1} x(iM+n), \quad n = 0,1,\ldots, M-1. \tag{5}$$

The resulting wrapped block

$$\mathbf{x}_{dec} = [x_{dec}(0), x_{dec}(1), \ldots, x_{dec}(M-1)] \tag{6}$$

has length $M = N_{dec}/B$, where N_{dec} is the size of original signal segment. The connection of the mod-M reduction to the DFT is the theorem that the length-M wrapped signal $x_{dec}(n)$ has the same M-point DFT as the original unwrapped signal $x(n)$, that is,

$$X_{dec}(k) = \sum_{n=0}^{M-1} x_{dec}(n) \exp\left(-j\frac{2\pi kn}{M}\right) = \tag{7}$$

$$= X(k) = \sum_{n=0}^{N_{dec}-1} x(n) \exp\left(-j\frac{2\pi kn}{M}\right) \quad k = 0,1,\ldots M\text{-}1$$

where $\mathbf{X}_{dec}$ is the M-point DFT of the length-M signal $\mathbf{x}_{dec}$ [7]. This approach can significantly decrease the size of used FFT algorithms. The block diagram of order analysis algorithm based on decimated power of two FFT algorithm is depicted on Fig.4.

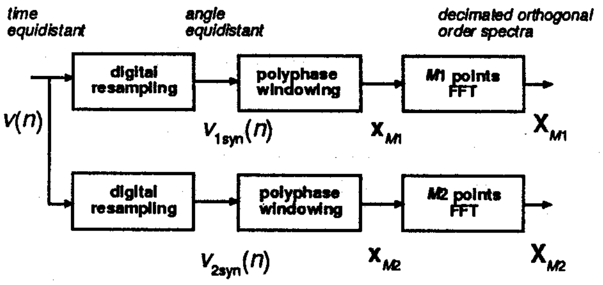

Fig.4 Orthogonal order analysis based on decimated power of two FFT algorithm

As an example we suppose gear pair from previous example. Let the sizes of wrapped blocks (are equal to the power of two FFT size) are M=$M1$=$M2$= 256. The sizes of $N1_{dec}$ and $N2_{dec}$ of angle equidistant unwrapped signals must *be*

$N1_{dec}$=2*44*256=22528,

$N2_{dec}$=2*23*256=11776.

Main disadvantages of this algorithm are:

- two times resampling is necessary
- number of resampling points $N1_{dec} + N2_{dec}$ is generally higher than N_{ord} in the orthogonal order analysis
- two window functions with generally non power of two length must be generated

A nice feature of this algorithm is much lower memory requirements (smaller FFT size, overlapping and adding of windowed signal can be done on-line). Although FFT size (as well as corresponding DSP implementation) can be significantly reduced, complete decimated order analysis algorithm implementation requires higher number of resampling points than in full orthogonal order analysis. Further reduction of number of resampling points is possible by using combination of both previously described methods [6].

3.3 Order Analysis based on Decimated non power of two DFT

This method is based on the same principle as previous decimated power of two FFT algorithm with the exception that *M1* and *M2* can be any integers. The block diagram of order analysis algorithm based on decimated non power of two DFT algorithm is depicted on Fig.5.

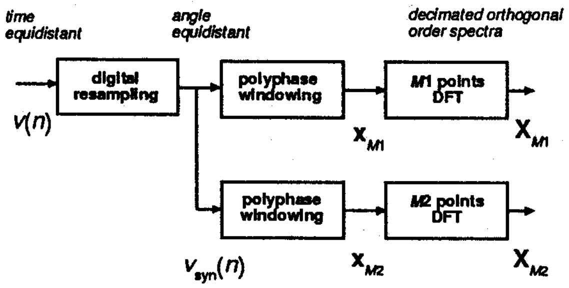

Fig.5 Orthogonal order analysis based on non power of two DFT algorithm

The gear box from previous example can be processed with the following parameters:

N_{dec} =23*44*3*4=12144,

$M1$ =23*6=138,

$M2$ =44*6=264.

Main advantages of this algorithm are:

- only one resampling is necessary
- resampling requires integer number of points per revolution
- number of resampling points N_{dec} is generally not higher than N_{ord} in the orthogonal order analysis (and typically can be lower)
- only one window function with generally non power of two length must be generated
- small memory requirements

The only disadvantage of the proposed algorithm is non power of two DFT algorithm which is generally less efficient than power of two FFT. Next chapters present possible real-time implementation of non power of two DFT algorithm based on chirp Fourier transform suitable for standard fixed point DSP implementation.

4. The Chirp Fourier Transform

A special technique, called chirp Fourier transform [8] uses convolutional approach to DFT computation. It can be used for implementation of spectral ZOOM effect or efficient non power of two DFT computation based on power of two FFT algorithm. The second property is used in the proposed real time decimated order spectral analysis.

Suppose we are given a sequence $x_{dec}(n)$, n=0,1..., M-1 and we wish to compute $X_{dec}(k)$ in (7) for k=0,1, ..., K-1 where $K \le M$. Direct computation of (7) would require MK complex multiply-add operations. The chirp Fourier transform reduces the number of operations to a small multiple of

$$(M+K)\log_2(M+K). \tag{8}$$

Define [7]:

$$W = \exp\left(j\frac{2\pi}{M}\right) \tag{9}$$

and rewrite (7) as:

$$X_{dec}(k) = \sum_{n=0}^{M-1} x_{dec}(n)\, W^{-nk}\,. \tag{10}$$

By using the identity

$$nk = 0.5\left[n^2 + k^2 - (k-n)^2\right] \tag{11}$$

we get: (12)

$$X_{dec}(k) = W^{-0.5k^2} \sum_{n=0}^{M-1} x_{dec}(n)\, W^{-0.5n^2} W^{0.5(n-k)^2}\,.$$

$$g(n) = x_{dec}(n)\, W^{-0.5n^2}, \qquad f(n) = W^{0.5n^2} \tag{13}$$

and rewrite (12) in the form

$$X_{dec}(k) = W^{-0.5k^2} \{g * f\}(k) \tag{14}$$

where * represents convolution. This convolution can be effectively implemented by circular convolution in the frequency domain. The sequence $\{g(n)\}$ is finite, having the same length as $\{x_{dec}(n)\}$, whereas $\{f(n)\}$ is infinite one. However, we are interested in $X_{dec}(k)$ only on a finite set of ks, so we need only the finite segment $\{f(n), n=-M+1, ...-1, 0, 1, ..., K-1\}$. We can therefore summarise the non power of two DFT algorithm implemented with power of two FFT algorithm as follows:

- find a number L that is an integer power of two and is not smaller than $M+K-1$
- form the sequence $\{g(n)\}$ as defined in (13) and zero pad it to a length L
- form the sequence $\{f(n), n=0, 1, ..., K-1\}$, as defined in (13); concatenate to it the sequence $\{f(n), n=-L+K, ...-2, -1\}$
- perform the operation
  IFFT{FFT($g(n)$).FFT($f(n)$)} (15)
- multiply the first K terms of the resulting sequence according to (14)

The computational complexity of the DFT is thus proportional to $L\log_2(L)$ plus a small multiple of M and another small multiple of K for the auxiliary sequences. Although this algorithm is not as efficient as power of two FFT it still has real-time performance as it will be documented in the next chapter.

5. Hardware Optimization

5.1 Polyphase Windowing

Proposed algorithm has been optimised for DSP board of ANOVIS measurement system [9] and is based on available optimised DSP libraries [5] for 24bit fixed point Motorola DSP56002 signal processor [10]. General (high precision, high-speed) C-callable assembler library function for generation of partial polyphase window function is based on P-term cosine series expansion [5]:

$$h(n)=\sum_{i=0}^{P} h_i \cos^i\left(\frac{2\pi n}{N_{dec}}\right),\ n=0,1,\ldots,N_{dec}-1. \quad (16)$$

Sequential on-line generation (this feature significantly decreases memory requirements) of generally long N_{dec} points sequence of *cos( )* values is accomplished by state space implementation of digital oscillator [6]:

$$\begin{bmatrix} c(n) \\ s(n) \end{bmatrix} = \begin{bmatrix} \cos(2\pi/N_{dec}) & -\sin(2\pi/N_{dec}) \\ \sin(2\pi/N_{dec}) & \cos(2\pi/N_{dec}) \end{bmatrix} \begin{bmatrix} c(n-1) \\ s(n-1) \end{bmatrix} \quad (17)$$

where $c(n)=\cos(2\pi n/N_{dec})$ for $c(0)=1$, $s(0)=0$.
State space implementation of digital oscillator ensures low sensitivity to the finite word length effects. In order to achieve 23 bit precision of output values it is necessary to implement digital oscillator in double precision (DSP56002 supports 48 bit double precision mode). The precision of this approach is illustrated in Fig.6 for digital oscillator output (48 bit values) and Hamming window output (24 bit values) for randomly chosen M and N_{dec} values.

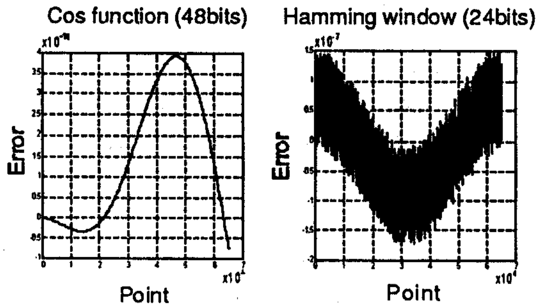

Fig.6 a/ Error of digital oscillator output for window length N_{dec}=65000 and 48 bit precision, b/ error of Hamming window function with N_{dec}=65000 and 24 bit output (1LSB ≅ 1.2E-7)

5.2 Polyphase Order Analysis

The speed and round-off error performance of the proposed algorithm was tested on the synthetic revolutions synchronous vibration signal $v_{syn}(n)$ as a mixture of the following signals (rpm=*const*, $f_{rot}=const/60$, $z_1=23$, $z_2=44$, $f_s=10f_{rot}z_1$):
Three harmonics (24th, 48th and 96th) of shaft1 and shaft2:

$s_1^{24th}(n)=0.01\sin(2\pi 24n/z_1/10)$ .... –40dB level

$s_1^{48th}(n)=0.1\sin(2\pi 48n/z_1/10)$ .... –20dB level

$s_1^{96th}(n)=1.0\sin(2\pi 96n/z_1/10)$ .... 0dB level

$s_2^{24th}(n)=1.0\sin(2\pi 24n/z_2/10)$ .... 0dB level

$s_2^{48th}(n)=0.1\sin(2\pi 48n/z_2/10)$ .... –20dB level

$s_2^{96th}(n)=0.01\sin(2\pi 96n/z_2/10)$ .... –40dB level

and interference error signal at $(24+1/z_2)^{th}$ order (should be removed from order spectra of shaft1 and shaft2):

$err(n)=10\sin(2\pi(24+1/z_2)n/z_1/10)$ ....+20dB level.

The input signal $v_{syn}(n)$ (simulates angle equidistant samples):

$$v_{syn}(n)=s_1^{24th}(n)+s_1^{48th}(n)+s_1^{96th}(n)+s_2^{24th}(n)+ +s_2^{48th}(n)+s_2^{96th}(n)+err(n) \quad (18)$$

was quantized to 16 bit precision (resolution of typical ΣΔ–AD converters) and processed with orthogonal DFT with N_{ord}=20240 and proposed algorithm with *M1*=230 and *M2*=440, *K*=100, *L1*=512, *L2*=1024. The signal (18) represents signal with quite large dynamical range and shaft signals are at least 20dB below maximal AD range. The complete orthogonal DFT spectrum of this signal computed by MATLAB is depicted on the Fig.7.

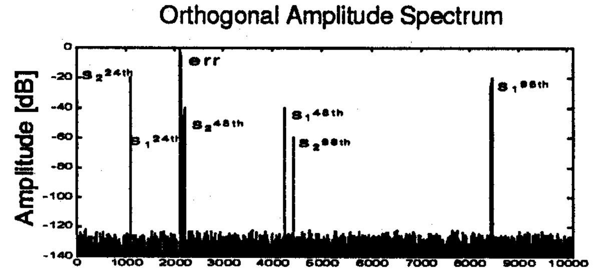

Fig.7 Orthogonal order spectrum of signal $v_{syn}(n)$ computed by 20240 point DFT (MATLAB precision)

The order spectra of shaft 1 and shaft 2 normalised to the maximal amplitude are depicted on the Fig.8 and Fig.9 respectively.

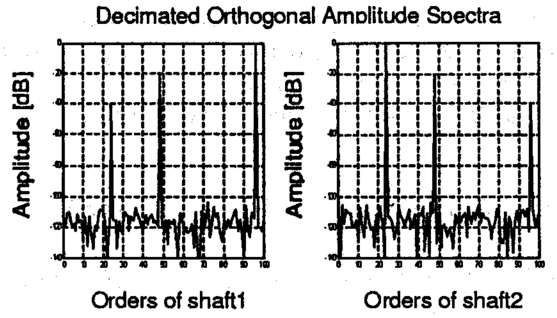

Fig.8 Decimated orthogonal order spectra of shaft1 and shaft2 (MATLAB 64 bit floating point precission)

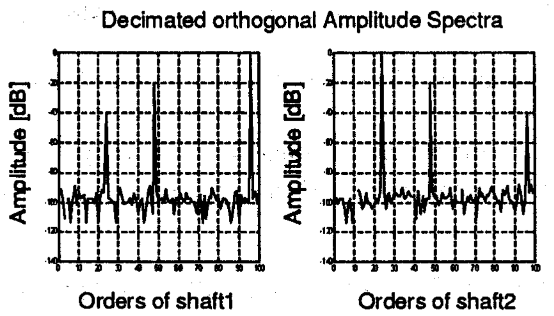

Fig.9 Decimated orthogonal order spectra of shaft1 and shaft2 (24bit fixed point DSP results)

6. Conclusion

The results clearly show that level of round-off noise of 24 bit fixed point implementation is quite low (although higher than for 64 bit floating point MATLAB arithmetic). Complete processing (without oversampling) of 20240 input samples requires about 3200000 cycles of MOTOROLA DSP56002 with 1 wait state access time to the external SRAM. It is less than 5% of total DSP one second processing power, which demonstrates real-time performance of this algorithm. Moreover the number of resampling points for the proposed algorithm is minimal possible one (for complete shaft1 and shaft2 order spectra with given resolution) and indirectly increases speed of complete algorithm (including oversampling).

Currently the most critical part of the polyphase order analysis algorithms is the resampling algorithm which consumes most of the DSP computational power.

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