New Canonical State Models of Chua's Circuit Family

Dedicated to the 80th birthday of Prof. Ing. Josef ČAJKA, DrSc.

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Abstract

Two new modified types of canonical state models simulating chaotic phenomena in piecewise-linear dynamical systems are derived. Both are topologically conjugate to Class C similarly as Chua's circuit family. Their state matrix equations and corresponding integrator-based circuit models are proposed including their relations with the first elementary canonical state model. As an example the phase portraits of typical chaotic attractor are shown

Keywords

dynamical systems, piecewise-linear systems, state models, canonical form, Chua's circuits, chaos

1. Introduction

Third-order piecewise-linear (PWL) dynamical systems belonging to Class C of vector fields in $\Re^3$ [1] can be described by the matrix state equation

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}\,h(\mathbf{w}^T\mathbf{x}) \qquad (1)$$

($\mathbf{A} \in \Re^{3x3}, \mathbf{b} \in \Re^3, \mathbf{w} \in \Re^3$) where PWL function

$$h(\mathbf{w}^T\mathbf{x}) = \tfrac{1}{2}\left(\left|\mathbf{w}^T\mathbf{x}+1\right| - \left|\mathbf{w}^T\mathbf{x}-1\right|\right) \qquad (2)$$

is continuous, odd-symmetric and partitioning $\Re^3$ by two parallel planes into the inner (origin) region and two outer regions (Fig. 1). The dynamical behaviour of such systems is determined by two sets of eigen values representing two characteristic polynomials associated with the corresponding regions [1], i.e.

$$D_0: \quad P(s) = \det(s\mathbf{1}-\mathbf{A}_0) = (s-\mu_1)(s-\mu_2)(s-\mu_3) = \\ = s^3 - p_1 s_2 + p_2 s - p_3 \qquad (3)$$

$$D_{+1}, D_{-1}: \quad Q(s) = \det(s\mathbf{1}-\mathbf{A}) = (s-\nu_1)(s-\nu_2)(s-\nu_3) = \\ = s^3 - q_1 s_2 + q_2 s - q_3 \qquad (4)$$

where $\mathbf{A}_0 = \mathbf{A} + \mathbf{b}\mathbf{w}^T$ and $\mathbf{1}$ is the unity matrix.

Any two systems having the same eigenvalues are qualitatively equivalent and their mutual relations can be expressed by the linear topological conjugacy conditions [2]

$$\tilde{\mathbf{x}} = \mathbf{T}\mathbf{x}, \quad \tilde{\mathbf{A}} = \mathbf{T}\mathbf{A}\mathbf{T}^{-1}, \quad \tilde{\mathbf{b}} = \mathbf{T}\mathbf{b} \qquad (5a,b,c)$$

where

$$\mathbf{T} = \tilde{\mathbf{K}}^{-1}\mathbf{K} \qquad (5d)$$

Variables $\tilde{\mathbf{x}}$ and $\mathbf{x}$, state matrices $\tilde{\mathbf{A}}$ and $\mathbf{A}$, vectors $\tilde{\mathbf{b}}, \tilde{\mathbf{w}}$ and $\mathbf{b}, \mathbf{w}$ belong to the first and second systems, respectively. Partial transformation matrices $\tilde{\mathbf{K}}$ and $\mathbf{K}$ are defined by the nonsingular form [2]

$$\tilde{\mathbf{K}} = \begin{bmatrix} \tilde{\mathbf{w}}^T \\ \tilde{\mathbf{w}}^T\tilde{\mathbf{A}} \\ \tilde{\mathbf{w}}^T\tilde{\mathbf{A}}^2 \end{bmatrix} \quad \text{and} \quad \mathbf{K} = \begin{bmatrix} \mathbf{w}^T \\ \mathbf{w}^T\mathbf{A} \\ \mathbf{w}^T\mathbf{A}^2 \end{bmatrix} \qquad (6a,b)$$

fulfilling the observability condition of pairs $(\tilde{\mathbf{A}}, \tilde{\mathbf{w}}^T)$ and $(\mathbf{A}, \mathbf{w}^T)$, respectively.

So called Chua's circuit family (e.g. Chua's canonical circuit, Chua's oscillator, etc.) represent dynamical systems which are canonical in the sense of the minimum of free parameters needed for their design [1]. Their elementary forms have quite simple relations between the network parameters and the corresponding equivalent eigenvalue parameters, i.e. the coefficients of two characteristic polynomials [3]. Many other state models can be derived using the linear topological conjugacy to utilize them for the study of various chaotic phenomena including the synchronization [4],[6]. The new proposed canonical models are based on the decomposition of the state matrix $\mathbf{A}$ into the block-diagonal [7] and block triangular forms, i.e. generally

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}' & 0 \\ 0 & a_{33} \end{bmatrix} = \left[\begin{array}{cc|c} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ \hline 0 & 0 & a_{33} \end{array}\right] \qquad (7a)$$

and

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}' & \mathbf{A}'' \\ 0 & a_{33} \end{bmatrix} = \left[\begin{array}{cc|c} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ \hline 0 & 0 & a_{33} \end{array}\right], \qquad (7b)$$

respectively. Their global design procedure and relation to the chosen simple reference system is described more in detail in the following part.

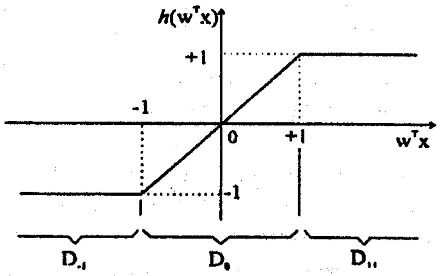


Fig. 1. Simple PWL feedback function

2. General Design Procedure

When substituting general forms of state matrix $\mathbf{A}$ from eqn (7a) or (7b) to eqn (4) and comparing the individual polynomial coefficients q_1, q_2, q_3 the following conditions are obtained for both models:

$$a_{11} + a_{22} = \nu_1 + \nu_2 \ , \qquad (8a)$$

$$a_{11}a_{22} - a_{21}a_{12} = \nu_1\nu_2 \ , \qquad (8b)$$

$$(a_{33})^3 - q_1(a_{33})^2 + q_2(a_{33}) - q_3 = 0 \qquad (8c)$$

Let $\nu_{1,2} = \nu' \pm j\nu''$ denote the complex conjugate eigenvalues and ν_3 the real eigenvalue in outer regions D_{+1}, D_{-1}. It follows directly from eqn (8c) that in both cases $a_{33} = \nu_3$ while conditions (8a,b) can evidently be fulfilled by various ways. The following two types of the second-order submatrix $\mathbf{A}'$ are considered in both models:

$$\mathbf{A}' = \begin{bmatrix} \nu_1 + \nu_2 & -1 \\ \nu_1\nu_2 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{A}' = \begin{bmatrix} \nu' & -\nu'' \\ \nu'' & \nu' \end{bmatrix} \qquad (9a,b)$$

i.e. the elementary canonical and the complex decomposed forms, respectively.

For the comparison the first elementary canonical form [3] is used as the simple reference system where

$$\tilde{\mathbf{A}} = \begin{bmatrix} q_1 & -1 & 0 \\ q_2 & 0 & -1 \\ q_3 & 0 & 0 \end{bmatrix}, \quad \tilde{\mathbf{b}} = \begin{bmatrix} p_1 - q_1 \\ p_2 - q_2 \\ p_3 - q_3 \end{bmatrix} \qquad (10a,b)$$

$$\tilde{\mathbf{w}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \tilde{\mathbf{K}}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ q_1 & -1 & 0 \\ q_2 & -q_1 & 1 \end{bmatrix} \qquad (10c,d)$$

so that the state equations (1) can be rewritten into the complete and modified form as

$$\dot{x}_1 = q_1[x_1 - h(x_1)] - x_2 + p_1 h(x_1) \ , \qquad (12a)$$

$$\dot{x}_2 = q_2[x_1 - h(x_1)] - x_3 + p_2 h(x_1) \ , \qquad (12b)$$

$$\dot{x}_3 = q_3[x_1 - h(x_1)] + p_3 h(x_1) \qquad (12c)$$

The corresponding integrator-based circuit structure is introduced in Fig. 2.

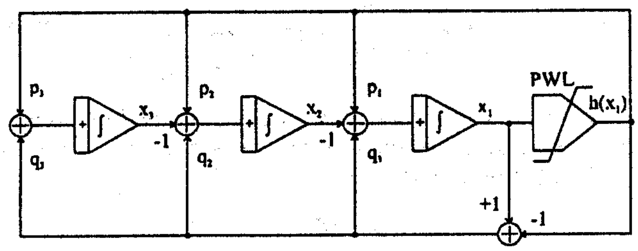


Fig. 2. Integrator-based model of the first elementary canonical form of the third-order dynamical system

Starting from the chosen type of submatrix $\mathbf{A}'$ complete state matrix $\mathbf{A}$ can easily be derived. Choosing vector $\mathbf{w}$ in accordance with [7] in the form $\mathbf{w}^T = [\,1 \ \ 0 \ \ 1\,]$ partial transformation matrix of the new system $\mathbf{K}$ is obtained from eqn (6b) and resultant transformation matrix $\mathbf{T}$ from eqn (5d). Then vector $\mathbf{b}$ is determined using condition (5c) as

$$\mathbf{b} = [b_1 \ \ b_2 \ \ b_3]^T = \mathbf{T}^{-1}\tilde{\mathbf{b}} \qquad (13)$$

where $\tilde{\mathbf{b}}$ is given by (10b). Finally the corresponding integrator-based circuit models can be derived. The main results are summarized in the next parts.

3. Canonical State Models with a Block-Diagonal State Matrix

3.1 Elementary Canonical Submatrix

State matrix and vectors in eqn (1) are given as

$$\mathbf{A} = \left[\begin{array}{cc|c} \nu_1 + \nu_2 & -1 & 0 \\ \nu_1\nu_2 & 0 & 0 \\ \hline 0 & 0 & \nu_3 \end{array}\right], \quad \mathbf{w} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \qquad (14a,b,c)$$

where

$$b_1 = p_1 - q_1 - b_3 \ , \qquad (15a)$$

$$b_2 = p_2 - q_2 - \nu_3 b_1 - (\nu_1 + \nu_2) b_3 \ , \qquad (15b)$$

$$b_3 = (\mu_3 - \nu_3)\frac{(\nu_3 - \mu_1)(\nu_3 - \mu_2)}{(\nu_3 - \nu_1)(\nu_3 - \nu_2)} \qquad (15c)$$

and

$$p_1 = (\mu_1 + \mu_2 + \mu_3) \ , \qquad q_1 = (\nu_1 + \nu_2 + \nu_3) \ ,$$

$$p_2 = \mu_1\mu_2 + \mu_3(\mu_1 + \mu_2) \ , \quad q_2 = \nu_1\nu_2 + \nu_3(\nu_1 + \nu_2)$$

Then the complete state equations can be rewritten as

$$\dot{x}_1 = (\nu_1 + \nu_2)x_1 - x_2 + b_1 h(x_1 + x_3) \ , \qquad (16a)$$

$$\dot{x}_2 = \nu_1\nu_2 x_1 + b_2 h(x_1 + x_3) \ , \qquad (16b)$$

$$\dot{x}_3 = \nu_3 x_3 + b_3 h(x_1 + x_3) \qquad (16c)$$

and the corresponding integrator-based circuit model is introduced in Fig. 3a. The partial and resultant transformation matrices have the form

$$\mathbf{K} = \begin{bmatrix} 1 & 0 & 1 \\ \nu_1 + \nu_2 & -1 & \nu_3 \\ \nu_1^2 + \nu_2^2 + \nu_1\nu_2 & -(\nu_1 + \nu_2) & \nu_3^2 \end{bmatrix}, \quad \mathbf{T} = \begin{bmatrix} 1 & 0 & 1 \\ \nu_3 & 1 & \nu_1 + \nu_2 \\ 0 & \nu_3 & \nu_1\nu_2 \end{bmatrix} \qquad (17a,b)$$

3.2 Complex Decomposed Submatrix

State matrix and vectors in eqn (1):

$$\mathbf{A} = \left[\begin{array}{cc|c} \nu' & -\nu'' & 0 \\ \nu'' & \nu' & 0 \\ \hline 0 & 0 & \nu_3 \end{array}\right], \quad \mathbf{w} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \qquad (18a.b.c)$$

where

$$b_1 = p_1 - (2\nu' + \nu_3) - b_3 \ , \qquad (19a)$$

$$b_2 = \frac{[p_2 - q_2 - (\nu_3 + \nu')b_1 - 2\nu' b_3]}{\nu''} \ , \qquad (19b)$$

$$b_3 = (\mu_3 - \nu_3)\frac{(\nu_3 - \mu_1)(\nu_3 - \mu_2)}{(\nu_3 - \nu')^2 + \nu''^2} \qquad (19c)$$

The complete modified state equations are

$$\dot{x}_1 = v' x_1 - v'' x_2 + b_1 h(x_1 + x_3)\ , \qquad (20a)$$

$$\dot{x}_2 = v'' x_1 + v' x_2 + b_2 h(x_1 + x_3)\ , \qquad (20b)$$

$$\dot{x}_3 = v_3 x_3 + b_3 h(x_1 + x_3) \qquad (20c)$$

and the corresponding integrator-based circuit model is shown in Fig. 3b. The partial and resultant transformation matrices are

$$\mathbf{K} = \begin{bmatrix} 1 & 0 & 1 \\ v' & -v'' & v_3 \\ v'^2 - v''^2 & -2v'v'' & v_3^2 \end{bmatrix}\ , \qquad (21a)$$

and

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 1 \\ v' + v_3 & v'' & 2v' \\ v' v_3 & v'' v_3 & v'^2 + v''^2 \end{bmatrix} \qquad (21b)$$

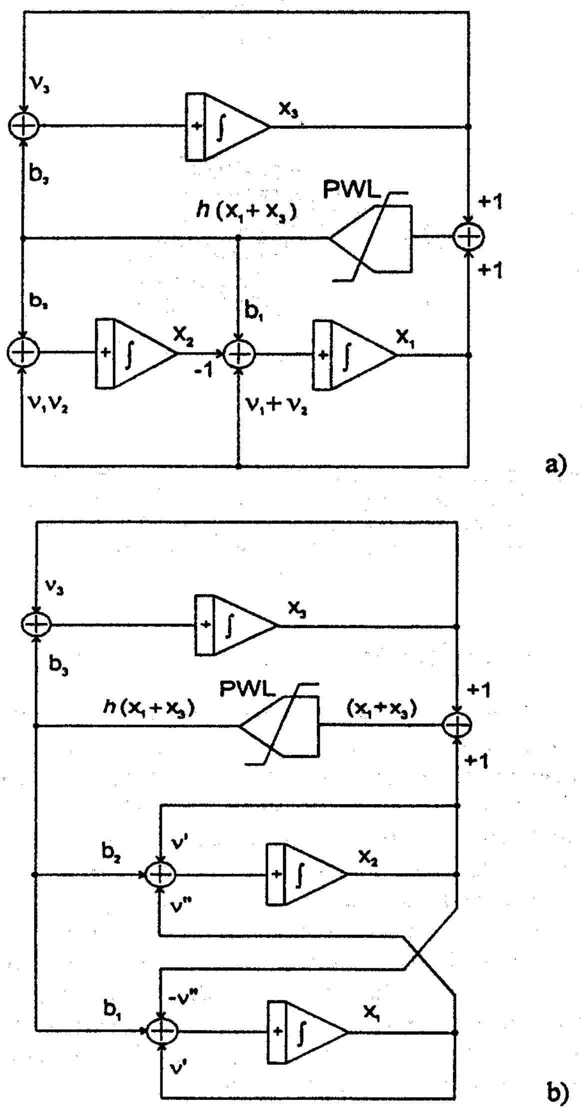


Fig. 3. Integrator-based circuit structures of the canonical state models with block-diagonal state matrices. (a) Elementary canonical submatrix, (b) Complex decomposed submatrix

4. Canonical State Models with a Block-Triangular State Matrix

4.1 Elementary Canonical Submatrix

State matrix and vectors in eqn (1) are given as

$$\mathbf{A} = \left[\begin{array}{cc|c} v_1 + v_2 & -1 & -b_1 \\ v_1 v_2 & 0 & -b_2 \\ \hline 0 & 0 & v_3 \end{array}\right],\quad \mathbf{w} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},\quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \qquad (22a,b,c)$$

where

$$b_1 = (\mu_1 + \mu_2) - (v_1 + v_2)\ , \qquad (23a)$$

$$b_2 = \mu_1 \mu_2 - v_1 v_2\ , \qquad (23b)$$

$$b_3 = (\mu_3 - v_3) \qquad (23c)$$

The complete state equations can be rewritten as

$$\dot{x}_1 = (\mu_1 + \mu_2)[h(x_1 + x_3) - x_3] - x_2 \\ -(v_1 + v_2)[h(x_1 + x_3) - (x_1 + x_3)]\ , \qquad (24a)$$

$$\dot{x}_2 = \mu_1 \mu_2 [h(x_1 + x_3) - x_3)] \\ -v_1 v_2 [h(x_1 + x_3) - (x_1 + x_3)]\ , \qquad (24b)$$

$$\dot{x}_3 = \mu_3 h(x_1 + x_3) - v_3 [h(x_1 + x_3) - x_3] \qquad (24c)$$

and the corresponding integrator-based circuit model is shown in Fig. 4a. The partial and resultant transformation matrices are

$$\mathbf{K} = \begin{bmatrix} 1 & 0 & 1 \\ v_1 + v_2 & -1 & v_3 - b_1 \\ v_1^2 + v_2^2 + v_1 v_2 & -(v_1 + v_2) & v_3^2 + b_2 + q_1 b_1 \end{bmatrix} \qquad (25a)$$

and

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 1 \\ v_3 & 1 & \mu_1 + \mu_2 \\ 0 & v_3 & \mu_1 \mu_2 \end{bmatrix} \qquad (25b)$$

4.2 Complex Decomposed Submatrix

State matrix and vectors in eqn (1) :

$$\mathbf{A} = \left[\begin{array}{cc|c} v' & -v'' & -b_1 \\ v'' & v' & -b_2 \\ \hline 0 & 0 & v_3 \end{array}\right],\quad \mathbf{w} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},\quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \qquad (26a,b,c)$$

where

$$b_1 = (\mu_1 + \mu_2) - 2v'\ , \qquad (27a)$$

$$b_2 = \frac{(v' - \mu_1)(v' - \mu_2)}{v''} - v''\ , \qquad (27b)$$

$$b_3 = \mu_3 - \mu_3 \qquad (27c)$$

The complete modified state equations are

$$\dot{x}_1 = v' x_1 - v'' x_2 + b_1 [h(x_1 + x_3) - x_3]\ , \qquad (28a)$$

$$\dot{x}_2 = v'' x_1 + v' x_2 + b_2 [h(x_1 + x_3) - x_3]\ , \qquad (28b)$$

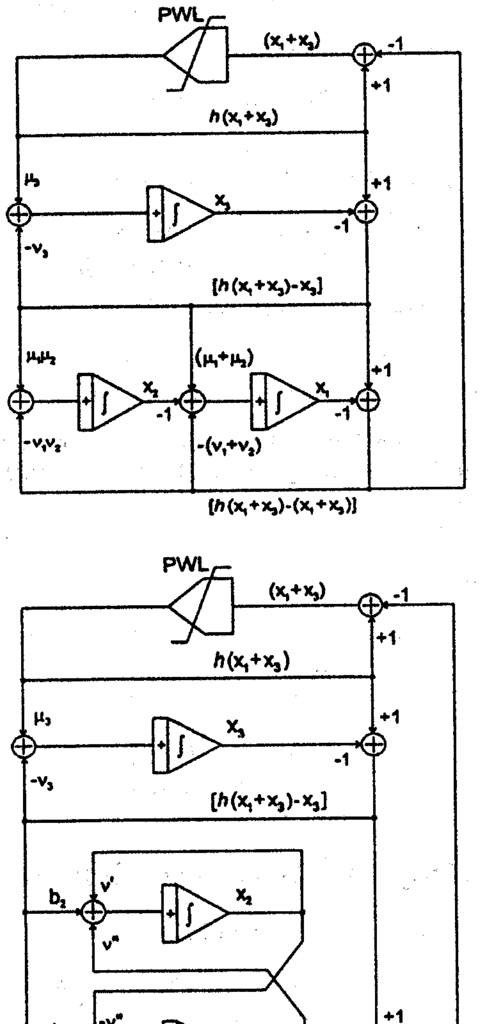
$$\dot{x}_3 = \mu_3 h(x_1 + x_3) - v_3 [h(x_1 + x_3) - x_3] \qquad (28c)$$

and the corresponding integrator-based circuit model is shown in Fig. 4b. The partial and resultant transformation matrices are

$$\mathbf{K}=\begin{bmatrix} 1 & 0 & 1 \\ \nu' & -\nu'' & \nu_3-b_1 \\ \nu'^2-\nu''^2 & -2\nu'\nu'' & \nu_3^2-b_1(\nu'+\nu_3)+b_2\nu'' \end{bmatrix} \qquad (29a)$$

and

$$\mathbf{T}=\begin{bmatrix} 1 & 0 & 1 \\ \nu'+\nu_3 & \nu'' & \mu_1+\mu_2 \\ \nu'\nu_3 & \nu''\nu_3 & \mu_1\mu_2 \end{bmatrix} \qquad (29b)$$



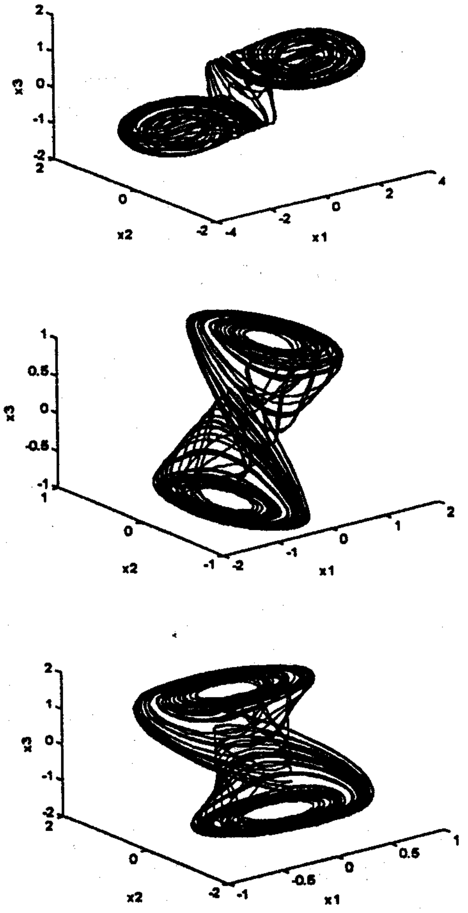
a)

b)

Fig. 4. Integrator-based circuit structures of the canonical state models with block-triangular state matrices. (a) Elementary canonical submatrix, (b) Complex decomposed submatrix

5. Example

As an example the well known double-scroll attractor [1] is chosen (related equivalent eigenvalue parameters: $p_1=0.09$, $p_2=0.43296$, $p_3=0.65332$ and $q_1=-1.168$, $q_2=0.84634, q_3=-1.2948;$ i.e. the eigenvalues: $\mu_{1,2}=-0.319\pm j0.892$, $\mu_3=0.728$ and $\nu_{1,2}=-0.061\pm j1.0$, $\nu_3=0.728$). For a comparison the phase portraits of three various qualitatively equivalent models are shown in Fig. 5.



a)

b)

c)

Fig. 5. Phase portraits of the double-scroll attractor. (a) First canonical state model (Fig.2), (b) Block-diagonal state matrix with complex decomposed submatrix (Fig. 3b), (c) Block-triangular state matrix with elementary canonical submatrix (Fig. 4a)

6. Conclusion

Two new types of the canonical state models topologically conjugate to Chua's circuit family, both based on the decomposed state matrix forms, are proposed. All these systems have been verified by numerical simulations including the applications which exhibit the synchronization phenomena [6]. Some of them have been realized using the integrator-based circuit models and also verified experimentally [5],[8],[9]. The theoretical and experimental results are in a good agreement. In the further research the modified models simulating various dynamical phenomena will be derived in order the system design could be optimized. All the models can evidently be extended also for higher-order systems [10].

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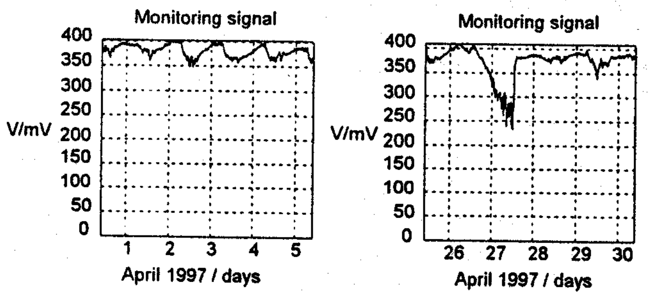
We apologize to the readers of our journal Radioengineering (April 1999, Vol. 8, No. 1) for a mistake in the article „Network Communication by Optical Directional Link" (Otakar WILFERT, Viera BIOLKOVÁ). In Fig.6 „Monitoring signals received over different weather conditions" on page 40 some parts are missing. The correct version of the figure is presented below:



a) b)

We thank you for your understanding.