

A DIELECTRIC-LOADED HORN WITH PERIODIC STRIP STRUCTURE

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Abstract

A dielectric-loaded horn with transverse strips is analysed theoretically. The method is based on a planar periodic strip structure and is used for a circular cylindrical and uniform waveguide model of the feed. The propagation constant and the total field can be determined by approximate solution for the „dominant“ Floquet mode with $n = 0$. Radiation pattern of the feed with small flare angle is computed as a superposition of the field components radiated from center of the aperture and from its region filled with dielectric.

Keywords

hybrid-mode feed, plane-wave model, symmetrical radiation pattern

1. Introduction

The corrugated horns have been superior to other feeds due to their excellent performance in terms of sidelobes and cross polarization. In recent years, some alternative high-performance feeds have been published in [1], [2], where first reference presents a comparison between different horns. Another class of horn antennas, which are used as the primary feeds of the Cassegrain reflector systems is presented in [3], where the "strip-loaded horns" are analysed theoretically. The analysis based on a circular, cylindrical and infinitely long waveguide model with a periodic zero-loss strip structure at the wall. Common to all these is that the interior of the horn is partially loaded with a dielectric material, which makes them potentially simple and inexpensive in production. These feed horns represent an interesting alternative to the corrugated horns in wide-band or dual-band applications in particular for millimeter waves and for lightweight applications onboard satellites. In the paper, a dielectric-strip-loaded horn is analysed as the plane-wave model. The propagation characteristics and the field components are determined by approximate solution for the dominant mode

with $n = 0$, while the higher order Floquet modes in the periodic structure are omitted. These simplifications can be used, because the higher order Floquet modes or space harmonics will only cause a slight modification of the field in vicinity of the strips. In the next part, the radiation patterns of the feed is analysed and numerically computed. A quadratic phase distribution across the aperture of the horn is supposed. The calculated results apply to horn antennas with small flare angles in the K_a - frequency band.

2. Propagation characteristics

We assume a circular, cylindrical and infinitely long waveguide with a periodic strip structure at the wall as shown in Fig.1.

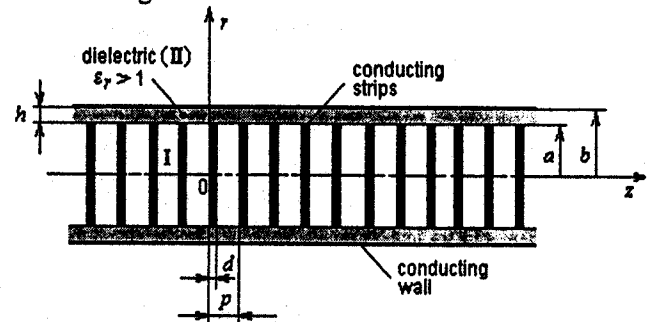


Fig.1 Circular cylindrical model of the strip-loaded feed

This waveguide consists of a central part I and dielectric region II. The field components can be written as an infinite series of space harmonics (Floquet modes) both in regions. For the lowest hybrid HE_{11} - mode one may write the solution of the wave equation in both regions:

$$E_{z1n} = A_n J_1(k_{1n}r) e^{-j\beta n z} e^{j\varphi}$$

$$Z_0 H_{z1n} = -j B_n J_1(k_{1n}r) e^{-j\beta n z} e^{j\varphi}$$

$$E_{\varphi 1n} = \left[A_n \frac{\beta_n r}{(k_{1n}r)^2} J_1(k_{1n}r) + B_n \frac{k}{k_{1n}} J_1'(k_{1n}r) \right] e^{-j\beta n z} e^{j\varphi} \quad (1)$$

$$Z_0 H_{\varphi 1n} = -j \left[A_n \frac{k}{k_{1n}} J_1'(k_{1n}r) + B_n \frac{\beta_n r}{(k_{1n}r)^2} J_1(k_{1n}r) \right] e^{-j\beta n z} e^{j\varphi}$$

$$E_{r1n} = -j \left[A_n \frac{\beta_n}{k_{1n}} J_1'(k_{1n}r) + B_n \frac{kr}{(k_{1n}r)^2} J_1(k_{1n}r) \right] e^{-j\beta_n z} e^{j\varphi} \quad (1)$$

$$Z_0 H_{r1n} = - \left[A_n \frac{kr}{(k_{1n}r)^2} J_1(k_{1n}r) + B_n \frac{\beta_n}{k_{1n}} J_1'(k_{1n}r) \right] e^{-j\beta_n z} e^{j\varphi}$$

in the air-filled region I ($0 < r < a$) and

$$E_{z2m} = C_m R_1(k_{2m}r) e^{-j\beta_m z} e^{j\varphi}$$

$$Z_0 H_{z2m} = -j D_m S_1(k_{2m}r) e^{-j\beta_m z} e^{j\varphi}$$

$$E_{\varphi 2m} = \left[C_m \frac{\beta_m r}{(k_{2m}r)^2} R_1(k_{2m}r) + D_m \frac{k}{k_{2m}} S_1'(k_{2m}r) \right] e^{-j\beta_m z} e^{j\varphi} \quad (2)$$

$$Z_0 H_{\varphi 2m} = \left[C_m \frac{\beta_m r}{(k_{2m}r)^2} R_1(k_{2m}r) + D_m \frac{k}{k_{2m}} S_1'(k_{2m}r) \right] e^{-j\beta_m z} e^{j\varphi}$$

$$E_{r2m} = -j \left[C_m \frac{\beta_m}{k_{2m}} R_1'(k_{2m}r) + D_m \frac{kr}{(k_{2m}r)^2} S_1(k_{2m}r) \right] e^{-j\beta_m z} e^{j\varphi}$$

$$Z_0 H_{r2m} = - \left[C_m e_r \frac{kr}{(k_{2m}r)^2} R_1(k_{2m}r) + D_m \frac{\beta_m}{k_{2m}} S_1'(k_{2m}r) \right] e^{-j\beta_m z} e^{j\varphi}$$

in the dielectric filled region II ($a \leq r \leq b$), where the symbols have the following meanings $\beta_n = \beta + 2\pi n/p$ is propagation constant in the zero-loss waveguide,

$$k_{1n}^2 = k^2 - \beta_n^2 \quad \text{and} \quad k_{2m}^2 = \epsilon_r k^2 - \beta_m^2$$

are transverse wavenumbers in the regions I and II, and

$$R_1(k_{2m}r) = N_1(k_{2m}b) J_1(k_{2m}r) - J_1(k_{2m}b) N_1(k_{2m}r)$$

$$S_1(k_{2m}r) = N_1'(k_{2m}b) J_1(k_{2m}r) - J_1'(k_{2m}b) N_1(k_{2m}r)$$

are cylindrical function which satisfy the boundary conditions at $r = b$ for the *TM* modes, and *TE* modes, respectively. The functions J_1 and N_1 denote Bessel functions of the first and second kind, respectively, and the indices n and m denote the integers from minus to plus infinity. These indices corresponding to the Floquet number. From the balanced hybrid condition

$$Z_\varphi Z_z = -Z_0^2$$

where Z_0 is the free space impedance, it is possible, principally, to design a feed with zero cross-polarization assuming a large aperture. The model gives only a qualitative value of anisotropic wall impedance, which may serve as a first-order estimate. The method of analysis is based on a circular, cylindrical model having a periodic strip structure, where the tangential fields are matched across the boundary $r = a$. The total field components can be found by summarizing each field component over all n and m , e.g.:

$$E_{z1}(r) = \sum_{n=-\infty}^{\infty} E_{z1n}, \quad \text{resp.} \quad E_{z2}(r) = \sum_{m=-\infty}^{\infty} E_{z2m}$$

The boundary conditions at $r = a$ can be expressed by the field components of the total field, given by

$$E_{z1}(a) = E_{z2}(a)$$

$$E_{\varphi 1}(a) = E_{\varphi 2}(a)$$

$$E_{z1}(a) = E_{z2}(a) = 0 \quad \text{for } 0 < z < d$$

$$E_{\varphi 1}(a) = E_{\varphi 2}(a) = 0 \quad 0 < z < d$$

$$H_{z1}(a) - H_{z2}(a) = \begin{cases} K_\varphi & \text{for } 0 < z < d \\ 0 & d \leq z \leq p \end{cases} \quad (3)$$

$$H_{\varphi 1}(a) - H_{\varphi 2}(a) = \begin{cases} K_z & \text{for } 0 < z < d \\ 0 & d \leq z \leq p \end{cases}$$

where K_φ and K_z represent the total currents on the strips. After applying the boundary conditions $E_{\varphi 1} = 0$ and $E_{\varphi 2} = 0$ at $r = a$, the relations between the field coefficients A_0 , B_0 and C_0 , D_0 , respectively are

$$B_0 = -A_0 \frac{\beta}{kx} \frac{J_1(x)}{J_1'(x)} \quad \text{and} \quad D_0 = -C_0 \frac{\beta}{ky} \frac{R_1(y)}{S_1'(y)} \quad (4)$$

where $x = k_1 a$ and $y = k_2 a$.

From the boundary condition $E_{z1}(a) = E_{z2}(a)$, the mutual relations between A_0 and B_0 coefficients follow

$$C_0 = A_0 \frac{J_1(x)}{R_1(y)} \quad (5)$$

After applying the second balanced hybrid condition $Z_z \rightarrow \infty$, then $H_{\phi 1} = 0$ and $H_{\phi 2} = 0$ at $r = a$, we obtain the characteristic equation [5]

$$\frac{J_1'(x)}{x J_1(x)} - \left(\frac{\beta}{kx}\right)^2 \frac{J_1(x)}{x J_1'(x)} = \epsilon_r \frac{R_1'(y)}{y R_1(y)} - \left(\frac{\beta}{ky}\right)^2 \frac{S_1(y)}{y S_1'(y)} \quad (6)$$

The numerical solution of the characteristic equation (6) is plotted in Fig. 2 as a dependence $\beta/k = f(ka)$.

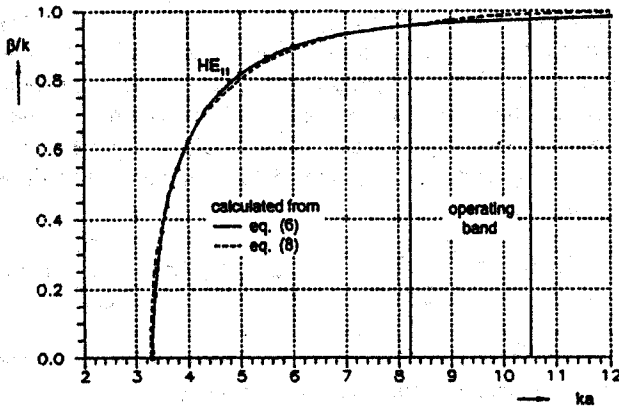


Fig. 2 Dispersion characteristics of the lowest HE_{11} -hybrid mode

When the propagation constant is known, the hybrid factor is given

$$\Lambda = \frac{B_0}{A_0} = -\frac{\beta}{kx} \frac{J_1(x)}{J_1'(x)} \quad (7)$$

The propagation constant can be approximate by calculated using a simple formula [3]

$$\beta \approx k \left\{ \left[1 - \frac{1}{b} \left(1 - \frac{1}{\epsilon_r} \right) \right]^{-1} - \left[\frac{\lambda}{1,64b} \right]^2 \right\}^{\frac{1}{2}} \quad (8)$$

From (8) the cut-off wavelength becomes

$$\lambda_{cut-off} \approx 1,64b \left[1 - \frac{1}{b} \left(1 - \frac{1}{\epsilon_r} \right) \right]^{-\frac{1}{2}} \quad (9)$$

Equation (9) is exact when $\epsilon_r = 1$ or when $h = 0$. This formula is based on the fact that the HE_{11} cutoff exhibits pure TM -properties, corresponding to a TM_{11} mode in the waveguide when the strips are absent. Numerical solution of the formula (8) for HE_{11} -mode in Fig.2 is drawn by the dotted line.

3. Calculation of the strip dimensions

For the design of the width and periodicity of the metal strips a *plane-wave model* [4] can be used, which is shown in Fig. 3a). The incident plane wave may be decomposed into a transverse electric (TE) and transverse magnetic (TM) mode with respect to the normal to the

surface. The transmission line equivalents for these two modes are shown in Fig. 3b).

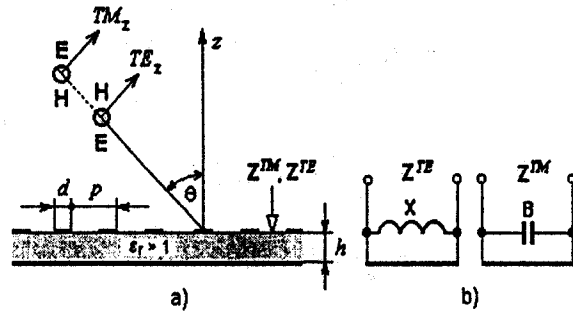


Fig. 3 Plan wave model of the strip-loaded feed
a) Incident plane waves
b) Transmission line equivalents

The strip structure behaves like a reactance X for TE mode, and like a susceptance B for TM mode. Then a input impedances of the equivalent transmission lines in Fig. 3b) become

$$Z^{TE} = \frac{j\bar{X} \frac{Z_0}{\sqrt{\epsilon_r}} \operatorname{tg}(\beta h)}{X + \operatorname{tg}(\beta h)} \quad \text{and} \quad Z^{TM} = \frac{j \frac{Z_0}{\sqrt{\epsilon_r}} \operatorname{tg}(\beta h)}{1 - B \operatorname{tg}(\beta h)} \quad (10)$$

where

$$\bar{X} = \frac{X}{Z_0^{TE}} = \frac{X}{Z_0} \sqrt{\epsilon_r - \sin^2 \theta}$$

$$\bar{B} = B Z_0^{TM} = \frac{B Z_0}{\sqrt{\epsilon_r - \sin^2 \theta} / \epsilon_r}$$

If assumed that the dielectric loaded horn is lossless then the propagation constant is defined as follows

$$\beta = \frac{2\pi}{\lambda} \sqrt{\epsilon_r - \sin^2 \theta}$$

After applying the balanced hybrid condition $Z^{TE} Z^{TM} = Z_0^2$ we obtain the characteristic equation

$$\operatorname{cotg}(\beta h) = -\frac{1}{2} \left[\frac{1}{\bar{X}} - \bar{B} \right] \left\{ 1 \pm \sqrt{1 + 4 \left[\frac{\bar{B}}{\bar{X}} - \frac{1}{\epsilon_r} \right] \left[\frac{1}{\bar{X}} - \bar{B} \right]} \right\} \quad (11)$$

Normally $\bar{B} \ll 1/\bar{X}$, so that (11) can be simplified. Classical formulas [6] are available for X and B

$$\frac{X}{Z_0} = \frac{p \cos \theta}{\lambda} F(u_1)$$

$$Z_0 B = \frac{4 p \cos \theta (\epsilon_r + 1)}{\lambda} \frac{F(u_2)}{2} \quad (12)$$

where

$$F(u) = \ln \frac{1}{u} + \frac{1}{2} \frac{(1-u^2) \left[(1-u^2/4)(A_+ + A_-) + 4u^2 A_+ A_- \right]}{(1-u^2/4) + u^2(1+u^2/2-u^4/8)(A_+ + A_-) + 2u^6 A_+ A_-}$$

$$A_{\pm} = \left[1 \pm 2 \frac{p}{\lambda} \sin \theta - \left(\frac{p}{\lambda} \cos \theta \right)^2 \right]^{1/2} - 1$$

$$u_1 = \sin \left(\frac{\pi d}{2p} \right) \quad \text{and} \quad u_2 = \sin \left(\frac{\pi p - d}{2p} \right)$$

These equations are valid provided $\frac{p}{\lambda} (\sqrt{\epsilon_r} + \sin \theta) < 1$.

When the frequency is far beyond cut-off, which is equivalent to $\theta = 90^\circ$, balanced hybrid condition is met when the electrical thickness of the wall is approximately 90° or

$$\sqrt{\epsilon_r} - 1 \frac{h}{\lambda} \approx 0,25$$

which is valid when the aperture diameter is large relative to wavelength.

Numerical solution of the characteristic equation (11), using an iterative method the dimensions of the strips can be calculated. The results for parameters $\epsilon_r = 2,5$ and $\theta = 10^\circ$ is plotted in Fig. 4.

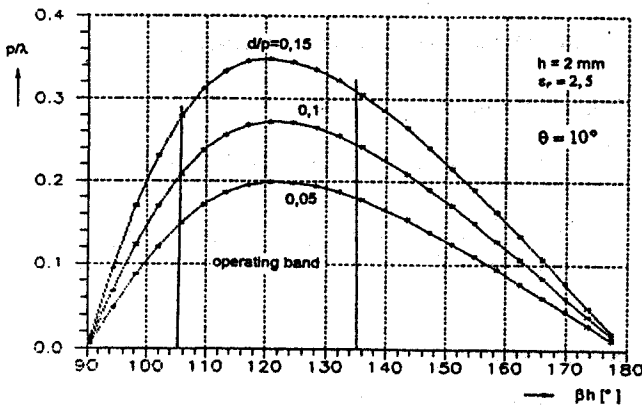


Fig.4 Design curves for the strip-loaded horn

At the center frequency 29,7GHz, parameter $p/\lambda = 0,2516$ width of the strips $d = 0,254$ mm and the period of the metal strip structure then $p \approx 2,54$ mm.

4. Radiation from a conical horn

The problem of radiation is solved using the aperture field method. The radiation patterns of conical horn with small flare angle ($\alpha = 10^\circ$) are numerically computed as a superposition of field components radiated from center of the aperture (I) and from its region field with dielectric (II). A quadratic phase distribution is supposed across the aperture of feed. Let us consider a conical horn antenna in the coordinates system in Fig. 5.

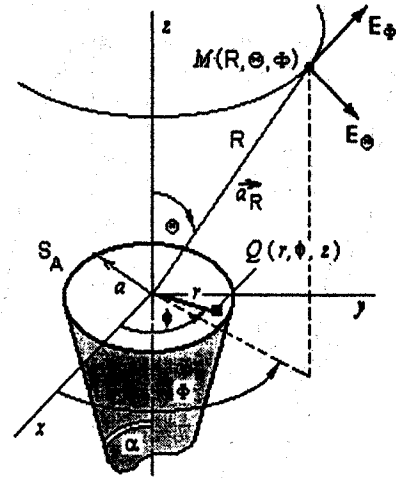


Fig. 5 Aperture in coordinate system

The application of the Kirchoff-Huygens integration over the aperture gives the following expressions for the electric field in the Fraunhofer zone [5]

$$E_{\theta} = \frac{jk}{4\pi R} e^{-jkR} \int_0^{r_2} \int_0^{2\pi} \{ [E_r + Z_0 H_{\phi} \cos \theta] \times \cos(\phi - \varphi) + [E_{\phi} - Z_0 H_r \cos \theta] \times \sin(\phi - \varphi) \} e^{jkr \sin \theta \cos(\phi - \varphi)} e^{-j\mathcal{N} \left(\frac{r}{r_1} \right)^2} r dr d\varphi \quad (14)$$

$$E_{\phi} = \frac{jk}{4\pi R} e^{-jkR} \int_0^{r_2} \int_0^{2\pi} \{ [E_{\phi} \cos \theta - Z_0 H_r] \times \cos(\phi - \varphi) - [E_r \cos \theta + Z_0 H_{\phi}] \times \sin(\phi - \varphi) \} e^{jkr \sin \theta \cos(\phi - \varphi)} e^{-j\mathcal{N} \left(\frac{r}{r_1} \right)^2} r dr d\varphi \quad (15)$$

where

$E_r, E_{\phi}, H_r, H_{\phi}$ are cross field components at the aperture in region I and II,
 r_1, r_2 - intergration boundaries
 for region I: $r_1 = 0, r_2 = a$,
 for region II: $r_1 = a, r_2 = b$,
 $v = kd$ - phase difference between the rim and the centre of aperture.

Total field in the Fraunhofer zone is defined by a superposition of the field components

$$E_{\theta} = E_{\theta I} + E_{\theta II} \quad E_{\phi} = E_{\phi I} + E_{\phi II}$$

In appendix the formulas for calculation of the radiation characteristics are derived. Results, the radiation patterns at the center frequency in the E - plane and H - plane of the conical unloaded and dielectric-strip loaded horn are shown Fig. 6.

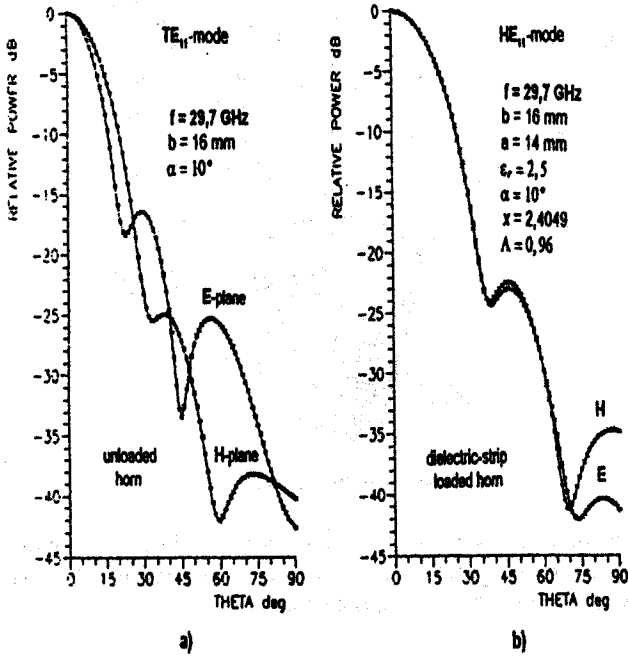


Fig.6 Calculated radiation patterns
a) unloaded horn excited by TE_{11} -mode,
b) dielectric-strip-loaded horn (HE_{11} -mode)

5. Conclusions

The dielectric-strip-loaded feed horn presented in this paper has the potential of very low weight, low price, and large bandwidth. For small aperture diameters it has a principally similar electrical performance to the dual depth corrugated horn with the same degree of freedom. It may also be cheaper to manufacture than the corrugated horn - in particular for millimeter wave application - if a combination of photolithography and etching is used. This feed horn has symmetrical radiation pattern, low sidelobes and low level of cross-polarization in the wide frequency band.

6. Appendix

After substitution of the field components at the aperture and by using the expressions

$$\int_0^{2\pi} \cos\varphi \frac{\cos(\phi-\varphi)}{\sin(\phi-\varphi)} e^{ju \cos(\phi-\varphi)} d\varphi = \pi \frac{\cos\phi}{\sin\phi} [J_0(u) \mp J_2(u)]$$

$$\int_0^{2\pi} \sin\varphi \frac{\sin(\phi-\varphi)}{\cos(\phi-\varphi)} e^{ju \cos(\phi-\varphi)} d\varphi = \mp \pi \frac{\cos\phi}{\sin\phi} [J_0(u) \pm J_2(u)]$$

where $u = kr \sin\theta$, we obtain simple integrals, and the radiation patterns from center region (I) of the horn then become

E-plane ($\Phi = 0$)

$$F_{\Theta I} = -\frac{k}{k_1} \left[\left(\frac{\beta}{k} + \cos\theta \right) (S1 + S2) + \Lambda \left(1 + \frac{\beta}{k} \cos\theta \right) (S1 - S2) \right] \quad (16)$$

H-plane ($\Phi = \pi/2$)

$$F_{\Theta I} = +\frac{k}{k_1} \left[\left(1 + \frac{\beta}{k} \cos\theta \right) (S1 - S2) + \Lambda \left(\frac{\beta}{k} + \cos\theta \right) (S1 + S2) \right] \quad (17)$$

If $\beta > k_1$, then k_1 is imaginary, and the radiation patterns from region (I) become

E-plane ($\Phi = 0$)

$$F_{\Theta I} = -\frac{k}{k_1} \left[\left(\frac{\beta}{k} + \cos\theta \right) (\overline{S1} - \overline{S2}) + \Lambda \left(1 + \frac{\beta}{k} \cos\theta \right) (\overline{S1} + \overline{S2}) \right] \quad (18)$$

H-plane ($\Phi = \pi/2$)

$$F_{\Theta I} = +\frac{k}{k_1} \left[\left(1 + \frac{\beta}{k} \cos\theta \right) (\overline{S1} + \overline{S2}) + \Lambda \left(\frac{\beta}{k} + \cos\theta \right) (\overline{S1} - \overline{S2}) \right] \quad (19)$$

Radiation patterns from the dielectric region (II)

E-plane ($\Phi = 0$)

$$F_{\Theta II} = -\frac{k}{k_2} \left\{ \begin{aligned} & C \left(\frac{\beta}{k} + \epsilon_r \cos\theta \right) [N_1(z)(V1 + V4) - J_1(z)(V2 + V3)] \\ & + D \left(1 + \frac{\beta}{k} \cos\theta \right) [N_1'(z)(V1 - V4) - J_1'(z)(V2 - V3)] \end{aligned} \right\} \quad (20)$$

H-plane ($\Phi = \pi/2$)

$$F_{\Theta II} = +\frac{k}{k_2} \left\{ \begin{aligned} & C \left(\epsilon_r + \frac{\beta}{k} \cos\theta \right) [N_1(z)(V1 + V4) - J_1(z)(V2 + V3)] \\ & + D \left(\frac{\beta}{k} + \cos\theta \right) [N_1'(z)(V1 - V4) - J_1'(z)(V2 - V3)] \end{aligned} \right\} \quad (21)$$

where $S1, S2, \overline{S1}, \overline{S2}, V1, V2, V3$ and $V4$ are the following integrals

$$S1 = \int_0^a J_0(k_1 r) J_0(kr \sin\theta) e^{-j\sqrt{\epsilon_r} (r/a)^2} r dr$$

$$S2 = \int_0^a J_2(k_1 r) J_2(kr \sin\theta) e^{-j\sqrt{\epsilon_r} (r/a)^2} r dr$$

$$\overline{S1} = \int_0^a I_0(|k_1| r) J_0(kr \sin\theta) e^{-j\sqrt{\epsilon_r} (r/a)^2} r dr$$

$$\overline{S2} = \int_0^a I_2(|k_1| r) J_2(kr \sin\theta) e^{-j\sqrt{\epsilon_r} (r/a)^2} r dr$$

$$V1 = \int_a^b J_0(k_2 r) J_0(kr \sin \theta) e^{-\nu(r/b)^2} r dr$$

$$V2 = \int_a^b N_0(k_2 r) J_0(kr \sin \theta) e^{-\nu(r/b)^2} r dr$$

$$V3 = \int_a^b N_2(k_2 r) J_2(kr \sin \theta) e^{-\nu(r/b)^2} r dr$$

$$V4 = \int_a^b J_2(k_2 r) J_2(kr \sin \theta) e^{-\nu(r/b)^2} r dr$$

with constants

$$\Lambda = -\frac{\beta J_1(x)}{kx J_1(x)} \quad C = \frac{J_1(x)}{R_1(y)} \quad D = -\frac{\beta J_1(x)}{ky S_1(y)}$$

$$x = k_1 a \quad y = k_2 a \quad z = k_2 b$$

The J_n , N_n represent Bessel functions of 1-st and 2-nd kind of the n -th order, and I_n is the modified Bessel function of 1-st kind of the n -th order. These integrals can be numerically calculated by using the 20 -point Gaussian quadrature approximation.

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