GSM CHANNEL EQUALIZATION ALGORITHM – MODERN DSP COPROCESSOR APPROACH

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Abstract

The paper presents basic equations of efficient GSM Viterbi equalizer algorithm based on approximation of GMSK modulation by linear superposition of amplitude modulated pulses. This approximation allows to use Ungerboeck form of channel equalizer with significantly reduced arithmetic complexity. Proposed algorithm can be effectively implemented on the Viterbi and Filter coprocessors of new Motorola DSP56305 digital signal processors. Short overview of coprocessor features related to the proposed algorithm is included.

Keywords
GSM channel equalization, GMSK modulation, derotation technique, DSP coprocessors, Viterbi algorithm

1. Introduction

The digital land mobile radio system as specified by the European Telecommunication Standards Institute/Group Spéciales Mobiles (ETSI/GSM) has been put into service in several European countries. It uses narrow band time-division multiple-access (TDMA) digital cellular system with compact spectrum constant envelope modulation – Gaussian minimum shift keying (GMSK). GMSK as partial response system and multipath nature of the mobile communication channel cause intersymbol interference (ISI) between adjacent digital symbols. The best theoretical performance for demodulating operations over channels with ISI and additive white noise is the maximum likelihood sequence estimation (MLSE) technique, which is typically implemented by means of the Viterbi algorithm (VA). The GSM communication protocol establishes reference (training) data sequences in the data packets to determine the channel characteristic. The paper presents an algorithm, which allows to use optimized VA, originally developed for linear carrier modulation techniques, also for nonlinear GMSK modulation. The paper is organized as follows. Section 2 describes GMSK modulation and its linear model. In Section 3 the structure of proposed equalization algorithm is described. ISI equalization in the GSM environment is analyzed in Section 4. Short overview of Motorola DSP56305 coprocessors features related to the proposed algorithm is included in Section 5. In Section 6 the performance of modern digital signal processor (DSP) implementation is demonstrated. Finally, concluding remarks can be found in Section 7.

2. GMSK Modulation Used in GSM

The GSM system [1] uses constant envelope partial response GMSK modulation specified in GSM recommendation R.05.04 [2]. GMSK as continuous phase modulated (CPM) signal with modulation index \( h = 1/2 \) can be defined by the continuous phase shift function \( \phi(t) \) and has the following complex baseband representation

\[
r_{\text{transmit}}(t) = A \exp \left( j \pi h \sum_{n} x_n \phi(t - nT) + \varphi_0 \right)
\]

where \( T \) is bit period, \( A \) is the amplitude, \( x_n = \pm 1 \) is the sequence of binary alphabet symbols, \( \varphi_0 \) is random initial phase and \( \phi(t) \) is so-called phase shift function. Constant envelope, continuous-phase modulation schemes are robust against signal fading as well as interference and have good spectral efficiency. The slower and smoother are the phase changes, the better is spectral efficiency, since the signal is allowed to change less abruptly, requiring lower frequency components. However, the effect of an input bit is spread over several bit periods, leading to so-called partial response system, which requires a channel equalizer in order to remove this controlled ISI, even in the absence of uncontrolled channel dispersion. Schematic diagram of GMSK modulator is depicted in Fig. 1, where GMSK signal is generated by modulating and adding two quadrature carriers with the frequency \( f_c \).

Phase changes are smoothed by a filter having a Gaussian impulse response [1]
\[
g(t) = \frac{1}{2T} \left[ Q\left(2\pi B \frac{t - T}{\sqrt{\ln 2}}\right) - Q\left(2\pi B \frac{t + T}{\sqrt{\ln 2}}\right) \right]
\]

where \( Q(t) \) is the \( Q \)-function

\[
Q(t) = \int_{t}^{\infty} \frac{1}{2\pi} \exp(-r^2/2)dr
\]

and phase shift function \( \varphi(t) \) in (1) is given by

\[
\varphi(t) = \int_{-\infty}^{t} g(t')dt'.
\]

![GMSK modulator diagram](image)

Fig. 1 GMSK modulator

The key parameter of GMSK in controlling both bandwidth and interference resistance is the 3-dB down filter bandwidth \( BT \) referred to as normalized bandwidth. GSM uses \( BT=0.3 \), which corresponds to spreading the effect of 1 bit over approximately 3 bit intervals. The channel separation (or bandwidth) at the TDMA burst rate of 271 kbit/s (\( T = 3.69 \mu s \)) is \( BW = 200 \text{ kHz} \), and the modulated spectrum must be 40 dB down at both adjacent carrier frequencies.

### 2.1 Linear Model of GMSK Signal

The phase shift function for GMSK signal is assumed to be zero for negative values of time \( t \) and to have constant value for \( t \) greater than \( (L-1)T \), \( L \) being a positive integer. As shown in [3], the \( h=1/2 \) CPM (including GMSK) signal can be very closely approximated by a sum of time and phase shifted pulses as

\[
r_{\text{transmit}}(t) \approx \sum_{n} j^{n}a_{n}g(t-nT) = \sum_{n} A_{n}q(t-nT)
\]

where \( A_{n} = j^{n}a_{n} \) are rotated binary symbols, symbols \( a_{n} \) are determined by the recursion

\[
a_{n} = x_{n}a_{n-1}
\]

and the pulse \( q(t) \) [3], which approximates the form of modulation pulse shape and has duration of less than or equal \( LT \), is defined by

\[
q(t) = \sum_{l=0}^{L-2} \sin \left[ \frac{\pi}{2} \psi(t+lT) \right]
\]

where

\[
\psi(t) = \begin{cases} \varphi(t) & t \leq (L-1)T \\ 1 - \varphi(t-(L-1)T) & t > (L-1)T \end{cases}
\]

The factor \( j^{n} \) in (5), which causes \( \pi/2 \) phase rotation on complex plane from symbol to symbol, can be avoided by means of derotation technique, e.g. by multiplying at the receiver by the complex sequence \( (j^{n}) \). If \( a_{n} \) is binary sequence to be modulated, the transmitter should build the phase in the exponent of (1) after a differential encoding of sequence \( a_{n} \) [3] that is

\[
x_{n} = a_{n}a_{n-1}.
\]

In this way, GMSK modulated signal can be depicted as a simple linear binary pulse amplitude modulation (PAM) signal and optimized receiver structure originally developed for linear carrier modulation techniques [4] can be used also for GMSK signals.

### 3. Modified MLSE Receiver Structure

The received signal \( r(t) \), after propagating through a dispersive medium with complex impulse response \( c(t) \), is given by

\[
r(t) = \sum_{n} A_{n}h(t-nT) + N(t)
\]

where

\[
h(t) = q(t) * c(t)
\]

is the overall complex channel impulse response (CIR), \( N(t) \) is the additive Gaussian noise (AWGN) and \( * \) means convolution. After A/D conversion and matched filtering, the discrete signal sample \( x_{n} \) at the output of matched filter (MF) can be expressed as the output of an equivalent transversal filter

\[
x_{n} = \sum_{k} A_{n-k}s_{k} + N_{n}
\]

where \( s_{k} = s(kT) \) and

\[
s(t) = h(t) \ast h^{*}(-t) = h(t) \ast h_{MF}(t)
\]
is the overall transmission channel plus MF impulse response. Theoretically, the use of MF makes the receiver insensitive to the carrier and clock phases (usually modeled as constant phase $\varphi_0$, in (1)) used to demodulate and to sample the received signal, provided that the MF coefficients are correctly adjusted and time span of the MF is long enough to include complete channel impulse response. We now assume that at the MF output ISI from a particular signal element is limited to $K$ preceding and $K$ following sampling instants, so

$$s_k = 0 \text{ for } |k| > K.$$  \hspace{1cm} (14)

Once the receiver is modeled according to (12), it is shown in [4] that survivor maximum likelihood (ML) metric $J_n(\sigma_n)$ used by VA can be computed as:

$$J_n(\sigma_n) = 2 \text{Re}(A_n^* z_n) +$$
$$+ \max_{\{\sigma_{n-1} \to \sigma_n\}} \left\{ J_n(\sigma_{n-1}) - F(\sigma_{n-1}, \sigma_n) \right\} \hspace{1cm} (15)$$

where the maximum is taken over all states $\{\sigma_{n-1}\}$ that have $\sigma_n$ as a possible successor state, and, so-called $F$ table is

$$F(\sigma_{n-1}, \sigma_n) = A_n^* s_0 A_n + 2 \text{Re} \left( A_n^* \sum_{k=1}^{K} s_k A_{n-k} \right).$$  \hspace{1cm} (16)

Equation (15) enables to compute survivor metrics and path histories used by VA in a recursive fashion. Because GMSK is binary modulation scheme, equations (15) and (16) can be simplified into the form (by removing constant term including $s_0$ and omitting factor 2)

$$J_n^\text{GMSK}(\sigma_n) = a_n \text{Re}\left[(-j)^n s_n\right] +$$
$$+ \max_{\{\sigma_{n-1} \to \sigma_n\}} \left\{ J_n^\text{GMSK}(\sigma_{n-1}) - F^\text{GMSK}(\sigma_{n-1}, \sigma_n) \right\} \hspace{1cm} (17)$$

$$F^\text{GMSK}(\sigma_{n-1}, \sigma_n) = a_n \sum_{k=1}^{K} \text{Re}[j^{-k} s_k] a_{n-k} =$$

$$= a_n \sum_{k=1}^{K} S_k a_{n-k} \hspace{1cm} (18)$$

where $\text{Re}()$ is the real part of () and the only complex operation is derivation of complex output of matched filter $z_n$ in (17). All other values (including so called $S$-table: $S_k, \ k = 1, \ldots K$ in (18)) are pure real values and can be pre-computed.

4. ISI Equalization in the GSM Environment

The GSM receiver has to operate adaptively, in order to cope with the time-varying nature of the mobile radio channel. GSM burst structures directly support block adaptivity by introducing training sequence into the most frequently used normal burst (NB) depicted in Fig. 2.

![Fig. 2 Structure of normal burst used in GSM system](image_url)

A normal transmission burst, occupying one slot of time, contains 58 message bits on each side of 26 bit midamble called a training or sounding sequence. The slot time duration is 0.577 ms. The purpose of midamble is to assist the receiver in estimating the impulse response of the channel in an adaptive way (during the time duration of each 0.577 ms slot). In order for the technique to be effective, the fading behavior of the channel should not change appreciably during the time interval of one slot. In other words, there should not be any fast fading degradation during a slot time when the receiver is using knowledge from the midamble to compensate for the channel's fading behavior. Consider the example of a GSM receiver used aboard a high-speed train [5], traveling with the constant velocity of $v = 200$ km/h. Assume the carrier frequency to be 900 MHz (the wavelength $\lambda = 0.33$ m). The half-wave length is traversed in approximately the same (so-called coherence time)

$$T_c = \frac{\lambda/2}{v} \approx 3 \text{ ms}. \hspace{1cm} (19)$$

Therefore, the channel coherence time is over five times greater than the slot time 0.577 ms, so the time needed for a significant change in fading behavior is relatively long compared to the time duration of one slot.

For ISI cancellation the most important parts of normal burst are training sequence and tail bits. Once a communication is set up, it is maintained using NB incorporating the 26-bit midamble in the center of the burst, of which 16 bits constitute the frame synchronization word $\text{sync}_k$ and 5 bits are quasi-periodically repeated at both ends to keep auto-correlation and frequency oscillations low. There are 8 training binary (regarded as ±1) sequences defined in the GSM specification [6], which have auto-correlation function

$$R_n = \sum_{k=1}^{16} \text{sync}_k \text{sync}_{k+n} \begin{cases} 16 & n = 0 \\ 0 & n \neq 0 \end{cases} \hspace{1cm} (20)$$
at least for $\tau = nT$ in the range of expected maximum echo delays ($|p| \leq 5$ and $T = 3.69$ $\mu$s). These special synchronization sequences have been found by computer search, evaluating the auto-correlation functions of all possible $2^{16}$ sequences. Based on linear model of GMSK signals, the training sequence is regarded as alternate complex sequence (odd symbols $\pm 1$, even symbols $\pm j$).

Training sequence enables proper estimation of CIR for each NB. The CIR estimation can be performed by correlating the received training sequence with the replica of the training sequence, which is known at the receiver according to the equation

$$\tilde{h}_k = r_k * (j^{-k} \text{sym}_k)$$  \hspace{1cm} (21)

where $r_k = r(kT)$ and the second term is matched complex digital FIR filter, which includes deotation compensation. In actual digital implementation the channel estimation is limited to $K$ terms. A reasonable basis on which to select the appropriate segment of the estimated channel impulse response $h_{\text{est}}$ is to slide the rectangular window of the length $K$ over the whole estimated response, calculating the energy contained within the window at each point and then to identify the window position where the energy is maximum [1]

$$h_{\text{est}} = h_{i+n}, \left( E_i = \sum_{k=0}^{K-1} |h_{i+k}|^2 = \max \right) \Rightarrow i$$  \hspace{1cm} (22)

$n = 0,1, \cdots K-1$, and $E_i$ is computed over all available values. The block diagram of complete MLSE Ungerboeck equalizer of GSM channel is depicted in Fig. 3.

5. Implementation on Optimised DSP Coprocessors

Traditionally the general purpose DSPs have performed all mathematical operations inside the arithmetic logic unit (ALU). High performance of specialized telecommunication algorithms (e.g., VA, cyclic redundancy computation, and error-correction) was later improved by introducing special optimised instructions and/or improving ALU architecture (e.g., adding barrel shifter). Relatively new approach how to improve performance of new 'general purpose' DSPs is to include new parallel module(s) - DSP coprocessors at the same chip as DSP core. From this point of view the MOTOROLA DSP56305 [7] is the most advanced DSP currently available.

The DSP56305 is a member of DSP56300 family of programmable CMOS DSPs and is optimised for GSM base station. It uses the DSP56300 core, a high performance, single clock cycle per instruction engine and set of three DSP coprocessors: Filter Co-Processor (FCOP), Cyclic Code Co-Processor (CCOP) and Viterbi Co-Processor (VCOP). All of these modules are highly optimised for GSM but in principle they are general-purpose reprogrammable DSP coprocessors and are designed to operate independently of the DSP56300 core.

The CCOP executes cyclic code calculations for data ciphering, deciphering, parity coding generation and checking and can be used in higher layers of GSM signals processing.

The FCOP is a peripheral module designed as a general purpose fully programmable 16 bit complex FIR filter, with up to 21 complex taps. It also has dedicated modes of operation optimised for GSM applications, taking advantages of special GMSK modulation scheme and GSM burst format:

- Special optimised mode for performing cross correlation between the received training sequence and a pre-defined midamble according to (21)
- Special optimised mode for performing matched filtering required for channel equalization using the Ungerboeck scheme according to Fig. 3
• Internal data and coefficient memory banks support
  GSM normal burst as well as GSM access burst

The VCOP is designed for a wide range of standard
applications requiring the Viterbi algorithm for
convolution encoding and decoding (used in higher layers
of GSM signals processing) and channel equalization. For
GSM channel equalization algorithm the most important
features of VCOP are:

• Direct computation of optimised Ungerboeck metric
  according to (17) and (18)
• Ability to specify initialization and end states of trellis
  diagram used by VA
• Access to intermediate path metrics for
  implementation of channels parameters adaptation
  models

6. Simulation Results

The performance of proposed DSP coprocessor based
implementation of Viterbi equalizer was tested on the
artificial 6-tap GSM equalizer test impulse response (EQx)
specified in recommendation R.05.05 [8] and depicted in
Fig. 5. In the name EQx, the x specifies the vehicle speed
in km/h.

![GSM EQx channel profile](image)

Fig. 5 GSM equalizer test echo profile

Although the type of the equalizer is not standardized, this
artificially contrived GSM channel impulse response EQx
enables to test equalizer’s performance and is constituted
by six equidistant unit-amplitude impulses representing
six equal-powered independent Rayleigh-fading paths with
a delay spread over 16 μs. Moreover in a typical multipath
mobile environment, the received signal arrives from
several reflected paths with different path distances and
different angles of arrival, and the Doopler frequency shift
of each arriving path is generally different from that of
another path. The effect on the received signal is seen as a
Doopler spreading or spectral broadening of the
transmitted signal frequency, rather than a shift. Classical
Doopler power spectral density (CLASS in [8])

\[
S(f) = \frac{1}{\pi f_d \sqrt{1 - \left(\frac{f - f_c}{f_d}\right)^2}}
\]

holds for frequency shifts \( f \) that are in the range ±\( f_d \) about
carrier frequency \( f_c \), and would be zero outside that
range. Frequency \( f_d \) is given by

\[
f_d = \frac{v}{\lambda}
\]

where \( v \) is relative velocity of moving object and \( \lambda \) is the
signal wavelength. The CLASS model is part of EQx
specification.

The performance of DSP coprocessor was tested with
MOTOROLA DSP6305 simulator. Actual DSP
implementation can fully use parallelism of DSP
coprocessors, independence of DSP core and availability of
six high performance DMA channels which are included
inside of DSP chip. Equalization of complete NB
(0.577 ms, but one communication channel uses only
every eighth NB) generates 2x58 data bits and takes less
than 7000 DSP cycles (87 μs for 80 MHz version of
DSP6305) which demonstrates real-time performance of
DSP coprocessor implementation.

The detection error histogram of 250 equalized
random normal bursts is depicted in Fig. 6 for EQ200 test
equalizer channel and AWGN noise with \( E_b / N_0 = 15 \) dB
(computed according to [9] for \( BW = 200 \) kHz,
\( T = 3.69 \) μs, 116 information bits/total of 156 bits of NB
and AWGN noise approximated by MATLAB function).

![Histogram of EQ200 channel equalization error](image)

Fig. 6 Histogram of equalization error of 116 data bits for 250 normal
bursts and \( E_b / N_0 = 15 \) dB

These errors are comparable with high precision
MATLAB simulation of proposed algorithm and confirm
that fixed point DSP implementation do not decrease
algorithm performance. GSM EQ200 bit error rate (BER)
performance of proposed algorithm versus available
\( E_b / N_0 \) is depicted in Fig. 7.
Fig. 7 GSM EQ200 channel – MLSE BER performance

7. Conclusion

A modern DSP coprocessor based implementation of GSM equalizer based on recent progress in VLSI DSP circuit technology has been presented. High throughput of such implementation can be used in GSM base station for concurrent real-time processing of several GSM channels and seems to be very promising for telecommunication applications. It is expected that DSP coprocessors will be more widely included in new generations of digital signal processors optimized for telecommunication applications.

Described equalizer algorithm uses fixed CIR estimation during complete burst. Current state of the art GSM equalization algorithms use decision directed mode. Once the CIR is estimated from training sequence, decision-directed adaptation is possible. In this mode, symbols estimated at the receiver can be used as the reference from which to measure the deviation error and subsequently adjust the equalizer parameters. With the equalizer trained, low decision-error rates make it possible to continue to adapt to small changes in channel conditions (especially progressive rotation of received pulse due to the presence of Doppler effects). Future work will try to analyze efficiency of DSP coprocessor implementation of decision directed GSM equalizer algorithms.

References


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Miloš DRUTAROVSKÝ was born in 1965 in Prešov, Slovak Republic. He received the Ing. (M.Sc.) degree and CSc. (Ph.D) degree in radioelectronics from Faculty of Electrical Engineering, Technical University of Košice, in 1988 and 1995, respectively. He is currently working as assistant professor with the Department of Electronics and Multimedial Communications of the Faculty of Electrical Engineering and Informatics, Technical University of Košice. His research interest is digital signal processing, especially in spectral analysis, digital signal processors and adaptive algorithms.