INVESTIGATION OF PREDICTABILITY OF CHAOTIC SIGNALS

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Abstract

Variables and functions for expressing quality of the prediction of the chaotic signal can serve as tools for simplified but useful description of the chaotic signal. In this paper the known generation mechanism of chaos is used as a predictor but using the perturbed signal as an input. Prediction performance of the predictor is compared with the estimate of the predictability based on a mutual information between known quantities and a predicted value. Mentioned two methods of investigation have different requirements for a priory knowledge of chaos. Certain constraints are given in consequence of an imperfection of numerical calculations and computer simulations.

Keywords

chaos, mutual information, prediction

1. Introduction

Chaotic motions lie between regular periodic time dependence and time courses considered as noise. One of the main properties of the chaotic signals is their limited predictability [4]. It can be expected that quantities or functions able to express or estimate predictability could describe the chaotic signal simply and effectively.

In the section 2 a chaotic system is used as the predictor. Prediction of the chaotic signal and prediction of the noisy chaotic signal are compared.

Using mutual information for the estimate of the predictor gain upper bound is proposed in [2]. Calculating the mutual information is connected with statistical and numerical difficulties. Certain improvement lies in the substitution of a regularisation independent additive noise by a quantization noise. Admissibility of such step is one of the problems discussed in the section 3.

2. The chaotic system as the predictor

The first elementary canonical model [5] of Chua's circuit family [1] has been chosen as an example of the chaotic system. Its mathematical description is given by the set of equations:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}h(\mathbf{w}^{\mathrm{T}}\mathbf{x}), \tag{1}$$

where

$$h(\mathbf{w}^{\mathrm{T}}\mathbf{x}) = \tfrac{1}{2}\left(\left|\mathbf{w}^{\mathrm{T}}\mathbf{x}+1\right| - \left|\mathbf{w}^{\mathrm{T}}\mathbf{x}-1\right|\right),$$

$$\mathbf{A} = \begin{bmatrix} q_1 & -1 & 0 \\ q_2 & 0 & 1 \\ q_3 & 0 & 0 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} p_1 - q_1 \\ p_2 - q_2 \\ p_3 - q_3 \end{bmatrix},$$

$$\mathbf{w} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

The individual coefficients are given as

$$\begin{aligned} p_1 &= \mu_1 + \mu_2 + \mu_3, \\ p_2 &= \mu_1\mu_2 + \mu_2\mu_3 + \mu_1\mu_3, \\ p_3 &= \mu_1\mu_2\mu_3, \\ q_1 &= \nu_1 + \nu_2 + \nu_3, \\ q_2 &= \nu_1\nu_2 + \nu_2\nu_3 + \nu_1\nu_3, \\ q_3 &= \nu_1\nu_2\nu_3, \end{aligned}$$

where μ_1, μ_2 and μ_3 represent the eigenvalues corresponding to inner region $(-1 \le \mathbf{w}^{\mathrm{T}}\mathbf{x} \le 1)$ while ν_1, ν_2 and ν_3 those for two outer regions, i.e. $(\mathbf{w}^{\mathrm{T}}\mathbf{x} \ge 1)$ and $(\mathbf{w}^{\mathrm{T}}\mathbf{x} \le -1)$. Assume the so-called double-scroll chaotic attractor [5] where μ_1=0.728, μ_2=-0.319+0.892j, μ_3=-0.319--0.892j, ν_1=-1.29, ν_2=0.061+j and ν_3=0.061-j.

Three quantities $x_1(t)$, $x_2(t)$ and $x_3(t)$ represent three chaotic signals.

Let $n_1(t)$, $n_2(t)$ and $n_3(t)$ are three independent white noises, each of them is uniformly distributed in the interval $\left\langle -\frac{1}{2}\Delta, \frac{1}{2}\Delta \right\rangle$.

The goal is to estimate $x_1(t+L)$ or $y_1(t+L) = x_1(t+L) + n_1(t+L)$, where L is the prediction time, when the vector $\mathbf{y}(t) = \{x_1(t)+n_1(t),\ x_2(t)+n_2(t),\ x_3(t)+n_3(t)\}$ is known.

If $\mathbf{x}(t)$ is substituted by $\mathbf{y}(t)$ in equation (1), the set of equations, which describe Chua's circuit, can be used as a predictor.

Predicted value $\hat{y}_1(t+L)$ can be compared either to $x_1(t+L)$, either to perturbed value $y_1(t+L)$. Thus two errors are defined by

$$\varepsilon_A(t+L) = \hat{y}_1(t+L) - y_1(t+L) \tag{2}$$

and

$$\varepsilon_B(t+L) = \hat{y}_1(t+L) - x_1(t+L) \tag{3}$$

and two appropriate prediction gains are given by

$$G_A(L) = 10\log\frac{E\{y_1^2(t+L)\}}{E\{\varepsilon_A^2(t+L)\}} \tag{4}$$

and

$$G_B(L) = 10\log\frac{E\{x_1^2(t+L)\}}{E\{\varepsilon_B^2(t+L)\}} \tag{5}$$

by analogue to [2].

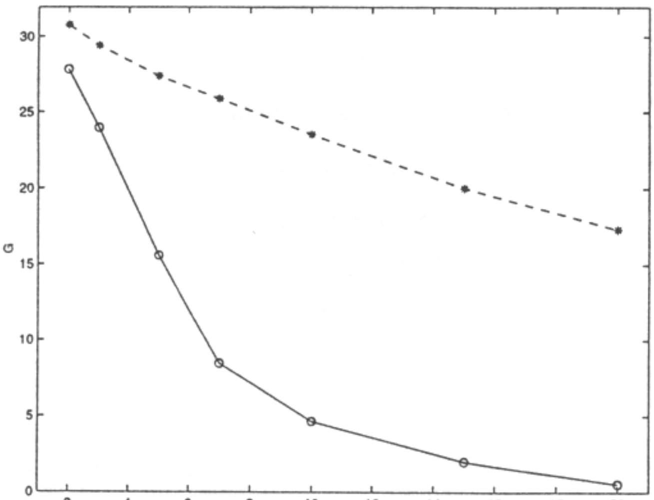

Fig. 1. The prediction gain G_A(L) and its upper bound.

A choice of a value of the variable Δ is a delicate matter. The naturally properties of the chaotic signal are cracked if the value of Δ is too high. We never have infinite accuracy of numeric simulation and calculated values of $x_1(t+L)$ and $y_1(t+L)$ could be slightly untrustworthy for higher values of prediction time L. Thus, Δ could not be too low.

The result of simulation experiment is shown in Fig. 1 by the circles and solid line. Prediction gain decreases with growing L as it was expected. The dashed line will be explained in section 4.

Difference between gains $G_A(L)$ and $G_B(L)$ is shown in Fig. 2 by x-marks and solid line for the considered type of chaos. The dashed line will be described in section 3.

3. The quantization noise and the additive noise

In the section 4 the values of **y** and **x** will be quantized. This is a motive for study of replacement of the additive noise $n(t)$ by the quantization noise and for investigation of admissibility of such substitution. Let variables $x_1(t)$, $x_2(t)$, $x_3(t)$ and $x_1(t+L)$ are quantized by rounding with quantization step Δ=0.0883.

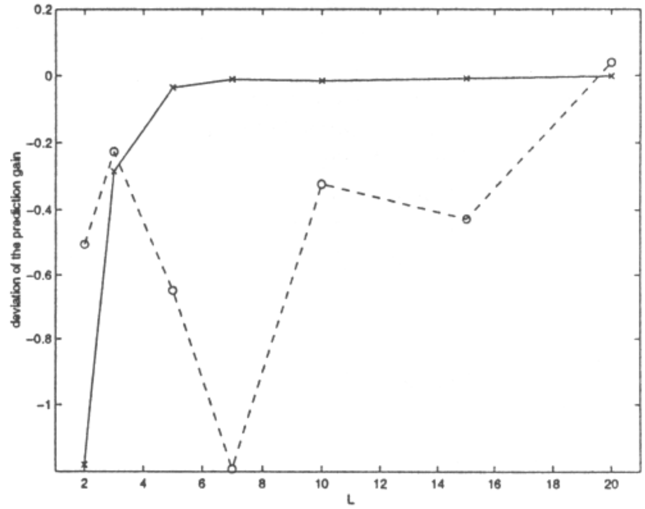

Fig. 2. Deviations of the prediction gains given by the different reference and the different type of noise.

In Fig. 2 circles and dashed line plot the difference between the prediction gain $G_A(L)$ and the gain $G_Q(L)$ calculated from quantized variables versus the prediction time L. It has been used 2500 different values of variable t with the step equal to 100 in simulation process.

Behaviour of the deviation $G_A(L)$ - $G_Q(L)$, which has been tested for different values of quantization step Δ, is irregular. But it is important that the deviation is not too significant.

4. The upper bound

The upper bound $G_{UB}(L)$ of the prediction gain $G_B(L)$ can be calculated from the mutual information between **y**(t) and $y_1(t+L)$ [2]. The gain $G_{UB}(L)$ is a tight bound, because it is equal to the $G_B(L)$ in the case of normal distribution of the $y_1(t+L)$.

The mutual information is calculated using quantized values of the variables $x_1(t)$, $x_2(t)$, $x_3(t)$ and $x_1(t+L)$. In spite of the calculation is rather difficult. Calculations need a large amount of data and a lot of memory. But, one significant advantage is there: no knowledge about the chaotic system is needed.

The set of data given for integer $t \in \langle 1, 262144 \rangle$ has been used for computation. The upper bound of the gain is plotted in Fig. 1 by the stars and the dashed line. Difference between the real gain and its upper bound is obviously consequence of the fact, that probability density function (pdf) of the signal $y_1(t)$ is unlike to Gaussian distribution (Fig. 3).

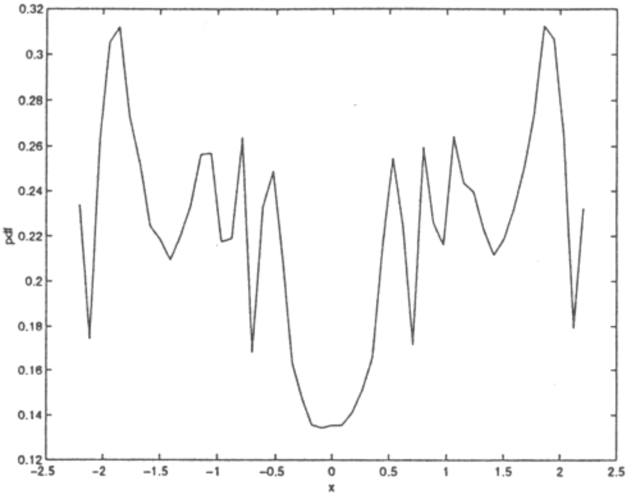

Fig. 3. Probability density function.

5. Conclusion

Determination of the prediction gain using the predictor is not extraordinary demanding to the needed amount of data. But it requires knowledge of the structure and all parameters of the chaos generating system. The choice of the maximum prediction distance L and the power of the additive noise could be done carefully.

The admissibility of the substitution of the additive noise by the quantization noise under given circumstances has been shown. Thanks to that, the estimation of the upper bound is accessible using calculation of the mutual information. Such estimate needs a big amount of data, but knowledge about the chaotic system is not needed.

This research is supported by the Grant Agency of the Czech Republic – Grant No. 102/99/0203 and by the Ministry of Education of Czech Republic – project No.: VS97060.

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