THE IMPORTANCE OF SIMULATION ACCURACY IN CHAOTIC CIRCUITS

Peter KVARDA
Dept. of Radioelectronics
Technical University of Bratislava
Ilkovičova 3, 812 19 Bratislava
Tel.: +421 (0)903 419310
E-mail: Peter.Kvarda@procus.sk

Abstract

In this paper an example of importance of the simulation accuracy in deterministic chaotic oscillators and other chaotic systems is briefly described and shown. Low accuracy in simulation may cause that the non-chaotic system may look chaotic or chaotic system as non-chaotic. How we can prevent it is shown also in this work.

Keywords

Deterministic chaos, Colpitts oscillator, Simulation, simulation accuracy.

1. Introduction

Deterministic chaotic circuits are very sensitive on a little change in initial condition what is also known as positive Ljapunov exponent. However, this sensitivity is not valid only for initial condition, but also for all parameters. Among these parameters also belongs the accuracy of simulation with the same influence as any other parameters. The accuracy of simulation can actually also cause that the non-chaotic system seems like a chaotic. This case is briefly discussed in the next paragraphs.

2. Colpitts Oscillator

This section describes an error that may cause garbled results in examining systems. In this paper the appearance of error and the accurate solution as an example how to prevent it in the future is also described.

In some simulation of deterministic chaotic circuits with low accuracy of simulation in DESIGN EVAL CENTER 6.1 (D.E.C.) or in DOS P-SPICE any non-deterministic chaotic circuit may seem as a deterministic chaotic. This fact may cause errors in analysis. In DOS P-SPICE the accuracy is set automatically (or manually in option by command *trtol*). In D.E.C., the accuracy can be set in *Analyses → Setup → Transient → Step ceiling*. If there is not set concrete value, the accuracy is set automatically. The set value means the maximal time step used during the simulation. The errors caused by low accuracy of simulation used in analyses are shown in figures below. To demonstrate the influence of used accuracy were realised example in D.E.C.

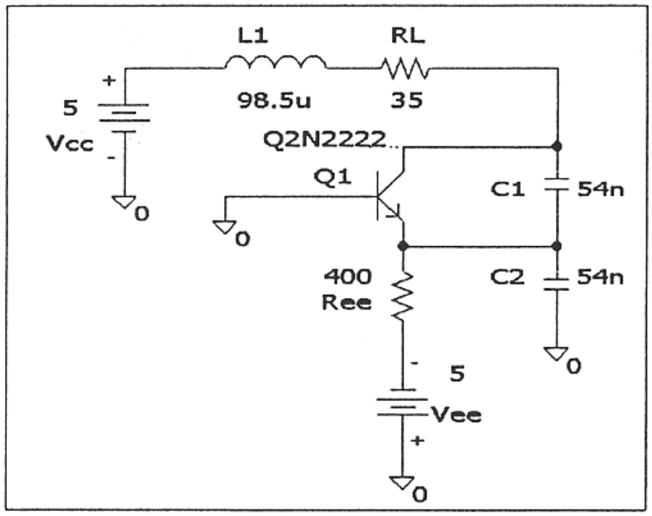

Fig. 1. Chaotic Colpitts oscillator by [1].

The tested circuit is chaotic Colpitts oscillator by [1] in Fig. 1 with changed resistor R_L. The R_L is equal to *50 Ω*, instead of *35 Ω*.

Fig. 2 shows the test of sensitivity on initial condition for voltage U_{BE}. The *step ceiling* was set to *50 ns*. This step is small enough so the accuracy is very good.

It is readily in Fig. 2 that the time characteristic is non-deterministic chaotic, but stable. In the top of the figure the two time characteristics with different initial condition are deducted. The difference in the initial condition is $IC_1 - IC_2 = 1{,}1\ V - 1\ V = 0{,}1\ V$. The sensitivity test readily shows that the Ljapunov exponent is negative. That means the small change in the initial condition does not arise but vice-versa declines. Thus the circuit with R_L=*50Ω* is not sensitive on little changes.

Fig. 3 shows the simulation of the circuit with same parameters but with different *step ceiling*. This *step ceiling* was in this case equal to *1 μs*.

The time characteristics in Fig. 2 have completely different character as characteristics in Fig. 3. The time characteristics in Fig. 3 have chaotic look and the deducted characteristic in the top shows that the Ljapunov exponent is positive.

The time characteristics in Fig. 2 and 3 have completely different character, however they are achieved from the same circuit, shown in Fig. 1. All the differences are caused, only by using the different *step ceiling*.

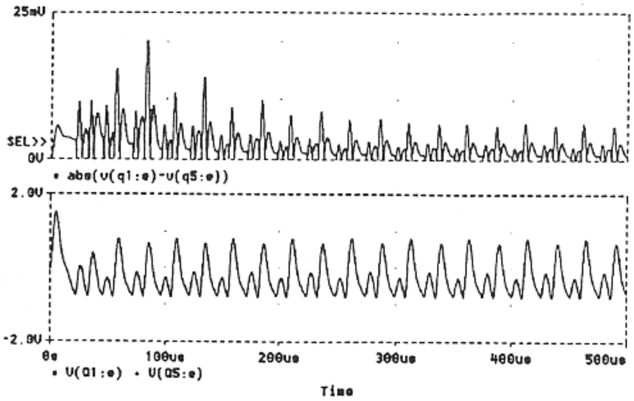

Fig.2. Sensitivity test for Colpitts oscillator with changed parameter R_L. The step ceiling is 50 ns.

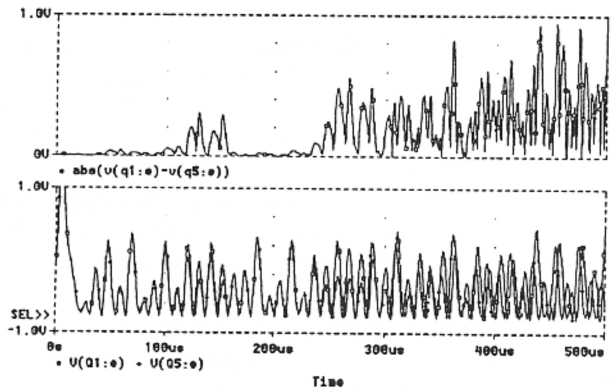

Fig. 3. Sensitivity test for Colpitts oscillator with changed parameter R_L. The step ceiling is 1 µs.

In Fig. 4 and 5 the phase space trajectories for simulated circuit occur. In Fig. 4 step ceiling equal to 50 ns was used and in Fig. 5 equal to 1 µs. In Fig. 4 the non-chaotic character of the circuit is evident. On the other hand Fig. 5 shows the chaotic character of the circuit, however they have the same time limit for drawing each trajectory.

3. Logistic Equation

The lower accuracy of simulation may also cause a different effect as it is mentioned in the text above. That means the simulated system may seem non-chaotic with lower accuracy of simulation and chaotic with higher one. As an example we can mention well known Logistic equation $x_{(n+1)}=k.x_n.(1-x_n)$, $k>0$, $x\in<0,1>$. This equation shows chaotic behaviour for $k > 3.57$. But everybody can try at home that this system has limit cycle or fixed point as solution for accuracy 2 digit after decimal point. But if we use accuracy 8 digit after decimal point the solution will be different and will show chaotic character. (This accuracy can be easily set on scientific calculator).

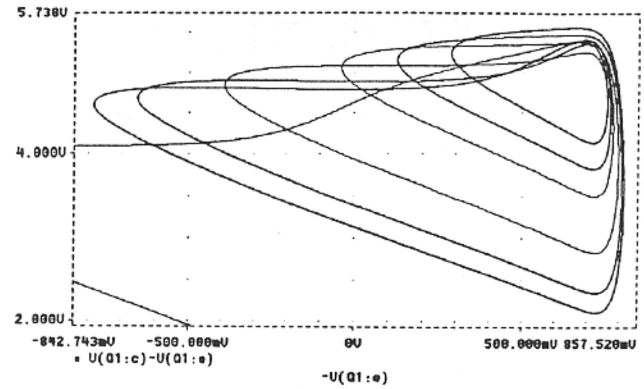

Fig. 4. Phase space trajectory for Colpitts oscillator with changed parameter R_L. The step ceiling is 50 ns.

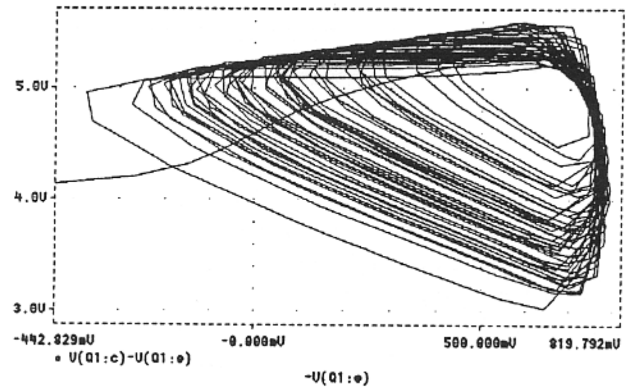

Fig. 5. Phase space trajectory for Colpitts oscillator with changed parameter R_L. The step ceiling is 1 µs.

4. Conclusion

This article showed the importance of used accuracy in theoretical simulation and numerical computing of deterministic chaotic systems.

The error may occur only when the accuracy of simulation is lower. If somebody is not sure in some simulated solution, the simulation must be iterated with better accuracy. Better accuracy of simulation means lower *step ceiling* of simulation or better digit precision. If the quality of the characteristic does not change with better accuracy of the simulation, the currently used accuracy of simulation is sufficient.

5. References

[1] Kennedy, M. P., "Chaos in the Colpitts Oscillator," IEEE Trans. Circuits Syst., Vol. 41, No. 11, 1994, pp. 771-774.

[2] Saber N. Elaydi, An Introduction to Difference Equations, Springer -Verlag New York, Inc. 1996