OPTIMISATION OF THE TRANSIENT RESPONSE OF A DIGITAL FILTER

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Abstract

The paper presents a theoretical analysis of the method that enables optimisation of the transient response of a digital filter or of an arbitrary discrete system, by pre-setting the initial conditions of the inner state description.

For a particular design of a filter it is enough to once evaluate coefficients, multiply them by the magnitude of the first sample of the signal and do filtration by using these initial conditions.

The method was verified using the simulated, and NMR signals to maintain spectrum baselines correction, and will also be used for the study of an optimum filtration of the NMR signal with a variable instantaneous frequency.

Keywords

Digital filtering, Signal Processing, Nuclear Magnetic Resonance

1. Introduction

One of the most frequently used methods in digital signal processing is linear filtration. Generally, it is a method used for selecting certain spectral components of a signal from a mixture of more signals, and for suppressing undesirable signal components.

Signal processing by linear digital filtration brings some problems, such as the transient response at the beginning of the signal. Its duration depends on the filter order. This causes problems when particularly short signals are filtered. In this case the useful signal can be considerably distorted owing to the transient response or it can be entirely lost in the transient response. This restricts, to a certain degree, the application of linear filtration to short signal processing [1].

The paper presents a theoretical analysis of the method that enables optimisation of the transient response of a digital filter in general or of an arbitrary discrete system by pre-setting the initial conditions of the inner state description.

Matlab [2] uses a similar method in one of its functions. It is implemented as a function to filter short signals. Here it is also used in combination with the digital filtration method that permits the design of a filter with a zero phase characteristic. This is possible by using the FIR or IIR filter, and with the suppression of the transient response that is due to the high dynamics of the signal and central symmetrisation of the beginning and end of the signal.

2. Method

In most technical applications, the digital filtration algorithm is designed as a convolution of the signal with the impulse characteristic of a digital filter. The resulting total response (output signal from the filter) is the sum of natural and enforced responses. Natural response is the response of the discrete system to initial conditions (the so-called inner states stored in the memory cells of the digital filter), and enforced response is its response to the input signal. Natural response is currently the response to zero initial conditions. Any unsuitable number located in the memory cell of the digital filter can cause marked prolongation of the transient response, and in the case of the IIR filter even oscillation.

From the analysis of the signals that cause the transient response during filtration it follows that the transient response is primarily influenced by:

– nonzero level of the first samples of the signal
– dc shift of the signal, and
– high dynamics signal.

Digital filters, a special part of discrete systems, can be described in many ways. The most frequently used descriptions are based on differential equations and transfer functions.

The differential equation adapted for use in digital signal processing has the form:

$$y(n+N) + a_1 y(n+N-1) + a_2 y(n+N-2) + \ldots\ a_N y(n) = b_0 x(n+M) + b_1 x(n+M-1) + b_2 x(n+M-2) + \ldots + b_M x(n) \quad (1)$$

where b_i and a_i are constant coefficients.

The transfer function has the form:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_M z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_N z^{-N}} \quad (2)$$

where b_i and a_i are again constant coefficients, the same as those with differential equation (1).

In digital filtration, the inner state description is rarely used. Information about inner variables in the individual steps is unnecessary and description by means of large matrixes is unpractical. To derive the calculation of the initial conditions, that allow removal of the transient response, it is necessary make oneself acquainted with the description.

Prior to writing state matrixes that describe the digital filter, it is important to choose a particular design of the digital filter because the form of state matrixes depends on it. For a discrete system described unambiguously by differential equation (1) or transfer function (2), different designs can be chosen. Then there is a description based on inner state matrixes that corresponds to the chosen kind of design. From this it follows that the system described unambiguously by the differential equation or the transfer function can be described in many ways by means of inner state description.

For the design of digital filters, the so-called canonical structures [4] are most frequently used. These structures have only such a number of retardation blocks that is identical with the order of the digital filter.

The general matrix form of state equations is as follows:

$$\begin{aligned} \boldsymbol{V}(n) &= \boldsymbol{A}\boldsymbol{V}(n-1) + \boldsymbol{B}\boldsymbol{X}(n) \\ \boldsymbol{Y}(n) &= \boldsymbol{C}\boldsymbol{V}(n-1) + \boldsymbol{D}\boldsymbol{X}(n) \end{aligned} \qquad (3)$$

The state equations describe the filter, which is defined by differential equation (1) or transfer function (2) and designed according to the first canonical structure written as components, have the form:

$$\begin{bmatrix} v_1(n) \\ v_2(n) \\ v_3(n) \\ \vdots \\ v_N(n) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & -a_N \\ 1 & 0 & 0 & \cdots & 0 & 0 & -a_{N-1} \\ 0 & 1 & 0 & \cdots & 0 & 0 & -a_{N-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 & -a_1 \end{bmatrix} \cdot \begin{bmatrix} v_1(n-1) \\ v_2(n-1) \\ v_3(n-1) \\ \vdots \\ v_N(n-1) \end{bmatrix} + \begin{bmatrix} b_M - b_0 \cdot a_N \\ b_{M-1} - b_0 \cdot a_{N-1} \\ b_{M-2} - b_0 \cdot a_{N-2} \\ \vdots \\ b_1 - b_0 \cdot a_1 \end{bmatrix} \cdot [x(n)] \qquad (4a)$$

$$[y(n)] = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} v_1(n-1) \\ v_2(n-1) \\ v_3(n-1) \\ \vdots \\ v_N(n-1) \end{bmatrix} + [b_0] \cdot [x(n)] \qquad (4b)$$

3. Theoretical analysis of evaluation of initial conditions

On the basis of the design of an arbitrary digital filter, described by differential equation (1) or transfer function (2), we can evaluate the initial conditions that will enable removal of the transient response in case of a signal with a dc shift or jump change at its beginning. We use the canonical structure and the state description resulting from it (4a,b) to realise this. It has turned out that this way is advantageous for further evaluations.

Prior to creation of state matrixes we complement the coefficients of the differential equation, or operator a_i b_i transfer, with zero coefficients so that

$$M = N. \qquad (5)$$

We will write matrix equation (4a) as a system of linear equations N containing N unknowns.

$$\begin{aligned} v_1(n) &= -a_N \cdot v_N(n-1) + [b_M - b_0 \cdot a_N] \cdot x(n) \\ v_2(n) &= v_1(n-1) - a_{N-1} \cdot v_N(n-1) + [b_{M-1} - b_0 \cdot a_{N-1}] \cdot x(n) \\ v_3(n) &= v_2(n-1) - a_{N-2} \cdot v_N(n-1) + [b_{M-2} - b_0 \cdot a_{N-2}] \cdot x(n) \\ &\dots \\ v_N(n) &= v_{N-1}(n-1) - a_1 \cdot v_N(n-1) + [b_1 - b_0 \cdot a_1] \cdot x(n) \end{aligned} \qquad (6)$$

We will write matrix equation (4b) in the form

$$y(n) = v_N(n-1) + b_0 x(n) \qquad (7)$$

Let us assume that the input signal $x(n)$ creates a sequence of ones $\{1, 1, 1, \dots, 1, 1\}$ or a unit jump.

The input signal $y(n)$ is required not to contain the transient response, i.e. to be instantaneously in the first step and the succeeding ones the same as the input sequence $x(n)$.

$$y(n) = x(n) = 1 \qquad n = 1, 2, 3, \tag{8}$$

By substituting this condition into equation (7), we calculate the value of the state variable $v_N(n-1)$.

$$y(n) = v_N(n-1) + b_0 x(n)$$

We put the condition:

$$\begin{aligned} y(n) &= x(n) = 1 \\ 1 &= v_N(n-1) + b_0 \\ v_N(n-1) &= 1 - b_0 \end{aligned} \tag{9}$$

This condition must be valid also in the next steps because we require that relation (8) be fulfilled also in them. Hence we can conclude that when the output $y(n)$ is to be equal to one, then the state variables in each step will keep having the same value.

$$v_i(n) = v_i(n-1) \tag{10}$$

where $i = 1, 2, 3, \ldots, N$.

Relation (10) can also be obtained by successive substitution of equation (9) into equations (6) and by their solution. After making several steps of filtration we arrive at the same conclusion (10). On the basis of this relation we can write:

$$\begin{aligned} v_1(n) &= v_1(n-1) \\ v_2(n) &= v_2(n-1) \\ v_3(n) &= v_3(n-1) \\ &\ldots \\ v_N(n) &= v_N(n-1) \end{aligned} \tag{11}$$

By substituting equations (11) and condition (8) into equations (6) we obtain the system of linear equations N containing N unknowns.

$$\begin{aligned} v_1(n) &= -a_N \cdot v_N(n) + b_M - b_0 \cdot a_N \\ v_2(n) &= v_1(n) - a_{N-1} \cdot v_N(n) + b_{M-1} - b_0 \cdot a_{N-1} \\ v_3(n) &= v_2(n) - a_{N-2} \cdot v_N(n) + b_{M-2} - b_0 \cdot a_{N-2} \\ &\ldots \\ v_N(n) &= v_{N-1}(n) - a_1 \cdot v_N(n) + b_1 - b_0 \cdot a_1 \end{aligned} \tag{12}$$

System of linear equations N (12) can be rewritten into a more clearly arranged matrix form.

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & a_N \\ -1 & 1 & 0 & \cdots & 0 & 0 & a_{N-1} \\ 0 & -1 & 1 & \cdots & 0 & 0 & a_{N-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & -1 & 1+a_1 \end{bmatrix} \cdot \begin{bmatrix} v_1(n) \\ v_2(n) \\ v_3(n) \\ \vdots \\ v_N(n) \end{bmatrix} = \begin{bmatrix} b_M - b_0 a_N \\ b_{M-1} - b_0 a_{N-1} \\ b_{M-2} - b_0 a_{N-2} \\ \vdots \\ b_1 - b_0 a_1 \end{bmatrix} \tag{13}$$

Matrix equation (13) can further be rewritten into a form that is more suitable for algorithmisation.

$$\begin{bmatrix} 1+a_1 & -1 & 0 & 0 & \cdots & 0 \\ a_2 & 1 & -1 & 0 & \cdots & 0 \\ a_3 & 0 & 1 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{N-1} & 0 & 0 & 0 & \cdots & -1 \\ a_N & 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \cdot \begin{bmatrix} v_N(n) \\ v_{N-1}(n) \\ v_{N-2}(n) \\ \vdots \\ v_2(n) \\ v_1(n) \end{bmatrix} = \begin{bmatrix} b_1 - b_0 a_1 \\ b_2 - b_0 a_2 \\ b_3 - b_0 a_3 \\ \vdots \\ b_{M-1} - b_0 a_{N-1} \\ b_M - b_0 a_N \end{bmatrix} \tag{14}$$

By solving this system of linear equations N containing N unknowns we obtain all initial conditions for memory cells of the digital filter under design. From this it follows that there is no need to use state equation (7) for the solution of initial conditions, and this is advantageous.

As regards the filtration application itself, prior to every new filtration it is necessary to multiply all evaluated coefficients by the magnitude of the first sample of the signal. Otherwise, filtration will not work correctly.

4. Experimental verification

The method was used for filtration of NMR signals. However in this paper an example is presented which better illustrates the capabilities of this method. As the input signal the function $s(t) = 4 + \sin(2 \cdot \pi \cdot f \cdot t)$ was used and in the other case it was the function $s(t) = \cos(2 \cdot \pi \cdot f \cdot t)$, where $f = 1\text{Hz}$ and the signal is sampled using the sampling frequency 1000Hz. Signals prepared this way are filtered using a 15th order low pass band digital filter, butterword approximation and cut-off frequency fm = 40Hz. Signals before and after filtration are shown in Fig.1a,d and their spectra in Fig.1b,e. The spectra before filtration contain only the spectral line at the frequency 1Hz, and the first signal contains the DC component in addition to that. Signals after filtration contain a greater number of harmonics. These are due to the transient response. In an ideal case, the output signal

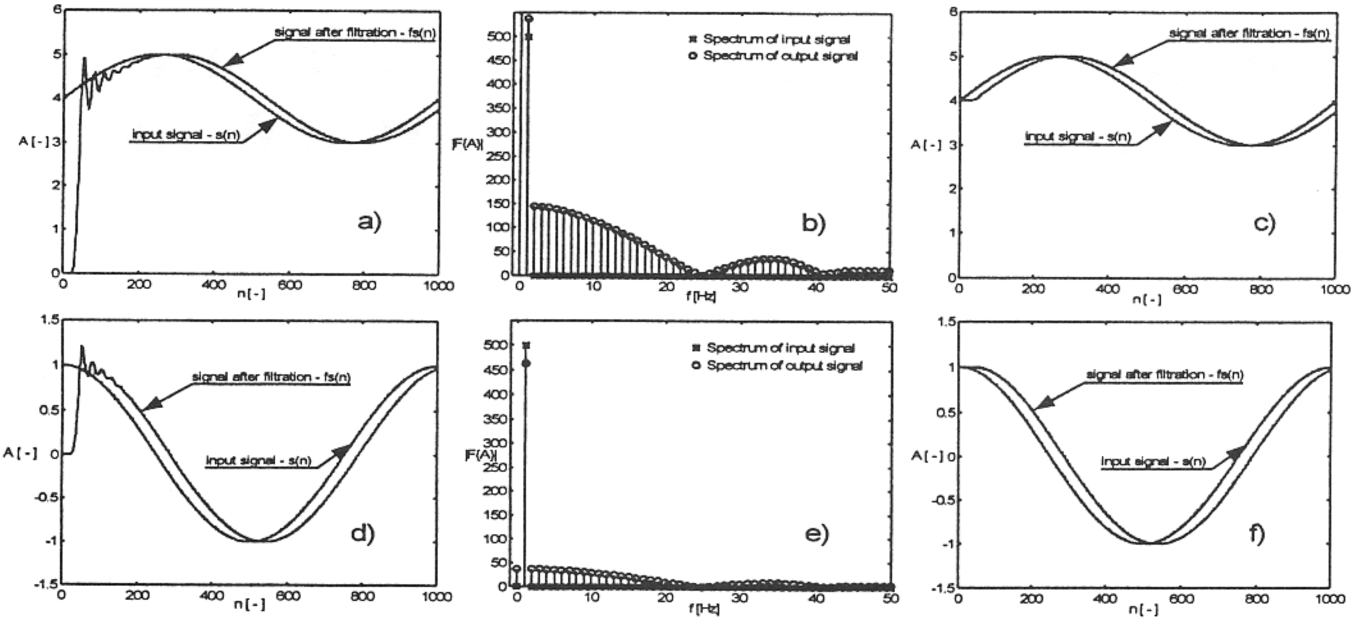

Fig.1 Upper are the simulated signals with DC shift. a) imput signals and signals after filtration, b) their spectra, and c) the signals after designed filtration. Under are the simulated signals with nonzero mean value. d) input signals and signals after filtration, e) their spectrum, and f) the signals after designed filtration.

should remain unchanged after passing through the filter because the cut-off frequency of the filter lies high above the frequency of the input signal.

The input signal and the signal after filtration under non-zero initial conditions are shown in Fig.1c and Fig.1f. When we remove the time lag caused by the order of the filter, we find that the output signal is identical with the input one. The transient response has been removed.

5. Conclusions

An analysis of the digital filter will enable optimisation of its transient response by pre-setting the initial conditions of the inner state description. This type of filtration is suitable, above all, for filtration of short signals where the useful signal can be markedly distorted by the transient response or entirely lost in it. The advantage of the method is that no auxiliary signal operations need to be made before filtration. For a particular design of a filter it is enough to once evaluate coefficients, multiply them by the magnitude of the first sample of the signal and do filtration by using these initial conditions.

The method was simulated using the signals described in the preceding chapter, and NMR signals to spectrum baselines correction. Also it will be used for the study of an optimum filtration of the NMR signal with a variable instantaneous frequency.

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