NEW REFERENCE STATE MODEL OF THE THIRD-ORDER PIECEWISE-LINEAR DYNAMICAL SYSTEM

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Abstract

Starting from the first elementary canonical state model, the new simple state model of the third-order dynamical systems that belong to Class C is derived. A typical property of this new model is a very simple form of its partial transformation matrix in the conditions of linear topological conjugacy, which represents the unity matrix. Complete state equations and the corresponding integrator-based block diagram of this model are shown and relation to the first canonical form is graphically illustrated.

Keywords

Piecewise-linear systems, dynamical systems, state models, topological conjugacy, chaos.

1. Introduction

Third-order piecewise-linear (PWL) dynamical systems belonging to Class C of vector fields in $\Re^3$ [1] can be described by the state equations in their matrix form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}\,h\,(\mathbf{w}^T\mathbf{x}) \tag{1}$$

$(\mathbf{A} \in \Re^{3x3}, \mathbf{b} \in \Re^3, \mathbf{w} \in \Re^3)$ where PWL function

$$h\,(\mathbf{w}^T\mathbf{x}) = \tfrac{1}{2}\left(\left|\mathbf{w}^T\mathbf{x}+1\right| - \left|\mathbf{w}^T\mathbf{x}-1\right|\right) \tag{2}$$

is continuous, odd-symmetric and partitioning $\Re^3$ by two parallel planes into the inner (origin) region D_0 and two outer regions D_{+1}, D_{-1}.

The dynamical behavior of such systems is determined by two sets of eigenvalues representing two characteristic polynomials associated with the corresponding region [1], i.e.

$$D_0: \quad P(\mathrm{s}) = \det(s\mathbf{1}-\mathbf{A}_0) = (s-\mu_1)(s-\mu_2)(s-\mu_3) = s^3 - p_1 s^2 + p_2 s - p_3 \tag{3}$$

$$D_{+1}, D_{-1}: \quad Q(s) = \det(s\mathbf{1}-\mathbf{A}) = (s-\nu_1)(s-\nu_2)(s-\nu_3) = s^3 - q_1 s^2 + q_2 s - q_3 \tag{4}$$

where $\mathbf{A}_0 = \mathbf{A} + \mathbf{b}\mathbf{w}^T$, $\mathbf{1}$ is the unity matrix, and coefficients p_i ,q_i. (i=1,2,3) are the so-called equivalent eigenvalue parameters.

Any two systems having the same eigenvalues are qualitatively equivalent and their mutual relations can be expressed by the linear topological conjugacy conditions [2] in the explicit form

$$\tilde{\mathbf{x}} = \mathbf{T}\mathbf{x} \tag{5a}$$

$$\tilde{\mathbf{A}} = \mathbf{T}\mathbf{A}\mathbf{T}^{-1} \tag{5b}$$

$$\tilde{\mathbf{b}} = \mathbf{T}\mathbf{b} \tag{5c}$$

where the complete transformation matrix is given as

$$\mathbf{T} = \tilde{\mathbf{K}}^{-1}\mathbf{K} \tag{5d}$$

Variables $\tilde{\mathbf{x}}$ and $\mathbf{x}$, state matrices $\tilde{\mathbf{A}}$ and $\mathbf{A}$, vectors $\tilde{\mathbf{b}}, \tilde{\mathbf{w}}$ and $\mathbf{b}, \mathbf{w}$ belong to the first and second systems, respectively. Partial transformation matrices $\tilde{\mathbf{K}}$ and $\mathbf{K}$ are defined by the nonsingular form [2]

$$\tilde{\mathbf{K}} = \begin{bmatrix} \tilde{\mathbf{w}}^T \\ \tilde{\mathbf{w}}^T\tilde{\mathbf{A}} \\ \tilde{\mathbf{w}}^T\tilde{\mathbf{A}}^2 \end{bmatrix} \quad \text{and} \quad \mathbf{K} = \begin{bmatrix} \mathbf{w}^T \\ \mathbf{w}^T\mathbf{A} \\ \mathbf{w}^T\mathbf{A}^2 \end{bmatrix} \tag{6a,b}$$

fulfilling the observability condition of pairs $(\tilde{\mathbf{A}}, \tilde{\mathbf{w}}^T)$ and $(\mathbf{A}, \mathbf{w}^T)$, respectively.

Any state model topologically conjugate to Class C can be used as the reference model, i.e. any other model or system can be then expressed by this qualitatively equivalent reference form. Suitable models with very simple form of partial transformation matrix represent Chua's model corresponding to Chua's oscillator [1] and especially its first canonical ODE's equivalent [3]. The new reference model introduced here is derived starting from this canonical model under requirement of the elementary form of its partial transformation matrix, i.e. the unity matrix

$$\tilde{\mathbf{K}} = \tilde{\mathbf{K}}^{-1} = \mathbf{1}$$

Then the partial transformation matrix of any other state model is equaled directly to complete transformation matrix $\mathbf{T}$ in formula (5d).

2. First Canonical form

The first elementary canonical state model [3] can be used as the basic initial system where

$$\mathbf{x} \mathrel{\hat=} \mathbf{x}_\mathrm{I} = \begin{pmatrix} x_\mathrm{I} \\ y_\mathrm{I} \\ z_\mathrm{I} \end{pmatrix}, \quad \mathbf{w} \mathrel{\hat=} \mathbf{w}_\mathrm{I} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{7a,b}$$

$$\mathbf{A} \mathrel{\hat=} \mathbf{A}_\mathrm{I} = \begin{bmatrix} q_1 & -1 & 0 \\ q_2 & 0 & -1 \\ q_3 & 0 & 0 \end{bmatrix}, \ \mathbf{b} \mathrel{\hat=} \mathbf{b}_\mathrm{I} = \begin{pmatrix} p_1 - q_1 \\ p_2 - q_2 \\ p_3 - q_3 \end{pmatrix} \tag{7c,d}$$

so that the complete state equations can be expressed as

$$\left.\begin{aligned} \dot{x}_\mathrm{I} &= q_1 x_\mathrm{I} - y_\mathrm{I} + (p_1 - q_1)\, h(x_\mathrm{I}) \\ \dot{y}_\mathrm{I} &= q_2 x_\mathrm{I} - z_\mathrm{I} + (p_2 - q_2)\, h(x_\mathrm{I}) \\ \dot{z}_\mathrm{I} &= q_3 x_\mathrm{I} + (p_3 - q_3)\, h(x_\mathrm{I}) \end{aligned}\right\} \tag{8}$$

The partial transformation matrix has the simple form in accordance with formula (6b), i.e.

$$\mathbf{K} \mathrel{\hat=} \mathbf{K}_\mathrm{I} = \begin{bmatrix} 1 & 0 & 0 \\ q_1 & -1 & 0 \\ q_1^2 - q_2 & -q_1 & 1 \end{bmatrix} \tag{9}$$

The corresponding integrator-based block diagram is introduced in Fig. 1.

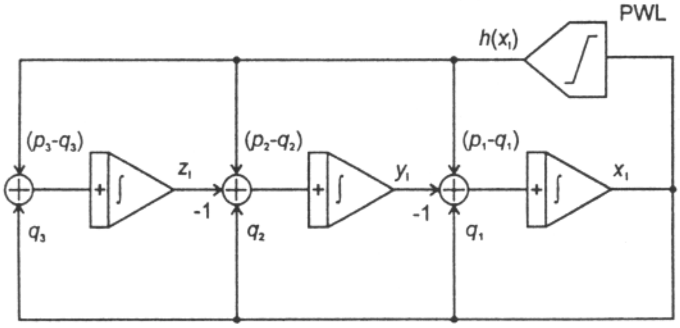

Fig.1. Integrator-based block diagram of the first canonical state model.

3. New Reference State Model

Starting from the basic requirement

$$\widetilde{\mathbf{K}} = \mathbf{K}_\mathbf{R} = \mathbf{1} \tag{10}$$

new reference state variables $\mathbf{x}_\mathbf{R}$ and vector $\mathbf{w}_\mathbf{R}$ are

$$\widetilde{\mathbf{x}} \mathrel{\hat=} \mathbf{x}_\mathbf{R} = \begin{pmatrix} x_\mathbf{R} \\ y_\mathbf{R} \\ z_\mathbf{R} \end{pmatrix}, \quad \mathbf{w} \mathrel{\hat=} \mathbf{w}_\mathbf{R} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{11}$$

and complete transformation matrix for expressing this new model by using the first canonical form is

$$\mathbf{T} \mathrel{\hat=} \mathbf{T}_{\mathbf{R}\text{-}\mathbf{I}} = \mathbf{K}_\mathrm{I} \tag{12}$$

Then the general conditions of linear topological conjugacy (5a) to (5c) have the following form

$$\widetilde{\mathbf{x}} \mathrel{\hat=} \mathbf{x}_\mathbf{R} = \mathbf{K}_\mathrm{I}\, \mathbf{x}_\mathrm{I} \tag{13a}$$

$$\widetilde{\mathbf{A}} = \mathbf{A}_\mathbf{R} = \mathbf{K}_\mathrm{I} \mathbf{A}_\mathrm{I} \mathbf{K}_\mathrm{I}^{-1} \tag{13b}$$

$$\widetilde{\mathbf{b}} \mathrel{\hat=} \mathbf{b}_\mathbf{R} = \mathbf{K}_\mathrm{I}\, \mathbf{b}_\mathrm{I} \tag{13c}$$

Then substituting form (7c,d) into (13b,c) state matrix $\mathbf{A}_\mathbf{R}$ and vector $\mathbf{b}_\mathbf{R}$ of the new reference model can be expressed as

$$\mathbf{A}_\mathbf{R} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ q_3 & -q_2 & q_1 \end{bmatrix}, \ \mathbf{b}_\mathbf{R} = \begin{pmatrix} p_1 - q_1 \\ b_2 \\ b_3 \end{pmatrix} \tag{14}$$

where

$$\left.\begin{aligned} b_2 &= q_1(p_1 - q_1) - (p_2 - q_2) \\ b_3 &= (q_1^2 - q_2)(p_1 - q_1) - q_1(p_2 - q_2) + (p_3 - q_3) \end{aligned}\right\} \tag{15}$$

The complete state equations have the following form

$$\begin{aligned} \dot{x}_\mathbf{R} &= y_\mathbf{R} + (p_1 - q_1)\, h(x_\mathbf{R}) \\ \dot{y}_\mathbf{R} &= z_\mathbf{R} + [q_1(p_1 - q_1) - (p_2 - q_2)]\, h(x_\mathbf{R}) \\ \dot{z}_\mathbf{R} &= q_1 x_\mathbf{R} - q_2 y_\mathbf{R} + q_3 z_\mathbf{R} + \\ &\quad + [(q_1^2 - q_2)(p_1 - q_1) - q_1(p_2 - q_2) + (p_3 - q_3)]\, h(x_\mathbf{R}) \end{aligned} \tag{16}$$

The integrator-based block diagram is introduced in Fig. 2. Its upper and lower parts evidently correspond to the first (Fig.1) and second canonical form [3], respectively.

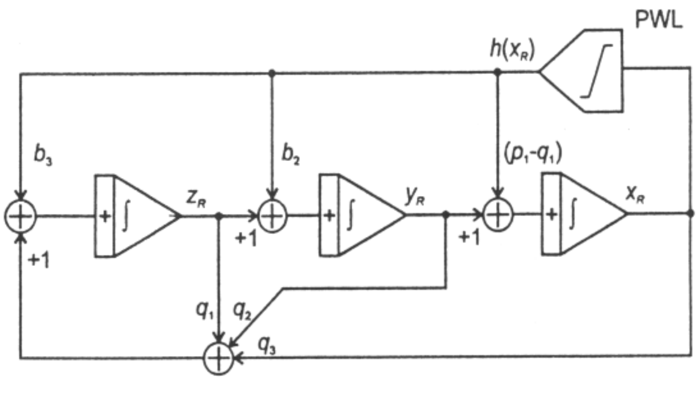

Fig. 2. Integrator-based block diagram of the new referen.ce state model

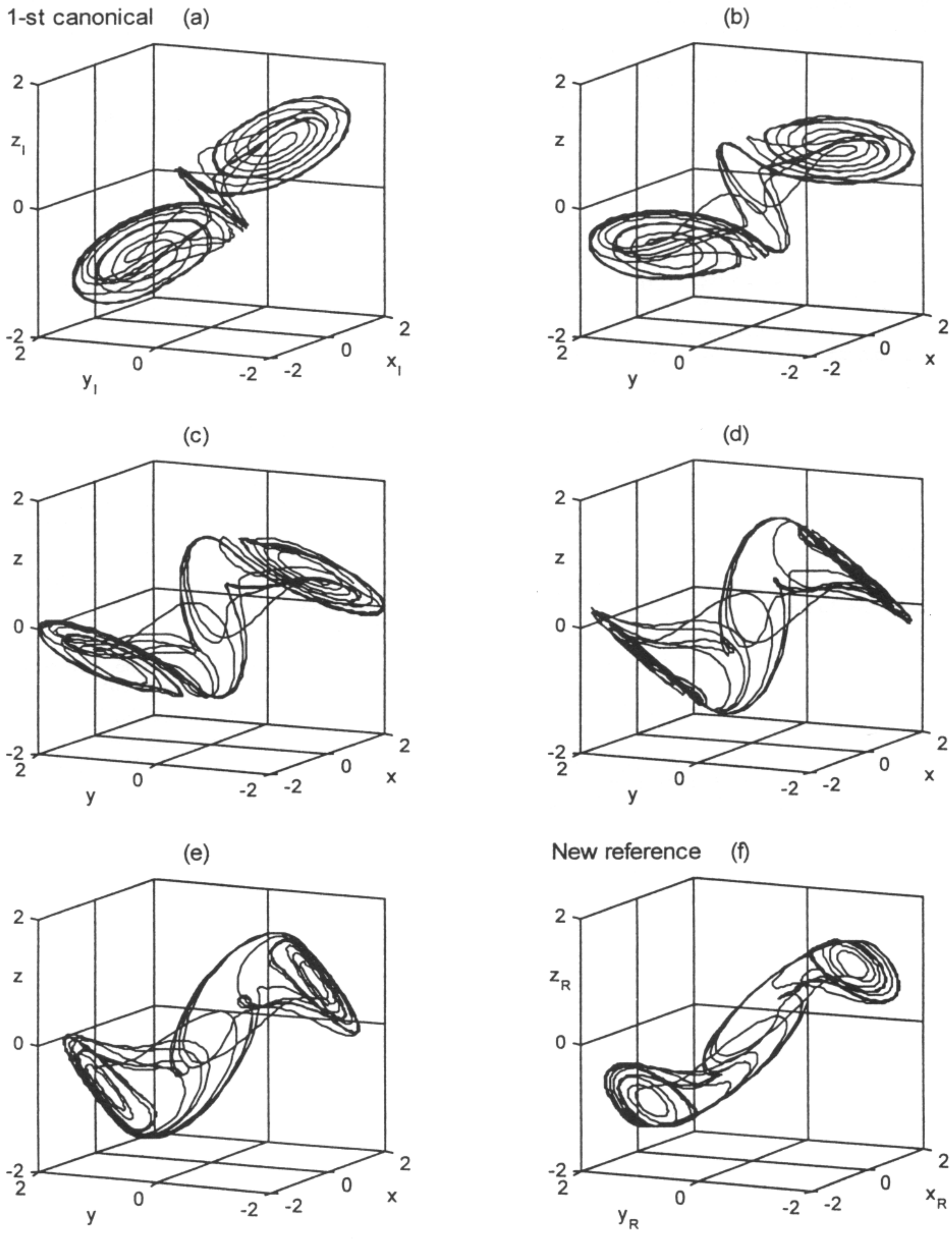

Fig. 3. Graphical illustration of linear topological conjugacy between first canonical form and new reference model using "*elastic space*" analogy. (a) First canonical form, (b)-(e) Transient steps, (f) New reference state model.
(Equivalent eigenvalue parameters: $p_1 = 0.09$; $p_2 = 0.432961$; $p_3 = 0.653325$; $q_1 = -1.168$; $q_2 = 0.846341$; $q_3 = -1.2948$.)

4. Conclusion

The new simple state model derived is topologically conjugate to Class C of vector fields in $\Re^3$, i.e. also to Chua's equations and their canonical equivalents. Similarly as both canonical forms it includes also the so-called "*set of measure zero*" which is excluded in Chua's equations. As the partial transformation matrix of the new reference model represents the unity matrix, the relation to any other qualitatively equivalent state model is determined directly by the partial transformation matrix of this model. Topological conjugacy between the first canonical form and new reference model is graphically illustrated using the so-called "*elastic space analogy*" [4] in Fig. 3. The integrator-based block diagram shown in Fig. 2 can also be used as prototype for the practical realization using various circuit techniques similarly as both canonical forms [5],[6],[7].

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