IDENTIFYING THE DETERMINISTIC CHAOS BY USING THE LORENZ MAPS

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Abstract

This paper presents an investigation of the deterministic and stochastic chaos. The modified Colpitts oscillator is used as an example of deterministic chaos in electronic circuits. The graphical method of Lorenz maps is used for graphical observation of both chaotic classes.

Keywords

Deterministic chaos, Lorenz map, Colpitts oscillator

1. Introduction

The observation of the deterministic chaos may be sometimes difficult task. In the time analyses the deterministic character of the signal cannot be seen. Some other tools are usually used for this purpose. A very simple but powerful tool for determination of recurrent character of deterministic chaos is here presented. The Lorenz plot discovered Lorenz to easier studying the simulated chaotic signals. This map is qualitatively similar to Poincaré map, but its construction is easier.

2. Lorenz Map

The mentioned tool for observation the deterministic character is Lorenz map constructed from extremes in time characteristic. The Lorenz map is a map, which exemplifies the interdependency of two neighbour extremes and can be described by recurrent equation

$$x_{(n+1)} = f(x_{(n)}), \qquad n = 0,1,2,\ldots \tag{1}$$

Thus, the next extreme $x_{(n+1)}$ is defined by function f and extreme $x_{(n)}$. The $\{x_{(n)}\}$ are then successive extremes of a concrete deterministic chaotic signal. Thereby, the Lorenz map for deterministic chaotic signal is a function, but does not have to be continuous function. The Lorenz map will be some type of area, which depends on distribution if the signal is stochastic.

3. Colpitt's Oscillator

As an example of deterministic chaotic circuit, the Kennedy's chaotic Colpitts oscillator by [1] is used. This oscillator has chaotic character for certain circuit's parameters (Fig. 1). In this type of oscillators, to get chaotic behaviour must be the resonant part of circuit strongly attenuated. In Kennedy's modification of Colpitts oscillator is this condition satisfied by resistor R_L in series with inductor L_1. More information is given in [2].

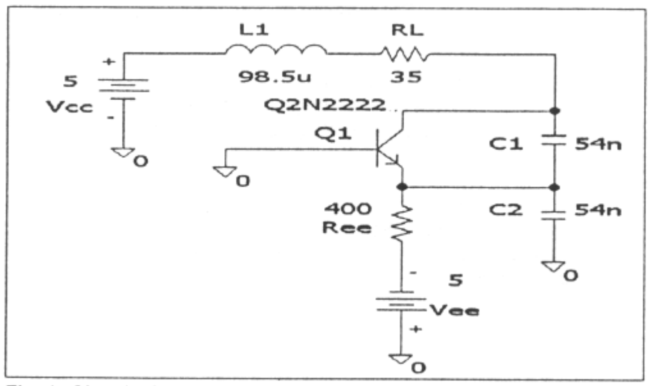

Fig. 1. Chaotic Colpitts oscillator by [1].

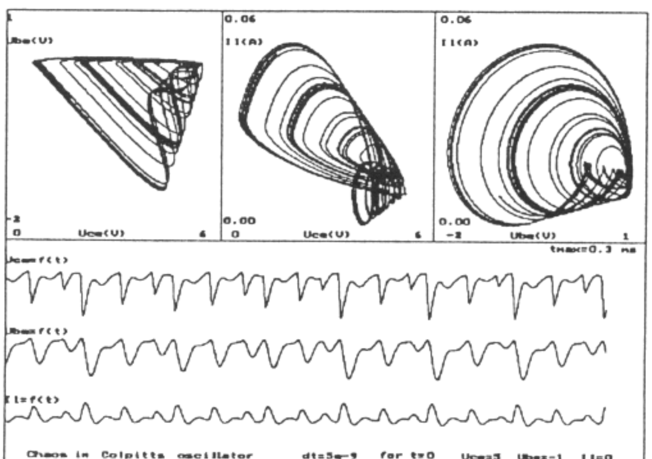

Fig. 2. Numerically calculated time characteristics for Colpitts oscillator

Numerically calculated time characteristics using Eurel's one step method of Colpitts oscillator are in Fig. 2. The chaotic character of time characteristics is also readily showed in Fig. 2.

3.1 Lorenz Maps of Colpitts Oscillator

Lorenz maps of Colpitts oscillator were constructed theoretically and also practically. The theoretical Lorenz maps were constructed using numerical simulation of Colpitts oscillator. The results occur in Fig. 3. These Lorenz maps have fractal character. In finite resolution can belong to one x-axes point more than one point from y-axes, like in Fig. 3. Mathematically exemplifies the Lorenz map the Eq. 1, where the $x_{(n+1)}$ is extreme following the extreme $x_{(n)}$.

The practically measured Lorenz maps occur in Fig. 5 and 6. By using special digital oscilloscope attached to the personal computer were these maps constructed. (This device named *Iteroscope* is in details described in [2]). The practically measured Lorenz map in Fig. 5 corresponds to the theoretical Lorenz map in the left up corner in Fig. 3. The practically measured Lorenz map in Fig. 6 corresponds to the theoretical Lorenz map in right up corner in Fig. 3.

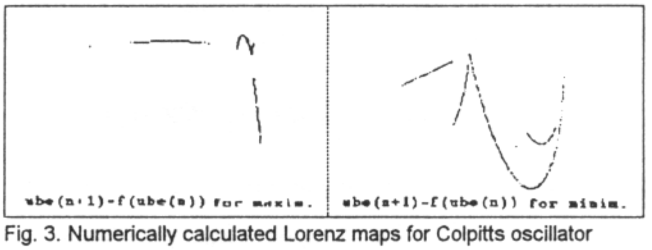

Fig. 3. Numerically calculated Lorenz maps for Colpitts oscillator

The similarities between theoretically computed and practically measured Lorenz maps are well visible in these figures. The deterministic character of time dependence of deterministic chaotic systems, which is not valid for stochastic signal is also proved in these Lorenz maps. All these extremes are exactly defined in deterministic chaotic systems what shows deadly the Lorenz maps, too.

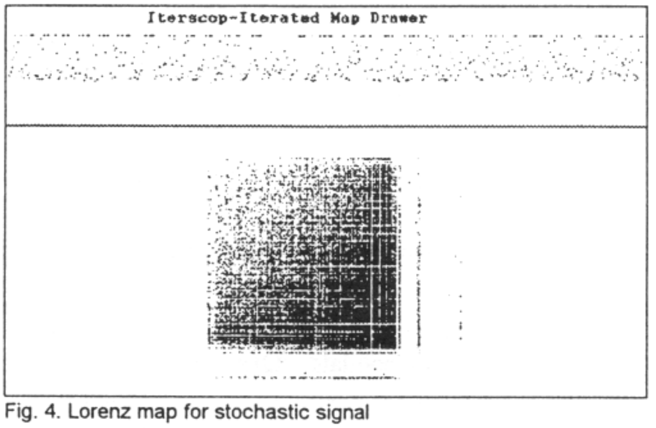

Fig. 4. Lorenz map for stochastic signal

3.2 Lorenz Maps of Stochastic Signal

The Lorenz map of stochastic signal has character given by the probability distribution of examined system. For example the Lorenz map for flat random noise is square because the probability of next extreme's amplitude is same for all extreme. In Fig. 4 is given an example of such signal but the noise is not ideal which shows the map.

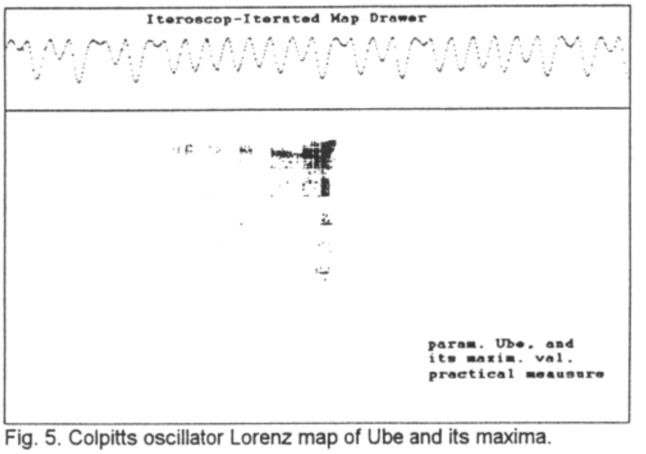

Fig. 5. Colpitts oscillator Lorenz map of Ube and its maxima.

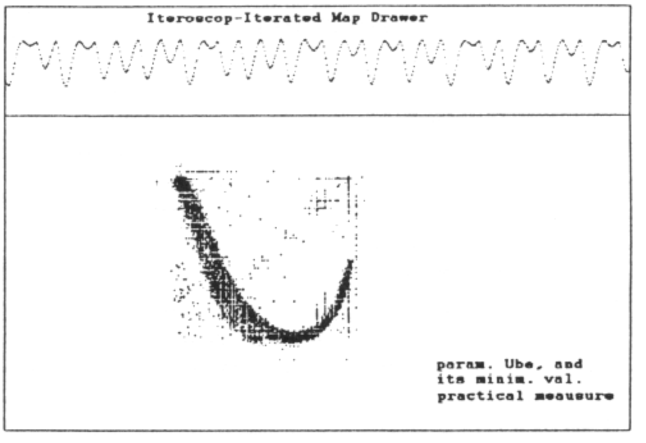

Fig. 6. Colpitts oscillator Lorenz map of Ube and its minima.

4. Conclusion

We showed on mentioned examples, how can be by easy way found out the real character of strange one-dimensional signal. The Lorenz maps give us a primitive tool for recognising the stochastic signals from non-stochastic. This principle can be also easily used in practice measuring what differentiate the Lorenz maps from other techniques used for same reason.

References

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