# EXTENSION OF IMPULSE DETECTORS TO SPATIAL DIMENSION AND THEIR UTILISATION AS SWITCH IN THE LMS L-SD FILTER

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## Abstract

*In this paper, one kind of adaptive LMS filters based on order statistics is used for two-dimensional filtration of noisy greyscale images degraded by mixed noise. The signal-dependent adaptive LMS L-filter (L-SD) consists of two normalised constrained adaptive LMS L-filters, because they have better convergence properties than simple LMS algorithm. Moreover, first filter suppresses the noise in homogeneous regions and second filter preserves the high components of filtered image. Some versions of spatial order statistic detectors were developed from the impulse detectors and were employed as switch between output these filters.*

## Keywords

LMS L-filters, adaptive filters, spatial impulse detector

## 1. Introduction

Adaptive filters have been applied in a wide range variety of problems including system identification, channel equalisation, echo cancellation in telephone channels. The most widely known adaptive filters are linear ones that have the form of finite impulse response (FIR) filters. Furthermore, linear filters may not be suitable for nonlinear applications. One of the best-known classes of nonlinear filters is based on the order statistics. The power of this family is well demonstrated by the median filter, which preserve edges and high frequencies in processed image. In applications, where the noisy image consists from combination of the white Gaussian and impulsive noise, problem of balance between edge preservation and maximal noise suppression in the homogeneous regions must by solved.

Several authors have used the Least Mean Square (LMS) algorithm to design nonlinear filters [1-2], [5-6]. There is a class of adaptive LMS L-filters [1-4], that very well approximate filter coefficients considering to noise model. The L-filters have found extensive applications in image processing, because they have elementary methodology of design. In this paper, the signal-dependent LMS L-filter (L-SD) is studied [1-2], [4]. L-SD filter consists of two normalised constrained adaptive LMS L-filter (L-NC) [3]. The outputs from these single filters are switched by employing the local signal to noise ratio measure. On the other hand, as switch can be used the spatial order statistic detector, which detects impulses or high image components in the observed image.

In the next section, the design theory of the adaptive LMS L-SD filter based on adaptive LMS L-NC filter is introduced. The spatial median detector (SMD), the spatial order statistic detector (SOSD) and the spatial central order statistic detector (SCOSD) are described in section 3. The experimental results are presented in section 4 and the last section contains the discussion about our results and future tasks, which can improve the filtration results.

## 2. Adaptive LMS L-SD filter

In this section, the adaptive LMS L-SD filter consists of two adaptive LMS L-NC filters is applied [1-2],[4]. The filter structure is shown in Fig.1. For simplicity, one general L-NC filter will be only described. The complete L-SD filter will be described at the end this section.

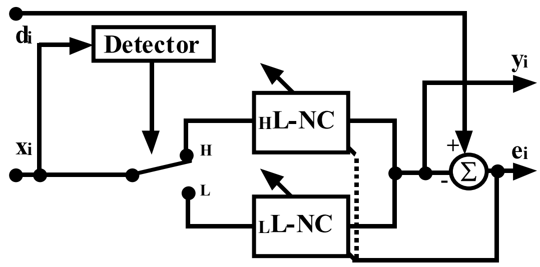


**Fig. 1** The scheme of Signal-Dependent L-filter

Let $\mathbf{x}_i = [x_i(1),...,x_i(N)]^{\mathrm{T}}$, $N = (2\xi+1)^2$ is a noisy observation vector. If these data are rearranged in ascending order to their magnitude, the order statistics result has form

$$ {}_r\mathbf{x}_i = sort\{\mathbf{x}_i\}. \quad (1)$$

Such as more filters we seek output in form

$$ y_i = \mathbf{w}_i^{\mathrm{T}}\, {}_r\mathbf{x}_i \quad (2)$$

by using of the constrain condition

$$\sum_{k=1}^{N} w_i(k) = 1 . \tag{3}$$

Let the filter coefficients of the L-filter vector is partitioned by employing (4) as follows

$$\mathbf{w}_i = \left( {}_1\mathbf{w}_i^T \middle| {}_\upsilon w_i \middle| {}_2\mathbf{w}_i^T \right)^T , \tag{4}$$

where ${}_\upsilon w_i$ is the central weight coefficient, ${}_1\mathbf{w}_i$, ${}_2\mathbf{w}_i$ are $(N\text{-}1)/2 \times 1$ vectors given by

$$\begin{aligned} {}_1\mathbf{w}_i &= \left(w_i(1),\ldots,w_i(\upsilon-1)\right)^T \\ {}_2\mathbf{w}_i &= \left(w_i(\upsilon+1),\ldots,w_i(N)\right)^T \\ {}_\upsilon w_i &= 1 - {}_{\upsilon-1}\mathbf{1}^T\,{}_1\mathbf{w}_i - {}_{\upsilon-1}\mathbf{1}^T\,{}_2\mathbf{w}_i \end{aligned} . \tag{5}$$

Such as the vector of the filter coefficients in form (4), the ordered input vector ${}_r\mathbf{x}_i$ can be similarly rearranged too. Thus, the rewritten vector has next form

$${}_r\mathbf{x}_i = \left( {}_{r1}\mathbf{x}_i^T \middle| {}_\upsilon x_i \middle| {}_{r2}\mathbf{x}_i^T \right)^T . \tag{6}$$

Let $\mathbf{w}'_i$ is the reduced L-filter coefficient vector

$$\mathbf{w}'_i = \left( {}_1\mathbf{w}_i^T \middle| {}_2\mathbf{w}_i^T \right)^T \tag{7}$$

and reduced ordered input vector ${}_r\mathbf{x}'_i$ is described as

$${}_r\mathbf{x}'_i = \begin{bmatrix} {}_{r1}\mathbf{x}_i - {}_\upsilon x_i \mathbf{1} \\ {}_{r2}\mathbf{x}_i - {}_\upsilon x_i \mathbf{1} \end{bmatrix} . \tag{8}$$

Finally, the LMS recursive relation for updating the reduced L-filter coefficient vector is given by

$$\mathbf{w}'_{i+1} = \mathbf{w}'_i + 2\mu\varepsilon_i\,{}_r\mathbf{x}'_i , \tag{9}$$

where $\mathbf{w}'_i$ is updated during search of minimal error by using certain criterion. $\mu$-denotes the convergence parameter and $\varepsilon_i$ is estimation error at *i-th* observation, i.e. $\varepsilon_i = d_i - y_i$. Because of normalisation of adaptation algorithm, the convergence parameter $\mu$ is particularly computed at each observation. The normalised convergence parameter $\mu_i$ has next form

$$\mu_i = \frac{1}{{}_r\mathbf{x}_i^T\,{}_r\mathbf{x}_i} = \frac{1}{\left\| {}_r\mathbf{x}_i \right\|^2} . \tag{10}$$

Furthermore, the constrained adaptation equation (9) is normalised by using definition (10). Updating formula for the coefficients of the adaptive LMS L-NC filter is

$$\mathbf{w}'_{i+1} = \mathbf{w}'_i + 2\frac{\mu_0}{\left\| {}_r\mathbf{x}_i \right\|^2}\varepsilon_i\,{}_r\mathbf{x}'_i , \tag{11}$$

where $\mu_0$ is normalised convergence parameter, it is chosen from interval

$$0 < \mu_0 \le 1 . \tag{12}$$

The structure consists of two independent L-filters, (e.g. 2 x L-NC filter or another filter combination) which can have a different filter window. Their outputs are called ${}_L y_i$ for low-frequency image components and ${}_H y_i$ for high-frequency image components. Thus, the L-SD output is obtained by using the signal-dependent weighting factor $\beta_i$ (13), as switch between ${}_L$L-NC and ${}_H$L-NC outputs by means of law in equation (14).

$$\beta_i = 1 - \frac{{}_n\sigma^2}{{}_x\sigma_i^2} , \tag{13}$$

where ${}_n\sigma^2$ is the noise variance and ${}_x\sigma_i^2$ is the variance of noisy input observation.

The filter output is obtained by using the signal-dependent weighting factor $\beta_i$, as switch between outputs the two LMS L-filters. This switch equation is given by

$$\mathrm{y_i} = \begin{cases} {}_{\mathrm{H}}\mathrm{y_i}, & \text{IF } \beta_{\mathrm{i}} > \beta_{\mathrm{t}} \\ {}_{\mathrm{L}}\mathrm{y_i}, & \text{OTHERWISE} \end{cases} \tag{14}$$

where $0 < \beta_t < 1$ is a threshold that determines a trade-off between noise suppression and edge preservation.

The final adaptation equations (15) with definitions (13) and (14) determine the adaptive LMS L-SD filter.

$$\begin{aligned} {}_{\mathrm{L}}\mathbf{w}'_{i+1} &= {}_{\mathrm{L}}\mathbf{w}'_i + 2\frac{\mu_0}{\left\| {}_r\mathbf{x}_i \right\|^2}\varepsilon_i\,{}_r\mathbf{x}'_i \\ {}_{\mathrm{H}}\mathbf{w}'_{i+1} &= {}_{\mathrm{H}}\mathbf{w}'_i + 2\frac{\mu_0}{\left\| {}_r\mathbf{x}_i \right\|^2}\varepsilon_i\,{}_r\mathbf{x}'_i \end{aligned} \tag{15}$$

## 3. Spatial Impulse Detectors

The spatial impulse detectors were derived from impulse detectors [7], and they are employed as impulse detector for all image pixels in the observed vector. If impulses are detected in the input sequence, these input data are processed by high-frequency segment. On other hand, if in the input sequence are not detected any impulses or high-frequency components, the input data are processed by low-frequency segment. The decision rule for L-SD filter in combination with spatial impulse detectors is given by

$$\textbf{IF} \quad \sum_{k=1}^{N} D_i(k) \geq Level \quad \textbf{THEN} \; {}_H L-NC \quad \textbf{ELSE} \; {}_L L-NC \tag{16}$$

where $D_i(k)$ is result of impulse detection for *k-th* image pixel in the *i-th* input vector. The value of level defines the number of detected impulses in the observed samples.

## 3.1 SMD

The spatial median order detector (SMD) is first from order statistic detectors family, which is based on the following rule:

$$\textbf{IF} \quad \left| med\{\mathbf{x}_i\} - x_i(k) \right| \geq Tol \quad \textbf{THEN} \; D_i(k)=1 \quad \textbf{ELSE} \; D_i(k)=0 . \tag{17}$$

If the different in magnitude between *k-th* and median input samples is more than value of tolerance, this input sample is marked as detected impulse. Its big advantage is very simple design and smaller computation complexity.

## 3.2 SOSD

The spatial order statistic detector (SOSD) was designed by using the order statistic detector (OSD) [7]. The sorted input vector not includes just the *k-th* sample of non-sorted input vector by

$${}_k\mathbf{z}_i = sort\{\mathbf{x}_i - \{x_i(k)\}\}, \tag{18}$$

where the vector ${}_k\mathbf{z}_i$ is computed for all input samples and $k$ is from $1$ to $N$. Two variations of SOSD were proposed, the SOSD1 and SOSD2. The different is only in the size of reduced ${}_k\mathbf{z}_i$ vector. The SOSD1 computes the reduced vector as follows

$${}_k\mathbf{zr}_i = \left( {}_k z_i\left(\frac{N+3}{4}\right), \ldots, {}_k z_i\left(\frac{3N-3}{4}\right) \right), \tag{19}$$

whereas the SOSD2 by equation

$${}_k\mathbf{zr}_i = \left( {}_k z_i(2), \ldots, {}_k z_i(N-2) \right). \tag{20}$$

Both SOSD1 and SOSD2 use the same decision rule:

$$\textbf{IF} \quad \left| mean\{{}_k\mathbf{zr}_i\} - x_i(k) \right| \geq Tol \quad \textbf{THEN} \; D_i(k)=1 \quad \textbf{ELSE} \; D_i(k)=0 . \tag{21}$$

## 3.3 SCOSD

The next developed spatial impulse detector is the spatial central order statistic detector (SCOSD). The SCOSD was derived from the central order statistic detector (COSD) [7]. This spatial detector employs only some samples of sorted input vector included with median sample.

As a SOSD detector family, the SCOSD has two modifications too. The SCOSD1 applies shorter sorted input vector considering to SCOSD2. Furthermore, the choice of $\mathbf{xr}_i$ vector length for SCOSD1 is defined as follows:

$$\mathbf{xr}_i = \left( {}_r x_i\left(\frac{N+3}{4}\right), \ldots, {}_r x_i\left(\frac{3N+1}{4}\right) \right). \tag{22}$$

For the SCOSD2, the reduced ordered input vector is defined by:

$$\mathbf{xr}_i = \left( {}_r x_i(2), \ldots, {}_r x_i(N-1) \right). \tag{23}$$

Finally, the SCOSD detectors are based on the following rule:

$$\textbf{IF} \quad \left| mean\{\mathbf{xr}_i\} - x_i(k) \right| \geq Tol \quad \textbf{THEN} \; D_i(k)=1 \quad \textbf{ELSE} \; D_i(k)=0 . \tag{24}$$

All of these spatial detectors operate in conjunction with law (16) and together they define the L-SD filter.

# 4. Experimental results

In this section will be described our experiments and achieved results. As training image was used the second frame of the Trevor sequence (Fig. 2a) that was corrupted by mixed noise (Fig. 2b). Moreover, the mixed noise consists of additive Gaussian white noise with standard deviation $\sigma$ = 20 and impulsive noise with probably $p$=10% (variable value noise).

Because of faster rate of convergence was found the optimal step-size $\mu_0$ for both L-NC filters. Thus, $\mu_0 \approx 0.1$ was used for all L-SD filters [4]. Spatial order statistic detectors replace the local signal-to-noise ratio measurement problem for L-SD filter described in [1]. Moreover, their optimal thresholds were detected. Filtration results were compared to the median and initial L-SD filters.

For objective comparison the mean absolute error (MAE), the mean square error (MSE), the noise reduction (NR) and the mean absolute error reduction (MAER) were used [1-2], [4].

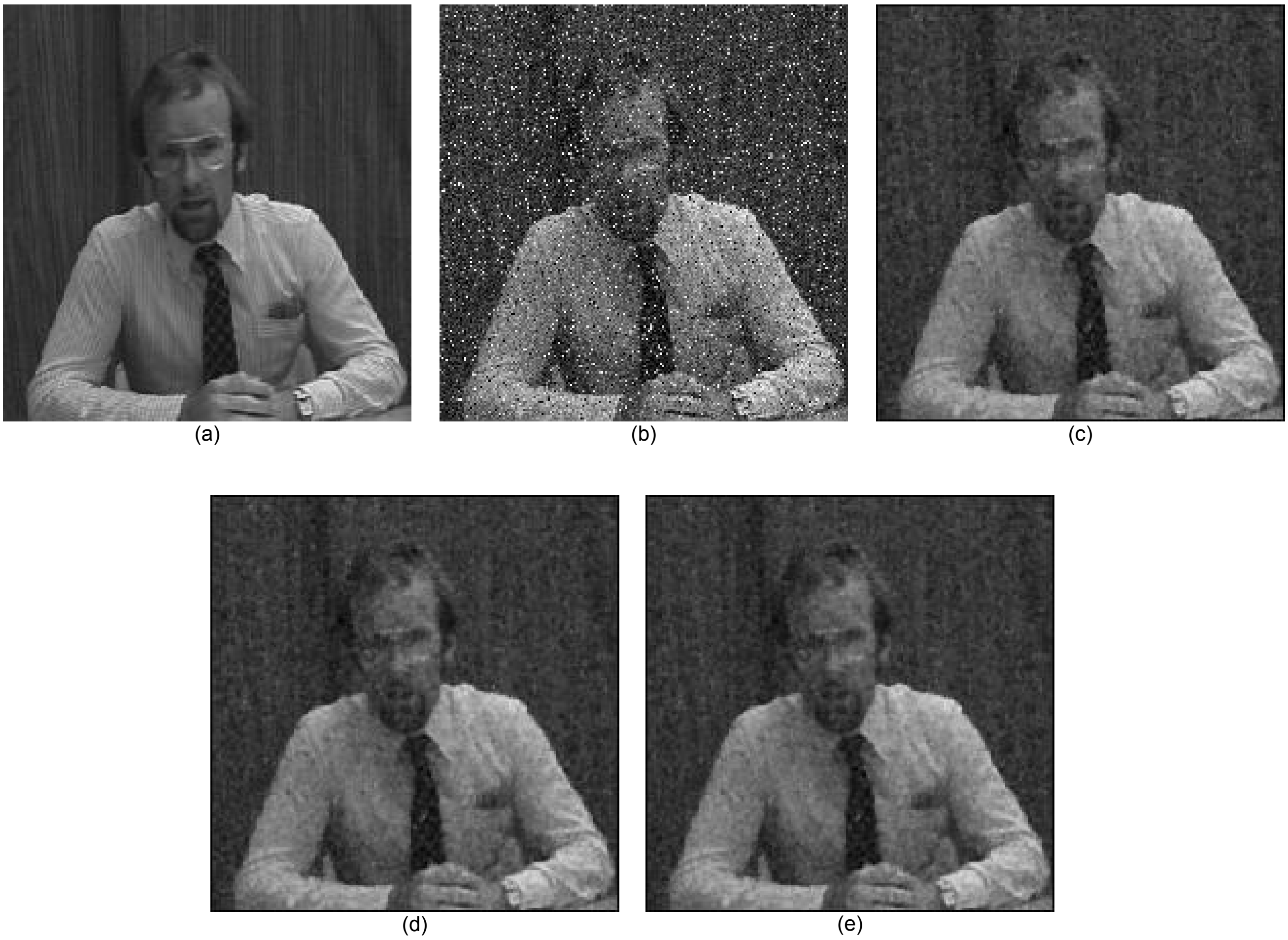


**Fig. 2** The 2nd frame of Trevor sequence. (a) Original Image, (b) Noisy image corrupted by mixed noise. Filtered image by (c) median filter (d) the adaptive L-SD filter, (e) the adaptive L-SD filter with SCOSD2 detector.

| Method | Performance indices | | | | |
|---|---|---|---|---|---|
| | MAE | MSE | NR | MAER | RANK |
| Noised | 21.489 | 1240.2 | - | - | - |
| Median | 8.156 | 113.01 | -10.403 | -8.414 | 7 |
| L-SD $\beta_t$=0.75 | 7.518 | 96.511 | -11.089 | -9.121 | 5 |
| L-SD-SMD threshold=64 | 7.441 | 94.726 | -11.170 | -9.211 | 3 |
| L-SD-SOSD1 threshold=19 | 7.542 | 97.481 | -11.045 | -9.094 | 6 |
| L-SD-SOSD2 threshold=19 | 7.514 | 96.514 | -11.089 | -9.126 | 4 |
| L-SD-SCOSD1 threshold=60 | 7.438 | 94.694 | -11.171 | -9.214 | 2 |
| L-SD-SCOSD2 threshold=62 | 7.436 | 94.642 | -11.174 | -9.216 | 1 |

**Tab. 1** The filter performance indices achieved for the $2^{nd}$ frame of Trevor sequence processing for $1^{st}$ detector level.

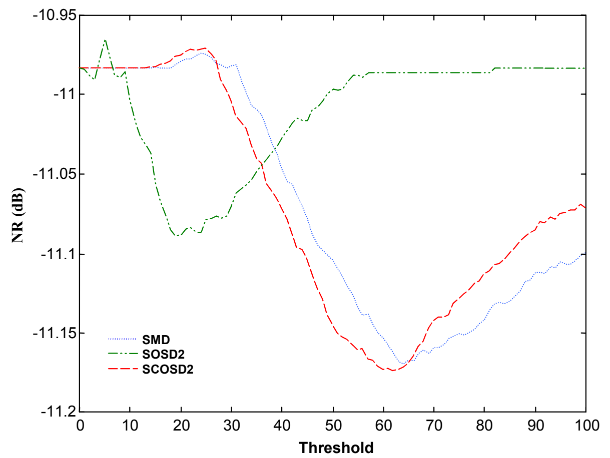


**Fig. 3** The NR dependence from threshold of spatial order statistic detectors at the $1^{st}$ detector level

The output from median filter is shown in the Figure 2c. For all filters, the same square 3 by 3 filter window was applied. Table 1 shows the results achieved by developed spatial detectors with best threshold.

Likewise, the Fig.3 shows the detailed NR dependence from SMD, SOSD2 and SCOSD2 thresholds.

The SCOSD2 detector achieved the best of results and the Fig.2e shows the filtered image. As can be seen, the improvement is only 0.1dB better than initial L-SD filter (Fig.2d). On the other hand, this result was not obtained by local signal-to-noise ratio measuring. These spatial detectors were successfully used for other images (e.g. Lena, Bridge).

# 5. Conclusion

In this paper several spatial order statistic detectors for filtering static images corrupted by mixed noise have been described. Furthermore, they were used as switch between two adaptive LMS L-NC filters. This class of detectors moves about the local signal-to-noise ratio measuring and offer the best filtration results than previous L-SD filter version. The SCOSD2 detector achieves the best of results in comparison with other designed spatial detectors. The better filtration results can be achieved by using other detector or filter shapes. It is well known that for maximal noise suppression in homogeneous regions is preferable larger filter window, but for edge preservation is preferable to employ only some pixels from filter window. Moreover, the application of some modified version of LMS algorithm improves the convergence properties and the filtration results too.

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