TRIGONOMETRIC APPROXIMATION IN FRACTAL IMAGE CODING

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Abstract

In this paper is presented a new approach in fractal image coding based on trigonometric approximation. The least square approximation method is used for approximation of blocks in standard fractal image compression algorithm. In the paper is shown that it is possible to use also trigonometric approximation for describing of blocks in fractal image coding. This approximation was implemented and analyzed from point of view of quality of reconstructed images. The experimental results of this method were tested on static grayscale images.

Keywords

fractal image coding, trigonometric approximation

1. Introduction

New and very popular method in image compression is fractal image coding. Algorithms for fractal image coding are based on the using of least square approximation method – linear model of polynomial approximation. Most decoding algorithms working on this linear model are iterative. At present a new approach based on trigonometric approximation is used in fractal image coding. Decoding of images by using proposed algorithm does not need iterative recursion.

2. Fractal image coding

Self-similarity, unique property of fractals, was introduced by Mandelbrot. This designation describes scale-similarity of objects. Fractals are hence scale-invariant objects. This property is interpreted from statistical point of view in fractal image coding. A fractal scheme has been developed by M. Barnsley. A. Jacquin, a former student of Barnsley´s, was the first who published a fractal image compression scheme. His algorithms were established on linear model of least square approximation and these algorithms are considered as standard algorithms.

First step of fractal image coding algorithm (as block-orientated lossy image compression) is dividing the image into blocks - non-overlapped R_i (Range) blocks that cover all image and overlapped D_i (Domain) blocks. The size of D_i blocks is bigger than size of R_i blocks.

Methods of image partitioning can be classified as [2]

- image-independent tiling,
- image-dependent segmentation.

Image-independent tiling is a partitioning scheme that does not take the structure of the image, but instead uses simple geometric shapes to tile the image. This method has the advantage of simplicity: shapes of regions can be simple, and data can be structured simply, so the decompressor (decoding algorithm) can be easy implemented.

Image-dependent segmentation is characterized by more complicated implementation, using of this approach achieves smaller distortion of decoded image. To image-dependent segmentation include

- horizontal-vertical partition,
- quadtree partition,
- Dealaunay triangulisation,
- segmentation into Voronoi regions, etc.

In the next step for each R_i block the most similar D_i block is searched. Due to size of D_i blocks which is bigger than size of R_i blocks we have to reduce the size of D_i to equal size of R_i blocks by using contraction. Reduced D_i block is denoted as B_i

$$B_i = \nu(D_i), \tag{1}$$

where *v(.)*- is operation of contraction e.g. subsampling or averaging of D_i block.

After contraction of D_i blocks the most similar B_i block is searched for each R_i block by using of metrics which describes distance between R_i and B_i blocks.

The Euclidian distance is used for calculation of distance. It means that parameter d in the following equation should be minimal for most similar blocks

$$d(R,B) = \sqrt{\sum_{i=1}^{n} (r_i - b_i)^2} \quad , \tag{2}$$

where r_i and b_i are elements of R_i block and B_i block resp.

The basic relation between $\underline{R_i}$ and B_i blocks expresses following equation

$$\underline{R}_i = s.\underline{B}_j + o.\underline{1} \;, \tag{3}$$

where s and o denote parameter for contrast and brightness transformation respectively.

From another point of view parameter s can be interpreted as measure of similarity between blocks and parameter o as an error of approximation.

Error of contraction E is described with Euclidian distance between blocks in following form

$$E = \sqrt{\sum_{i=1}^{n} \left[r_i - (s.b_i + o) \right]^2} \tag{4}$$

Products of minimization error E by partial derivatives

$$\frac{\partial E^2}{\partial o} = -2 \sum_{i=1}^{n} (r_i - s.b_i - o) = 0 \tag{5}$$

$$\frac{\partial E^2}{\partial s} = -2 \sum_{i=1}^{n} (r_i - s.b_i - o).b_i = 0 \tag{6}$$

is linear equation system

$$\begin{bmatrix} n & \sum_{i=1}^{n} b_i \\ \sum_{i=1}^{n} b_i & \sum_{i=1}^{n} b_i^2 \end{bmatrix} . \begin{bmatrix} o \\ s \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} r_i \\ \sum_{i=1}^{n} r_i b_i \end{bmatrix} . \tag{7}$$

Solving (7), parameter for contrast transformation (8) and brightness one (9) are found [1]

$$s = \frac{\left[n \sum_{i=1}^{n} r_i b_i - \sum_{i=1}^{n} r_i b_i \right]}{\left[n \sum_{i=1}^{n} b_i^2 - \left(\sum_{i=1}^{n} b_i \right)^2 \right]} \tag{8}$$

$$o = \frac{1}{n} \left[\sum_{i=1}^{n} r_i - s. \sum_{i=1}^{n} b_i \right] \tag{9}$$

The aim of comparison R_i and D_i blocks is to find scale-invariant blocks in image, i.e. for each Range block to find the most similar Domain block.

For each R_i block we save

- parameters for position of found Domain block;
- parameters for contrast and brightness transformation;
- information of used isometry of Domain block.

Fractal image compression takes a long time, because it is necessary for each Range block to find the most similar Domain block. If also isometric transformations of each Domain block are considered, compression time is very long. Therefore in practical implementations, (insufficient quality of reconstructed images), isometric transformations aren't considered and output of fractal image coding acquires a form shown on following figure (Fig. 1).

x_1	y_1	s_1	o_1
x_2	y_2	s_2	o_2
	$\vdots$		
x_n	y_n	s_n	o_n

Fig. 1 An output of fractal image coding

Decoding of original image is an iterative process of reconstruction $\hat{R}_i$ blocks from the set of fractal coding parameters by using Fig.1. The decoding process of each R_i block can be described in following form

$$\begin{aligned} R_i^1 &= s.B_j + o.\underline{1} \\ R_i^2 &= s.(s.B_j + o.\underline{1}) + o.\underline{1} \\ &\vdots \\ R_i^k &= s^k.B_j + \left(o \sum_{p=0}^{k-1} s^p \right).\underline{1} \end{aligned} \tag{10}$$

Start condition for decoding algorithm is a zero matrix (black image). Result of decoding process after k-th iteration is the reconstructed image called attractor.

3. Trigonometric approximation

An alternative way in fractal image coding is the approximation based on trigonometric polynomials [3]

$$R_i = \frac{a_0}{2} + \sum_{j=1}^{n} \left(a_j \cos jB_i + b_j \sin jB_i \right). \tag{11}$$

For practical implementation suffices case $j = 1$ then we obtain following formula

$$R_i = \frac{a_0}{2} + a_1 \cos B_i + b_1 \sin B_i \;, \tag{12}$$

where approximation coefficients are [4]

$$\begin{bmatrix} n & \sum_{i=1}^{n} \cos b_i & \sum_{i=1}^{n} \sin b_i \\ \sum_{i=1}^{n} \cos b_i & \sum_{i=1}^{n} \cos^2 b_i & \sum_{i=1}^{n} \cos b_i \\ \sum_{i=1}^{n} \sin b_i & \sum_{i=1}^{n} \cos b_i \sin b_i & \sum_{i=1}^{n} \sin^2 b_i \end{bmatrix} . \begin{bmatrix} \frac{a_0}{2} \\ a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} r_i \\ \sum_{i=1}^{n} r_i \cos b_i \\ \sum_{i=1}^{n} r_i \sin b_i \end{bmatrix}$$

Using trigonometric approximation changes fractal output code. It is necessary to save computed approximation coefficients of block. Then fractal output code obtains following form (Fig.2)

$$\begin{matrix} x_1 & y_1 & \left(\frac{a_0}{2}\right)_1 & (a_1)_1 & (b_1)_1 \\ x_2 & y_2 & \left(\frac{a_0}{2}\right)_2 & (a_1)_2 & (b_1)_2 \\ & & & & \\ x_n & y_n & \left(\frac{a_0}{2}\right)_n & (a_1)_n & (b_1)_n \end{matrix}$$

Fig. 2 Modified fractal output code

4. Experimental results

Trigonometric approximation in fractal image coding was implemented and compared with standard (Jacquin´s) method. Experiments were orientated on testing of image quality and compression ratio. Quantitative measure of image quality was analyzed with Peak-Signal-to-Noise-Ratio

$$PSNR = 10\log_{10}\frac{255^2}{\left(\frac{1}{N_1.N_2}\right)\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}\left(Original(i,j) - Attractor(i,j)\right)^2}$$

and compression ratio

$$C_r = \frac{N_1}{N_2} ,$$

where N_1 and N_2 are number of bits of image before and after compression respectively.

Proposed algorithm was tested on static grayscale images GIRL, CLAUDIA, EINSTEIN and ZEBRA in pgm format; some reconstructed images show Fig.3 – Fig.6.

Image	*Polynomial approximation*		*Trigonometric approximation*	
	PSNR [dB]	C_r	*PSNR* [dB]	C_r
Girl	28,191	4		
			26,716	5,33
Girl	25,990	**16**	26,710	**16**
Claudia	29,841	4		
			28,040	5,33
Claudia	27,403	**16**	28,039	**16**
Einstein	26,044	4		
			23,772	5,33
Einstein	23,635	**16**	23,770	**16**
Zebra	21,552	4		
			18,885	5,33
Zebra	18,537	**16**	18,881	**16**

Original image
GIRL

Fractal image coding
polynonomial approximation
$C_r = 4$
PSNR = 28,19 dB

Fractal image coding
trigonometric approximation
$C_r = 5,33$
PSNR = 26,72 dB

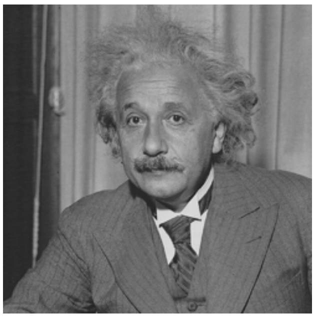
Fig. 3 Comparing of the methods - Girl image

Original image
CLAUDIA

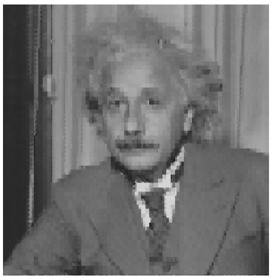
Fractal image coding
polynonomial approximation
$C_r = 4$

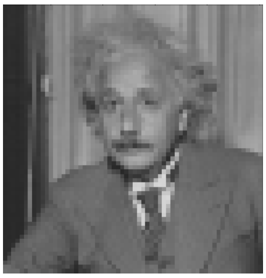
Fractal image coding
trigonometric approximation
$C_r = 5,33$

Fig. 4 Comparing of the methods - Claudia image



Original image
EINSTEIN

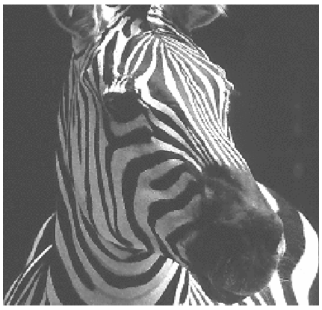




Fractal image coding
polynonomial approximation
$C_r = 4$
PSNR = 26,04 dB

Fractal image coding
trigonometric approximation
$C_r = 5{,}33$
PSNR = 23,77 dB

Fig. 5 Comparing of the methods - Einstein image



Original image
ZEBRA

Fractal image coding
polynonomial approximation
$C_r = 4$

Fractal image coding
trigonometric approximation
$C_r = 5{,}33$

Fig. 6 Comparing of the methods - Zebra image

5. Conclusion

The using of trigonometric approximation is an alternative way in fractal image compression. The using of proposed method eliminates an iterative process in decoding.

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Marek ČANDÍK was born in Košice, Slovakia, 1974. He graduated from the Technical University (TU) in Košice, 1997, then he started Ph.D. study at Department of Electronics and Multimedia Communications, Faculty of Electrical Engineering and Informatics, TU of Košice. His work includes digital image processing and transmission.