VOLTERRA FILTERING FOR ADC ERROR CORRECTION

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Abstract

Dynamic non-linearity of analog-to-digital converters (ADC) contributes significantly to the distortion of digitized signals. This paper introduces a new effective method for compensation such a distortion based on application of Volterra filtering. Considering an a-priori error model of ADC allows finding an efficient inverse Volterra model for error correction. Efficiency of proposed method is demonstrated on experimental results.

Keywords

Analog-to-digital converters, Volterra filtering, dynamic non-linearity correction

1. Introduction

Improvement of digitization chain performance is constantly in the center of many scientific studies. The crucial part of digitization systems is the Analog-to-Digital Converter (ADC) or the system of a Sample and Hold Circuit and an Analog-to-Digital Converter (S/H-ADC).

A big effort has been devoted to study of ADC error modeling on dependence of the particular ADC architecture [1]. Typically, the non-idealities of an ADC are predominated by its dynamic non-linear behavior. The main motivation for study of ADC error models is to reduce the ADC distortion. The main effort of many studies is developing a method, which enables compensation of ADC nonlinearity by post inverse “distortion” of recorded digitized samples [2-6].

The simplest approach is to subtract the value of integral non-linearity function *INL*(k) from each output code $k(n)$. In this case, the *INL* is considered as a one-dimensional function of output code k, and it describes the static features of ADC, i.e. the behavior of the ADC with constant or slowly varied input signal. For proper characterization of ADC behavior with higher frequency input signal, a more complex model is introduced (phase plane approach), where a two-dimensional *INL* is considered, which is the function of output code k, and time slope s of the input signal [3]. Error correction is then achieved by subtracting the corresponding value of the phase plane from the output code, where the time slope can be estimated from values of previous and subsequent samples. A similar method uses current sample-previous sample phase plane [4].

2. Volterra filtering for ADC error correction

Conceptually different methods utilize mathematical models of dynamic non-linear system, such as Volterra or Wiener models [5-7], [11]. The Volterra model is an exact mathematical approach for description of causal time-invariant systems, where dynamic and non-linear phenomena are present simultaneously [7]. According to this model, the output signal of the non-linear system can be expressed as series of Volterra functionals [5-12]:

$$\hat{y}(t) = h_0 + \sum_{i=1}^{\infty}\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\dots\int_{-\infty}^{\infty} h_i(\mu_1,\mu_2,\dots,\mu_i)\prod_{j=1}^{i}x(t-\mu_j)\prod_{j=1}^{i}d\mu_j\right\} \quad (1)$$

where $x(t)$ is the input signal, $\hat{y}(t)$ is the output signal, and $h_i(\mu_1, \mu_2, \dots, \mu_i)$ is the Volterra kernel of the i-th order.

Several studies have been devoted to Volterra kernel identification [5-7], [11,12]. In the conventional methods, usually an adaptive Volterra filter is utilized in a configuration called identifier of non-linear systems (Fig. 1) [5], [8-10]. Once adapted, the filter provides the Volterra kernels modeling a specific non-linear system under test.

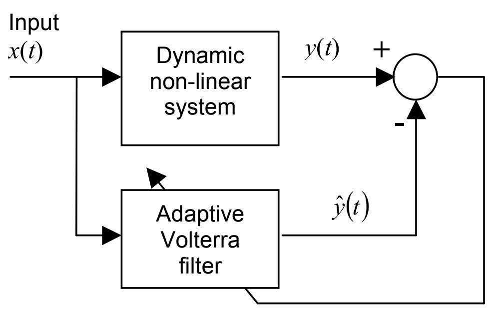

Fig. 1 Volterra kernel identification by an adaptive method

From the measured kernels, the inverse Volterra kernels can be derived analytically in order to determine an inverse model of the system [5]. This allows the non-linear system error to be corrected according to the inverse-model, by applying post-distortion. Inverse Volterra kernels can be also obtained directly by using the configuration presented in Fig. 2.

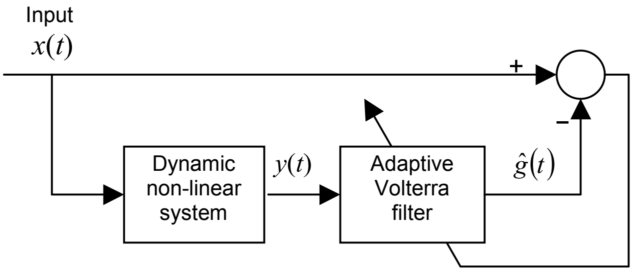

Fig. 2 Inverse Volterra kernel identification by an adaptive method

However, the principle of non-linear system identification by means of adaptive Volterra filters requires a stationary input signal [5], [8], [10]. This causes the two-fold problems of trying to find a proper calibration signal. The calibration signal has to:

(i) map the phase plane *k*, *s* and the "memory" plane (e.g. the state-previous state *k*, *k*-1 plane) simultaneously,

(ii) be stationary over the adaptation process in order to have invariant conditions during the adaptation.

Moreover, this signal should be generated with sufficient accuracy with relation to the tested ADC. Generation such a signal establishes a serious problem.

2.1 Volterra kernel identification using deterministic calibration signal

For reasons described above, a different approach was proposed by application of a known deterministic calibration signal on the input of the tested ADC (Fig. 3) [11].

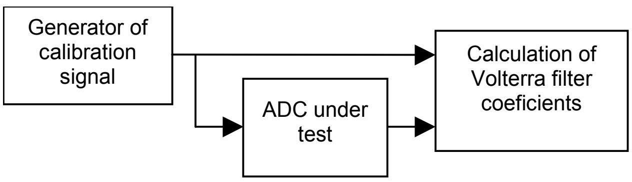

Fig. 3 Direct Volterra kernel calculation

Volterra filter coefficients were calculated on the base of the sequence of input signal values in the sampling instants, and the corresponding sequence of distorted output codes of the ADC, according to the least-mean-squares error optimization criterion. As calibration signal a sequence of low amplitude sinusoidal waveforms was used, superimposed on DC level varying by steps smaller than the double of sinewave amplitude, thus gradually mapping the whole ADC input range. The problems concerning calibration signal selection and generation can be easily overcome because of possibility to generate simply the sinewave with high accuracy. Moreover, the adaptive Volterra filter is not required, since the time invariant Volterra filter coefficients are obtained directly.

2.2 Volterra filter kernels calculation

The discrete equivalent of the general formula (1) is according to [8-10]:

$$\hat{y}(n) = h_0 + \sum_{i=1}^{\infty}\sum_{m1=0}^{\infty}\sum_{m2=0}^{\infty}\cdots\sum_{mi=0}^{\infty} h_{i_{m1,m2,\dots,mi}} \prod_{j=1}^{i} x(n-mj) \tag{2}$$

where $x(n)$ is the discrete input signal, $\hat{y}(n)$ is the filter output sequence, and h_i is the element of i^{th} order Volterra kernel. The equation (2) is the mathematical model of a Volterra filter of infinite order. In particular, the truncated model with a finite order M, and a finite memory of samples N+1 is considered:

$$\hat{y}(n) = h_0 + \sum_{i=1}^{M}\sum_{m1=0}^{N}\sum_{m2=0}^{N}\cdots\sum_{mi=0}^{N} h_{i_{m1,m2,\dots,mi}} \prod_{j=1}^{i} x(n-mj) \cdot \tag{3}$$

Organizing the Volterra kernel elements h_0 and h_i, and the input sample products

$$\prod_{j=1}^{i} x(n-mj)$$

into convenient block vectors $\mathbf{H}(n)$ and $\mathbf{X}(n)$, respectively, the (3) can be rewritten as:

$$\hat{y}(n) = \mathbf{H}^T(n)\mathbf{X}(n) = \mathbf{X}^T(n)\mathbf{H}(n) \tag{4}$$

The design of an optimal Volterra filter requires the search for a set of Volterra kernels that minimizes the error function between the desired signal $y(n)$, and the filter output $\hat{y}(n)$, according to a given optimization criterion. Mean-square error optimization criterion can be considered:

$$E\left[e^2(n)\right] = E\left[\left(y(n) - \hat{y}(n)\right)^2\right] \rightarrow \min \tag{5}$$

The solution leads to the equation:

$$\mathbf{R}_{XX}\mathbf{H}^* = \mathbf{R}_{XY}\,, \tag{6}$$

providing that

$$\det \mathbf{R}_{XX} \neq 0, \tag{7}$$

$$\mathbf{H}^* = \mathbf{R}_{XX}^{-1}\mathbf{R}_{XY} \tag{8}$$

where: $\mathbf{H}^*$ is the vector of the optimum Volterra kernels (optimal filter coefficients),

$$\mathbf{R}_{XY} = E\left[y(n)\mathbf{X}(n)\right] \tag{9}$$

is the higher-order mutual correlation vector of the input and the desired signals, and

$$\mathbf{R}_{XX} = E\left[\mathbf{X}(n)\mathbf{X}^T(n)\right] \tag{10}$$

is the higher-order autocorrelation matrix of input signal [8], [10]. Although $\mathbf{R}_{XX}$, and $\mathbf{R}_{XY}$ are statistical characteristics, primarily defined on the base of the statistical properties of the input and the desired signals, their value can be estimated from the finite sequence of deterministic input values and corresponding output codes:

$$\mathbf{R}_{XY} \approx \frac{1}{L}\sum_{n=1}^{L} x(n)\mathbf{X}(n) \tag{11}$$

$$\mathbf{R}_{XX} \approx \frac{1}{L}\sum_{n=1}^{L} \mathbf{X}(n)\mathbf{X}^{T}(n) \tag{12}$$

where $\mathbf{X}(n)$ is the vector of output code products obtained similarly as in (4), L is the length of the data sequence.

2.3 Verification of method efficiency

Method efficiency is evaluated in terms of signal-to-noise ratio by applying a test sinewave signal according to Fig. 4. The signals from tested ADC before and after Volterra filtering are recorded. Then the ideal signal is computed by four parameters best fitting method (sine-wave amplitude, DC offset, phase and normalized signal frequency to sampling frequency) [16]. Subtracting the calculated ideal sine-wave and recorded data the additive noise is determined and consecutively analyzed.

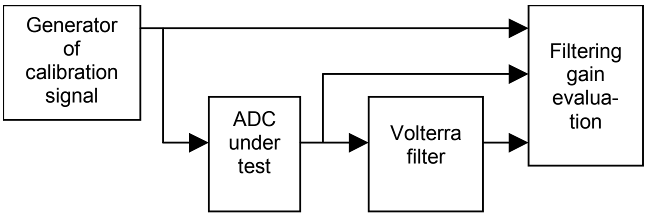

Fig. 4 Verification of developed method efficiency

3. Modification of Volterra series considering an a-priori ADC

Mathematical complexity of Volterra series approach is a cost for its general ability of dynamic nonlinear system description. In many cases, this is a serious limit of practical application. However, knowledge of structure of the system, where Volterra filtering has to be applied, sometimes allows assuming symmetry properties of Volterra kernels, and on this base simplifying the Volterra kernels. In the following section, modified Volterra filter expression is derived by exploiting the mathematical knowledge of the error in a given ADC architecture [1], [2].

The method can be applied for ADC error correction, when the integral non-linearity $INL_Q(k,s)$, considered in terms of a two-dimensional error function of the output code k and the input slope s presents a smooth surface and can be modeled with a low-order polynomial. This is particularly fulfilled for integrating ADCs (IADCs) [1], [3], [4]. In typical IADCs, INL_Q error function can be modeled [1]:

$$INL_Q(k,s) = B_0 + B_1 k + B_2 k^2 + B_3 s + B_4 ks + B_5 s^2 + \ldots \tag{13}$$

Considering order of filter M=2, and a memory of samples N+1=3, according to the conventional modified Volterra model [8] based on equation (3) we obtain:

$$\begin{aligned}\hat{y}(n) &= s_0 + s_{1_0}k(n) + s_{1_1}k(n-1) + s_{1_2}k(n-2) + \\ &+ s_{2_{00}}k^2(n) + s_{2_{01}}k(n)k(n-1) + s_{2_{02}}k(n)k(n-2) + \\ &+ s_{2_{11}}k^2(n-1) + s_{2_{12}}k(n-1)k(n-2) + s_{2_{22}}k^2(n-2)\end{aligned} \tag{14}$$

where the subscripted coefficients s denote conventional modified Volterra kernels.

Since $ING_Q(k, s) \approx \Delta k = k - k_{id}$ [1], where k_{id} is the k-th ideal code, thus $k_{id} \approx k - ING_Q(k, s)$. On this basis, the mathematical model of the Volterra filter is expressed in the following form:

$$\begin{aligned}\hat{y}(n) \overset{!}{=} k_{id}(n) &= -B_0 + (-B_1 + 1)k(n) - B_2 k^2(n) - \\ &- B_3 s(n) - B_4 k(n)s(n) - B_5 s^2(n) - \ldots\end{aligned} \tag{15}$$

The slope is estimated by central difference equation [3]:

$$s(n) = [k(n+1) - k(n-1)]/2T_S\ , \tag{16}$$

where T_S is the sampling period.

Using the substitution n=m-1, the second-order Volterra series corresponding to the following inverse non-linear model $\mathbf{G'}_n$ can be stated:

$$\begin{aligned}\hat{y}(m-1) &= g_0 + g'_{1_0}k(m) + g'_{1_1}k(m-1) + \\ &+ g'_{1_2}k(m-2) + g'_{2_{00}}k^2(m) + \\ &+ g'_{2_{01}}k(m)k(m-1) + g'_{2_{02}}k(m)k(m-2) + \\ &+ g'_{2_{11}}k^2(m-1) + g'_{2_{12}}k(m-1)k(m-2) + g'_{2_{22}}k^2(m-2)\end{aligned} \tag{17}$$

Substituting (16) to (17), assuming from symmetry that:

$$g'_{1_0} = -g'_{1_2},\ g'_{2_{01}} = -g'_{2_{12}},\ g'_{2_{00}} = -g'_{2_{02}}/2 = g'_{2_{22}}, \tag{18}$$

the following equation is obtained:

$$\begin{aligned}\hat{y}(m-1) &= g'_0 + g'_{1_1}k(m-1) + 2T_S g'_{1_0}s(m-1) + \\ &+ g'_{2_{11}}k^2(m-1) + 2T_S g'_{2_{01}}k(m-1)s(m-1) + \\ &+ 4T_S^2 g'_{2_{00}}s^2(m-1)\end{aligned} \tag{19}$$

After further modification, it is obtained:

$$\begin{aligned}\hat{y}(n) &= g_0 + g_1 k(n) + g_2 s(n) + g_3 k^2(n) + \\ &+ g_4 k(n)s(n) + g_5 s^2(n)\end{aligned} \tag{20}$$

Comparing (15) and (20), eqn. (20) provides sought formula for the Volterra filter necessary for the correction of an ADC according to the inverse-model principle. Vector $\mathbf{X}(n)$ takes the following form:

$$\mathbf{X}(n) = \left[1, k(n), s(n), k^2(n), k(n)s(n), s^2(n)\right]^T \tag{21}$$

$\mathbf{R}_{XX}$, and $\mathbf{R}_{XY}$ are calculated according to (11), (12). The vector of the optimal filter coefficients is obtained analogously as in (8):

$$\mathbf{G} = [g_0, g_1, g_2, g_3, g_4, g_5]^T = \mathbf{R}_{XX}^{-1} \mathbf{R}_{XY} \tag{22}$$

The final filter expression for corrected output calculation is the following:

$$\hat{y}(n) = \mathbf{G}^T \mathbf{X}(n) \tag{23}$$

4. Experimental results

For experimental verification of the method, the following configuration was chosen: The non-ideal ADC is substituted by an analogue circuit modelling dynamic non-linearities, and a subsequent "almost ideal" ACD. (Fig. 5). The “almost ideal” ADC was implemented by using a precise 12-bit ADC and considering the upper 9 bits only.

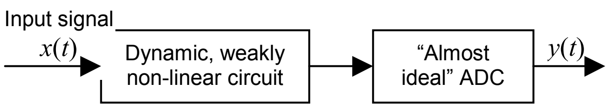

Fig. 5 Realized model of non-ideal ADC

This method allows including a controlled distortion, and studying the filtering effects on a physical model with known properties. The measurement was realized in configuration according to Fig. 6.

Both calibration, and filter efficiency verification signals were generated with precision direct digital synthesis generator (HP – Agilent). The dynamic non-linear circuit was created using TL071 operational amplifier with non-linear elements in the feedback loop. For data acquisition a National Instruments AT-MIO-16E-10 multifunction I/O board was used.

The needed testing and data processing software was developed partially in software developing environment LabWindows/CVI (National Instruments) and partially in Matlab.

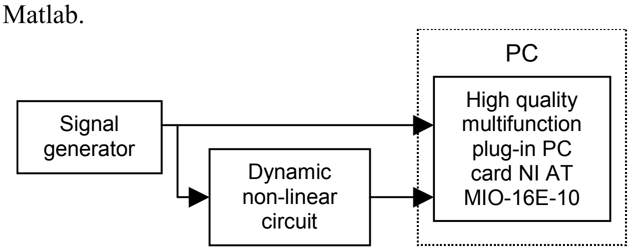

Fig. 6a Configuration of test setup

The calibration signal was created as a sequence of sinusoidal signals of frequency 5 Hz, and amplitude 1V superimposed on DC level that gradually varied from -4V to +4V with step 1V. Actual values on the input were estimated according to four-parametric method of best-fitting sine-wave [15], [16]. Volterra filter coefficients were calculated according to (22), and are presented in Tab. 1.

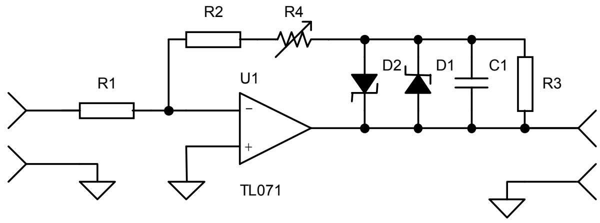

Fig. 6b Applied dynamic non-linear circuit

Filtering effectiveness is demonstrated on sinusoidal test signal with frequency 6.4 Hz, amplitude 2.5V, and offset -2V. Figure 7a shows a detail from noise before filtering, i.e. the difference signal between best fitting sine-wave estimation, and measured data on the output in the time domain. Figure 7b shows noise after filtering, i.e. the difference signal between input, and filtrated output.

Coefficient	Value
0	-0.50842288904812
1	+0.99505439577477
2	+0.12137028370907
3	+0.00000033775896
4	-0.00000477141689
5	+0.00134132122041

Tab. 1 Correcting Volterra filter coefficient calculated from results of measurement

The main improvements are

(i) decreasing noise peak value almost 2 times
(ii) a notable correction of DC offset error

Precise analysis of data before and after filtering indicates that the filtering improved signal-to-noise ratio by 6.5dB. The exact points of signal improvements can be found out from the spectrum after transformation the experimental results into frequency domain (Fig. 8).

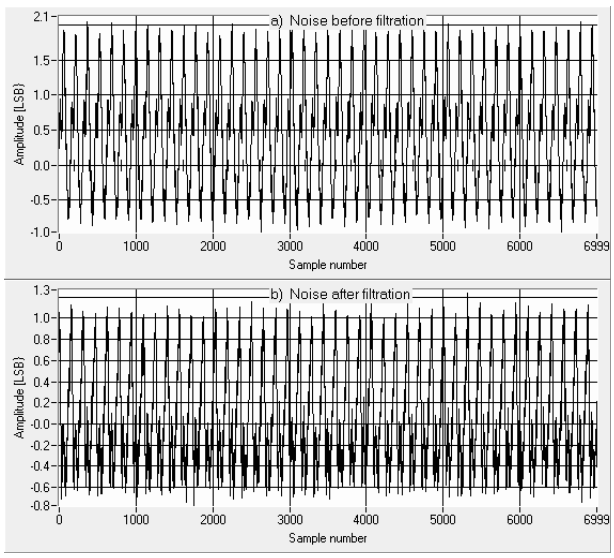

Fig. 7 Details of experimental results in time domain: a) noise before filtering, b) noise after filtering

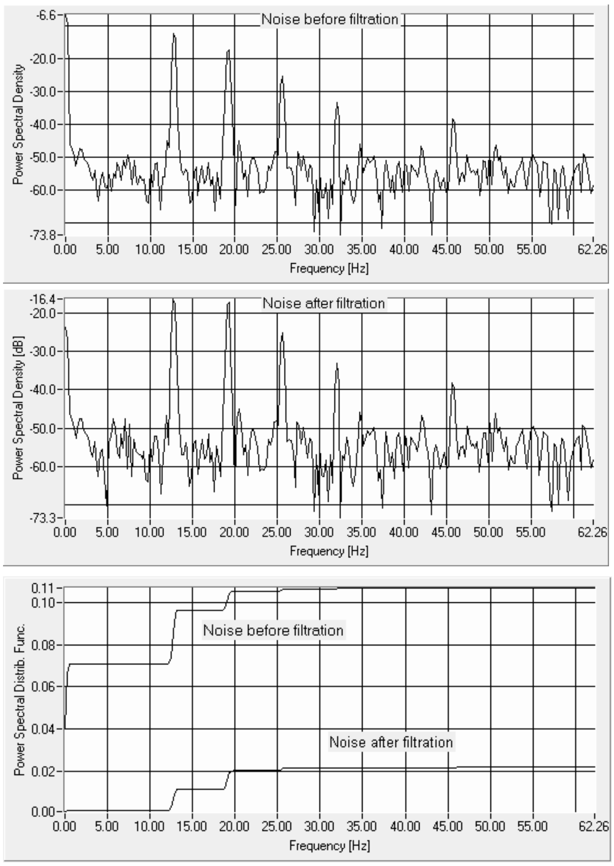

Fig. 8 Details of experimental results in frequency domain: power spectral densities and power spectral distribution functions of noise before and after filtering

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Experimental results transformed into frequency domain are shown in Fig. 8. The additive noise before and after filtering is expressed in form of power spectral density function and power spectral distribution function. Especially the power spectral distribution function very clearly indicates the advance in signal after filtration. The most evident improvements are:

- compensation of DC offset by 15.5 dB,
- decreasing distortion caused by the second harmonic (12.8 Hz) by 4.1 dB.

5. Conclusions

A new method based on Volterra filtering for inverse-model correction of the ADC error has been proposed. The Volterra filter models effects of dynamic non-linear error. The method can be considered as the next evolution step of the a-priori approach to ADC error modelling [1-2], [11], [14] toward to application of an error correction. The a-priori approach makes the analytical derivation of filter expression easy by taking specific characteristics of the ADC error into account. This allows:

(i) a more compact expression to be used in model definition
(ii) avoiding to a complicate adaptive scheme in model determination (in each adaptation step a non-linear equation system had to be solved),
(iii) easy computation of the Volterra filter coefficients in filter identification,
(iv) using simpler signal in experimental calibration.

The theoretical considerations were verified on real experimental stand. Volterra kernels of a non-linear circuit, simulating the behavior of an IADC, were measured, and consequently efficiency of the developed method was evaluated by applying the measured kernels for error correction on test signal. A significant performance improvement in terms of signal-to-noise ratio was achieved.

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