# ANALYSIS OF APERTURE-COUPLED MICROSTRIP ANTENNA USING METHOD OF MOMENTS

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## Abstract

A microstrip patch antenna that is coupled to a microstripline by an aperture in the intervening ground plane is analyzed by using the method of moments. Integral equation is formulated by considering the exact dyadic Green's function in spectral domain for grounded dielectric slab so that the analysis includes all coupling effects and the radiation and surface wave effects of both substrates. The combination of the reciprocity method analysis and a Galerkin moment method solution seems to be suitable for a number of planar antenna problems, especially when coupling slots in the ground plane are included. Results for antenna input impedance are compared with other authors and verified by experimental results.

## Keywords

Microstrip antenna, method of moments, reciprocity theorem

## 1. Introduction

Microstrip antennas are commonly fed by one of three methods: coaxial probe, microstripline connected directly to the edge of a patch, and stripline coupled to the patch through an aperture in the ground plane [1]. The latter method was first proposed by Pozar in 1985 and it becomes very popular in recent years as a method for feeding planar phased arrays in millimeter wave band. Some of its advantages are as follows: a) the feed network is isolated from the radiating element by ground plane that prevents spurious radiation, b) active devices can be fabricated in a feed substrate with high dielectric constant for size reduction, c) the radiating element can be placed on a thick substrate with low dielectric constant in order to achieve higher bandwidth.

The method of analysis of aperture coupled microstrip patches presented here is basically a full-wave method,

which uses exact Green's functions in spectral domain in order to find necessary field components from electric and magnetic currents in the presence of a grounded dielectric slab. Reciprocity theorem [2] is used to derive expressions for the amplitudes of reflected and transmitted waves on the microstrip feeding line, and equivalent circuit representing the slot discontinuity is found.

The combination of the reciprocity analysis and a moment method solution is very versatile technique that should find application in a number of planar antenna problems. This method avoids the more "brute-force" approach of modeling the nonuniform currents on a feed line, as was done in [3].



Fig. 1 Geometry of an aperture coupled microstrip patch antenna

## 2. Theory

The geometry of the aperture coupled microstrip patch antenna is shown in Fig. 1. Assuming a microstrip patch antenna with element size PL, PW placed on the surface of the antenna substrate and a coupling aperture with size L, W in the ground plane located below the centre of the patch. Feed line of width  $W_f$  is placed on the surface of the feed substrate on the opposite side of the ground plane, along the axis of the patch.

The metal surfaces are assumed to be made of a perfect conductor. The dielectric losses can be readily accounted by introducing complex permittivity

$$\widetilde{\varepsilon}_r = \varepsilon_r (1 - j \tan \delta)$$

## 2.1 Reciprocity analysis

The feed microstripline is assumed to be infinitely long and propagating a quasi TEM mode with fields [2]

$$\vec{E}^{\pm} = \vec{e}(y, z) e^{\pm j\beta x}, \qquad (1a)$$

$$\vec{H}^{\pm} = \pm \vec{h}(y, z) e^{\pm j\beta x}, \qquad (1b)$$

where  $\vec{e}$ ,  $\vec{h}$  are normalized transverse modal fields and  $\beta$  is the propagation constant of the line. If the aperture is centred at x = 0, total microstrip line fields can be written as

$$\vec{E} = \begin{cases} \vec{E}^+ + R\vec{E}^-, & x < 0\\ T\vec{E}^+, & x > 0 \end{cases}$$
(2)

$$\vec{H} = \begin{cases} \vec{H}^{+} + R\vec{H}^{-}, & x < 0\\ T\vec{H}^{+}, & x > 0 \end{cases}$$
(3)

where R and T are the voltage reflection and transmission coefficients on the line, respectively. Unknown aperture electric field is taken as a piecewise sinusoidal (PWS) mode of the form

$$V_0 e_x^a(x, y) = V_0 \frac{\sin k_e (L/2 - |y|)}{W \sin k_e L/2},$$
(4)

for |x| < W/2, |y| < L/2. In (4)  $V_0$  is the unknown amplitude of the aperture field, and  $k_e = k_0 [(\varepsilon_r + 1)/2]^{1/2}$  is the effective wavenumber in the substrate. The total power flow through the aperture can be evaluated as

$$\Delta v = \int_{S_a} e_x^a(x, y) h_y(x, y) ds$$
<sup>(5)</sup>

where  $S_a$  is the aperture surface. Applying the reciprocity theorem the reflection and transmission coefficients can be found

$$R = 0.5V_0 \,\Delta v \,, \ T = 1 - R \,. \tag{6}$$

Now define a Green's function  $G_{yy}^{HM}$  to account for the  $H_y$  fields on both sides of the aperture (z = 0) due to a y directed magnetic aperture current. An external admittance of the slot seen by the feed line can be written as

$$Y^{e} = \iint_{S_{a}S_{a}} e_{x}^{a}(x, y) G_{yy}^{HM}(x, y; x_{0}, y_{0}) e_{x}^{a}(x_{0}, y_{0}) ds ds_{0}.$$
 (7)

Expressions for the unknowns  $V_0$  and R can be written as

$$V_{0} = \frac{2\Delta v}{\Delta v^{2} + 2Y^{e}}, \ R = \frac{\Delta v^{2}}{\Delta v^{2} + 2Y^{e}}$$
(8), (9)

The slot discontinuity appears as a simple series impedance Z to the microstripline, that can be found as

$$Z = Z_c \frac{2R}{1-R} = Z_c \frac{\Delta v^2}{Y^e}$$
(10)

where  $Z_c$  is characteristic impedance of the microstripline.

## 2.2 Moment method formulation

Following the standard moment method procedure, the aperture field and surface patch currents are expanded in a set of N PWS modes and matrix equations are solved for unknown complex expansion coefficients [4].

Considering only equivalent admittance of the aperture seen by the microstripline and enforcing continuity of  $H_y$  through the aperture one can write the following matrix equation [2]

$$\mathbf{Y}^{e}\mathbf{V} = (1-R)\Delta\mathbf{v}, \qquad (11)$$

where **V** is column vector of aperture field expansion coefficients. From (11) R and consequently Z can be found using (10). Components of matrix **Y**<sup>*e*</sup> and vector  $\Delta$ **v** can be computed in spectral domain as

$$Y_{mn}^{e} = \frac{1}{4\pi^{2}} \int_{-\infty-\infty}^{\infty} \widetilde{f}_{u}^{2}(k_{x},W) \widetilde{G}_{yy}^{HM}(k_{x},k_{y}) \widetilde{f}_{p}^{2}(k_{y},L)$$
$$\times \cos k_{y}(y_{m}-y_{n}) dk_{x} dk_{y}, \qquad (12)$$

$$\Delta v_n = \frac{1}{2\pi\sqrt{Z_c}} \int_{-\infty}^{\infty} \widetilde{f}_u(k_y, W_f) \widetilde{G}_{yx}^{HJ}(k_x = -\beta_f, k_y) \widetilde{f}_p(k_y, L)$$
$$\times \cos k_y y_n dk_y, \qquad (13)$$

where  $k_x$ ,  $k_y$  are spectral domain variables,  $y_n$  is the center point of the *n*th expansion mode,  $f_u^{\sim}$  and  $f_p^{\sim}$  are Fourier transforms of the expansion mode (see Appendix) and  $G^{\sim}(k_x, k_y)$  are various components of spectral Green's functions which can be found for example in [2]. The wavy line in  $G^{\sim}(k_x, k_y)$ ,  $f_u^{\sim}$  and  $f_p^{\sim}$  indicates, that these functions are defined in spectral domain variables as opposed to  $G(\overline{r}, \overline{r_0})$ ,  $f_u$  and  $f_p$  defined in space variables.

Next, the patch current is expanded in a series with a column vector of current expansion coefficients I. The patch contribution to the aperture admittance seen by the slot is then

$$\mathbf{Y}^{a} = \mathbf{V}^{\mathrm{T}} \mathbf{Z}^{-1} \mathbf{V}, \qquad (14)$$

where components of moment method impedance matrix  ${\bf Z}$  and voltage vector  ${\bf V}$  are given in spectral domain as

$$Z_{mn} = \frac{-1}{4\pi^2} \int_{-\infty-\infty}^{\infty} \widetilde{f}_p^2(k_x, PL) \widetilde{G}_{xx}^{EJ}(k_x, k_y) \widetilde{f}_u^2(k_y, PW)$$
$$\times e^{jk_x(x_m-x_n)} dk_x dk_y, \qquad (15)$$

$$V_{n} = \frac{1}{4\pi^{2}} \int_{-\infty-\infty}^{\infty} \widetilde{f}_{p}(k_{x}, PL) \widetilde{f}_{p}(k_{y}, L) \widetilde{G}_{xy}^{EM}(k_{x}, k_{y})$$
$$\times \widetilde{f}_{u}(k_{y}, PW) \widetilde{f}_{u}(k_{x}, W) e^{jk_{x}x_{n}} dk_{x} dk_{y}, \qquad (16)$$

 $x_n$  is the center point of the *n*th expansion mode of the current on the patch,  $f_u^{\sim}$  and  $f_p^{\sim}$  are Fourier transforms of the

current expansion mode (electric current on the patch and magnetic current in the aperture). The equivalent series impedance seen by the microstrip feed line is

$$Z = Z_c \frac{\Delta v^2}{Y^e + Y^a}.$$
(17)

The aperture coupled patch antenna is usually tuned with an open-circuited stub of microstrip line, approximately  $\lambda_g/4$  long. If stub length is  $L_s$ , input impedance seen looking into the microstrip feed line referenced to aperture is

$$Z_{in} = Z - jZ_c \cot \beta L_s.$$
<sup>(18)</sup>

More accurate results can be obtained by adding a length extension to  $L_s$  to account for fringing fields at the end of the open stub, the length extension is approximately 0.4  $d_f$ , where  $d_f$  is the feed substrate thickness.

When numerically evaluating the infinite integrals in  $k_x$ ,  $k_y$  plane, in (12), (15), (16), it is convenient to transform the rectangular spectral variables  $k_x$ ,  $k_y$  to their polar form  $\alpha$ ,  $\beta$  and special care must be taken to perform the quadrature integration in the vicinity of surface wave poles, as described for example in [1].

Various components of the spectral Green's functions  $G^{\sim}(k_x, k_y)$  needed in computation, can be found for example in [2]. In the case, that the feed and antenna substrates have different parameters, by evaluating Green's function in computation of  $\Delta v$  and  $Y^e$ , parameters  $\varepsilon_{rf}$  and  $\tan \delta_f$  should be used and in computation of  $Y^a$  parameters  $\varepsilon_{ra}$  and  $\tan \delta_a$  should be used.



**Fig. 2** Smith chart plot of the input impedance of a stub-tuned aperture coupled patch antenna versus frequency in GHz.  $\varepsilon_{ra} = 2.54$ ,  $d_a = 0.16$  cm, PL = 4 cm, PW = 3 cm,  $\varepsilon_{rf} = 2.54$ ,  $d_f = 0.16$  cm, L = 1.12 cm, W = 0.155 cm,  $W_f = 0.442$  cm,  $L_s = 2$  cm.

#### 3. Results

We have implemented the above mentioned moment method in MATLAB and numerically computed input impedance of two configurations of aperture-coupled stubtuned microstrip antennas and compared our results with experimental and theoretical data published elsewhere.



**Fig. 3** Smith chart plot of the input impedance of a stub-tuned aperture coupled patch antenna versus frequency in GHz.  $\varepsilon_{ra} = 2.22$ ,  $d_a = 0.16$  cm, PL = 4 cm, PW = 3 cm,  $\varepsilon_{rf} = 10.2$ ,  $d_f = 0.127$  cm, L = 1 cm, W = 0.11 cm,  $W_f = 0.116$  cm,  $L_s = 1.1$  cm.

The case in Fig. 2 has the same substrate parameters for both the feed and antenna substrates. Our theory shows a small shift in the resonant frequency compared to experiment, which may be caused by neglecting the effect of nonuniform current distribution along the feed microstripline on the low dielectric constant substrate and by neglecting the actual dielectric losses in the substrate.

Fig. 3 shows results for a low dielectric constant ( $\varepsilon_{ra}$ =2.22) substrate for the antenna and a high dielectric constant ( $\varepsilon_{rf}$ =10.2) substrate for the feed line. This configuration simulates the monolithic phased array application, where the feed substrate would be Gallium Arsenide for phase shifters and other active circuitry. Results fit considerably well with both experimental and other authors data.

#### 4. Conclusion

Calculation of input impedance of aperture coupled microstrip antenna using moment method in spectral domain have been presented and compared with experimental and theoretical results published elsewhere. The method is based on the reciprocity theorem and uses exact Green's functions for dielectric slab. The agreement between measured and computed results supports validity of method.

## 5. Appendix

1D Fourier transforms of following expansion modes are required:

a) pulse function

$$f_u(x,W) = \begin{cases} 1/W, & |x| < W/2\\ 0, & |x| > W/2 \end{cases},$$
(19)

b) piecewise sinusoidal (PWS) mode

$$f_{p}(x,L) = \begin{cases} \frac{\sin k_{e}(L/2 - |x|)}{\sin k_{e}L/2}, & |x| < L/2 \\ 0, & |x| > L/2 \end{cases}$$
(20)

The Fourier transform is defined as

$$\widetilde{f}(k_x, W) = \int_{-\infty}^{\infty} f(x, W) e^{jk_x x} dx, \qquad (21)$$

and the transforms of the above functions are

$$\widetilde{f}_u(k_x, W) = \frac{\sin k_x W/2}{k W/2},$$
(22)

$$\widetilde{f}_{p}(k_{x},L) = \frac{2k_{e}[\cos k_{x}L/2 - \cos k_{e}L/2]}{(k_{e}^{2} - k_{x}^{2})\sin k_{e}L/2}.$$
(23)

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