RC AUTONOMOUS CIRCUITS WITH CHAOTIC BEHAVIOUR

Peter BERNÁT, Igor BALÁŽ
Department of Radio and Electronics
Faculty of Electrical Engineering and Information Technology
Slovak University of Technology in Bratislava
Ilkovičova 3, 812 19 Bratislava
Slovak Republic

Abstract

The paper presents obtained results with applying the "Semi-systematic procedure for producing the chaos from quasi-sinusoidal oscillators". We applied this procedure to RC autonomous circuit and obtained two new autonomous circuits with chaotic behaviour. The two obtained circuits are presented. The usage of various non-linear devices is examined.

Keywords

RC oscillators, chaotic behaviour, chaos

1. Introduction

Today, four types of steady state behaviour in deterministic non-linear systems are known: DC steady state, periodic state, quasi-periodic state and chaotic state also known as either deterministic chaos or simpler chaos.

Since begin of 1980s' these types of behaviour have been also known in non-linear electronic circuits. Plenty of autonomous and non-autonomous electronic circuits with chaotic behaviour have been presented and analysed. The many of these circuits were designed by "trial and error" approach. This joyless fact is due to the reality that the sufficient conditions for existence of chaotic behaviour in a system are still unknown. Only necessary conditions for existence of chaotic behaviour in a system are known. A little better approach is based on designing of circuit according to known set of differential equations which solution is chaotic (e.g. Lorenz system, Rössler system, etc.). Limitation in this case is the type of the non-linear term used in equations. As the non-linear term is often used the multiplication of variables and various complicated piecewise-linear functions [1]. These non-linear terms are not typical in electronic circuits. Their circuit implementations could be problematic and limit usage of such designed circuits. Examples of autonomous electronic circuits with chaotic behaviour designed by above described approach are in [4].

In [3] Elwakil deals with the possible simplification of designing circuits with chaotic behaviour by suggested procedure. This procedure is shortly described in second part of this paper. In the third part of this paper are presented RC oscillators with chaotic behaviour. We have verified the functionality of suggested circuits by simulation in MC6 simulation program.

2. Semi-systematic procedure for chaos generation in autonomous circuits

Authors in [3] introduced the semi-systematic procedure for design the autonomous circuits with chaotic behaviour by modification of sinusoidal oscillators. Here is necessary to point out, that the authors in [3] used the term sinusoidal oscillator for simple oscillator with the Wien-bridge. It is well known that signals in this circuit are not sinusoidal but more quasi-sinusoidal. So since this point we will use the more exact term quasi-sinusoidal.

The suggested procedure has three steps. The first step is design the appropriate quasi-sinusoidal oscillator that fulfils all circuit requirements like type of active component (transistor, operational amplifier, current feedback operational amplifier, etc.), type of the energy storage components (inductors, capacitors), their connection to circuit (floating, grounded), etc. Then the relations for necessary condition of oscillations and frequency of oscillations are derived by routine linear analysis.

The second and third steps are not as systematic as the first step. In the second step designers according to their previous experiences and derived relations for necessary condition of oscillations and frequency of oscillations choose the appropriate position to insert a non-linear component with appropriate voltage-current characteristic. If the designed oscillator is the circuit of second order the designers have to insert either inductor or capacitor into the designed circuit to obtain the circuit of third order. The value and position of connected inductor or capacitor also depend on experiences of designers.

The third step is adjusting the values of components until the circuit behaves chaotically.

The authors in [3] also suggest to use the passive non-linear component and recommend to use the either JFET or diode. JFET is appropriate to connect in serial combination with a capacitor and a diode in parallel combination with inductor. The active component should work linear.

Evidently, the procedure is not straightforward to desired circuit and greatly depends on previous experiences of designers. We tried to use the procedure to modify RC quasi-sinusoidal oscillator with current feedback op-amp (CFOA) to autonomous circuits with chaotic behaviour.

3. Investigated circuits

As a base RC quasi-sinusoidal oscillator we chose the circuit presented in Fig. 1. The frequency of oscillation is determined by eqn. 1. The condition of oscillation is determined by eqn. 2.

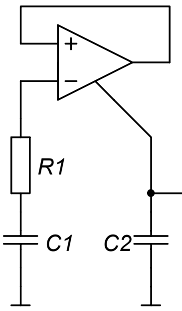

Fig. 1 Quasi-sinusoidal oscillator with CFOA

$$f = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} \tag{1}$$

$$1 = \frac{C_2}{C_1} + \frac{R_1}{R_2} \tag{2}$$

This circuit had been already used in [2] to design circuit with chaotic behaviour. In [2] the resistor R_1 is replaced by JFET which works as voltage controlled resistor. As the third energy storage device the authors used inductor in serial connection with resistor R_2. In our modifications we use capacitor instead of inductor. Of course its connection to circuits have to be different.

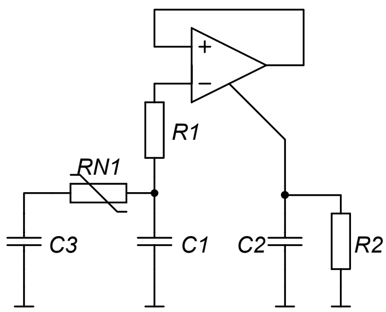

Fig. 2 First investigated circuit

As a first possible modification we investigated the circuit shown in Fig. 2. We considered using as a non-linear component either JFET, which operates as voltage controlled resistor or ordinary diode. The voltage-current characteristics of these devices are presented in Fig. 3. In [2] authors in the mathematical models of their circuits approximated the voltage-current characteristic of JFET by piecewise-linear function with one breakpoint (3). V_P is breakpoint of characteristic (approximately – 0.7 V), R_J is small signal resistance close to coordinate system origin (approximately 750 Ω). These mathematical models reliable represented behaviour of their circuits.

$$I = \frac{1}{R_J}\begin{cases} V & V \geq V_P \\ V_P & V < V_P \end{cases} \tag{3}$$

The shape of characteristic with one breakpoint is similar to voltage-current characteristic of diode. The difference is in the values of resistance for the first harmonic for small voltages and large voltages. In case of diode the resistance for small voltages is greater than resistance for large voltages. In case of JFET the situation is reverse.

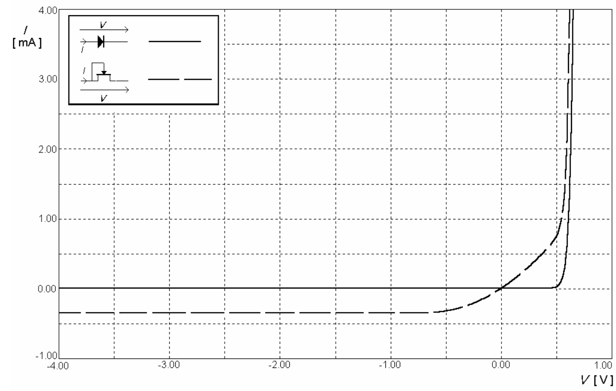

Fig. 3 Voltage-current characteristics of considered non-linear devices

The appropriate shape of non-linearity should enable birth of oscillation in circuit and should limit their unbounded rise. The proper type of characteristic is possible to find after analysis of magnitude-frequency characteristic and phase-frequency characteristic of open loop circuit with connected non-linear resistor and third capacitor. This analysis is possible to perform by computer simulation (AC Analysis in MC6) or exact analytical technique. Using the analytical technique leads to enormous mathematical expressions. The obtained results are complicated and blind. Therefore we used the simpler computer simulations. We disconnected the circuit in non-inverting input of CFOA. We chose the values of R_1, R_2, C_1, C_2 according to linear condition for oscillation and required frequency area. The value of capacitor C_3 we chose close to values of capacitors C_1 and C_2. Instead of non-linear resistor we used a linear resistors R with different values. We observed influence of resistor R values to open loop gain. In investigated circuit in case $R < 1.5$ kΩ open loop gain of circuit is greater than 1 (birth of oscillation), in case $R > 1.5$ kΩ open loop gain is lesser than 1 (limitation of oscillation). According to the obtained results we chose JFET as the non-linear component in oscillator. Of course, this analysis and chosen non-linear device do not guarantee the existence of chaotic behaviour in circuit.

In the following step by computer simulation (MC6 program) of suggested circuit we were able to quickly find a chaotic behaviour for various values of components presented in Tab. 1. Fig. 4 shows projection of investigated circuit state trajectory onto plane V_{C1},V_{C2} for the case $C_1 = 10$ nF, $C_2 = 10$ nF, $C_3 = 18$ nF. Open loop gain equals

1.29. In simulation we used model of JFET type 2N4338 and model of CFOA type AD844.

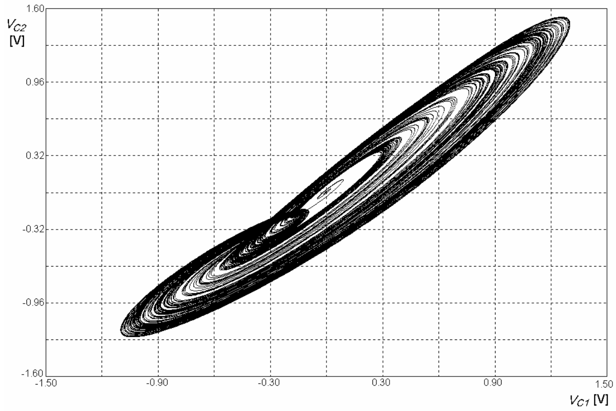

Fig. 4 Projection of state trajectory of the oscillator in Fig. 20

As the following modification we investigated the circuit depicted in Fig. 5. We performed analysis of open loop circuit by manner similar to analysis of previously analysed circuit.

C_1	C_2	C_3	R_1	R_2
100 nF	100 nF	180.0 nF	220 Ω	1500 Ω
10 nF	10 nF	18.0 nF	220 Ω	1500 Ω
1 nF	1 nF	1.8 nF	220 Ω	1500 Ω
100 pF	100 pF	180.0 pF	220 Ω	1500 Ω

Tab. 1

According to the obtained results we chose the JFET as a proper non-linear device. In the subsequent computer simulation by MC6 program we were able to find the chaotic behaviour in circuit. The values of components for chaotic behaviour are shown in Table 2. Chaotic behaviour is presented in Fig. 6 by projection of state trajectory of analysed circuit onto plane V_{C1},V_{C2} for case $C_1 = 10$ nF, $C_2 = 12$ nF,

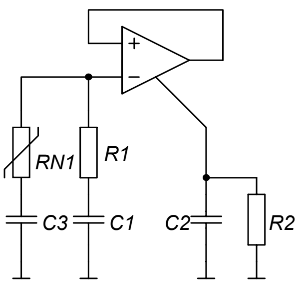

Fig. 5 Second investigated circuit

$C_3 = 5.6$ nF. Open loop gain equals 1.17. We used model of JFET type 2N4338 and model of CFOA type AD844.

Two presented RC autonomous circuits with chaotic behaviour suggest that it is not complicated to design RC autonomous circuits with chaotic behaviour. In the following part of study we investigated the possibility to use the diode and resistor connected in parallel combination instead of JFET by the similar manner as we used JFET in oscillators described above. According to our best knowledge a modification of quasi-sinusoidal oscillator to RC oscillator with chaotic behaviour by usage one diode has not been presented yet. As a base RC quasi-sinusoidal oscillator we chose the circuit in Fig. 1.

C_1	C_2	C_3	R_1	R_2
100 nF	120.0 nF	56.0 nF	220 Ω	2700 Ω
10 nF	12.0 nF	5.6 nF	220 Ω	2700 Ω
1 nF	1.2 pF	580.0 pF	220 Ω	2700 Ω
100 pF	120.0 pF	68.0 pF	220 Ω	2700 Ω

Tab. 2

We tried various possible positions for connection of non-linear two-terminal network built up by serial connection of capacitor with parallel connection of diode and resistor. The possibility of origin and limitation of generated signal amplitude we investigated by analysis of magnitude-frequency characteristic and phase-frequency characteristic of open loop circuit with connected non-linear resistor and third capacitor. Instead of non-linear resistor we used linear resistors *R* with different values. The process was same as a process that we described above. The result of this analysis was design of 2 circuits presented in Fig. 7, Fig. 8.

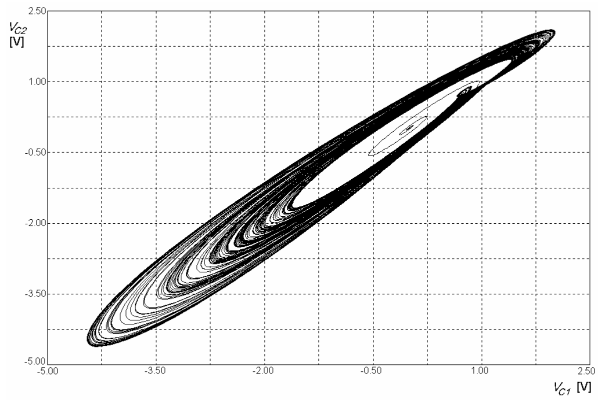

Fig. 6 Projection of state trajectory of oscillator in Fig. 5

$$\begin{aligned}
\frac{dv_{C1}}{dt} &= -\frac{1}{(R_{CFOA}+R_1)C_1}\,v_{C1} + \frac{1}{(R_{CFOA}+R_1)C_1}\,v_{C2} \\
\frac{dv_{C2}}{dt} &= -\frac{1}{(R_{CFOA}+R_1)C_2}\,v_{C1} + \left[\frac{1}{(R_{CFOA}+R_1)C_2} - \frac{1}{R_3 C_2} - \frac{1}{R_2 C_2}\right] v_{C2} + \frac{1}{R_3 C_2}\,v_{C3} - \frac{f(v_{C2}-v_{C3})}{C_2} \\
\frac{dv_{C3}}{dt} &= \frac{1}{R_3 C_3}\,v_{C2} - \frac{1}{R_3 C_3}\,v_{C3} - \frac{f(v_{C2}-v_{C3})}{C_3}
\end{aligned} \tag{4}$$

As a first candidate for possible chaos generation we investigated the circuit shown in Fig. 7. We tried to find the chaotic behaviour in designed circuit by ”trial and error” approach by MC6 simulation program. We did not find chaotic behaviour. Therefore we decided to find suitable values of components by systematic method. We described this circuit by set of state equations (4). We considered linear model of CFOA with input resistance of inverting input equals $R_{CFOA} = 20\ \Omega$. We approximated the voltage-current characteristic of diode by relation (5). $I_S = 1$ pA, $\alpha = 0.875$, $V_T = 25.8$ mV, V is voltage on diode, I is current through diode

$$I = f(V) = I_s\left[\exp(\alpha V/V_T) - 1\right] \qquad (5)$$

Then we investigated behaviour of circuit for all possible combinations of the components' values that are shown in Tab. 3. The value of capacitor $C_3 = 150$ nF was constant during all simulations.

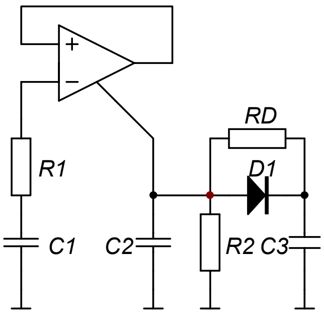

Fig. 7 Third investigated circuit

The initial conditions for all 5400 simulations were the same: $V_{C1} = 1$ mV, $V_{C2} = 0$ V, $V_{C3} = 0$ V. The type of behaviour in circuit was automatically identified according to the distribution of intersections of state trajectory with Poincaré plane during a long time interval. We used one-sided Poincaré plane dedicated by equation: $v_{C1} = 0$. For mentioned values of components the open loop gain was in interval <0.13, 4.5>. We did not investigated the cases with open loop gain lesser than 1.

C_1	C_2	R_1	R_2	R_D
50 nF	50 nF	100 Ω	470 Ω	1.0 kΩ
80 nF	80 nF	160 Ω	680 Ω	2.2 kΩ
100 nF	100 nF	220 Ω	820 Ω	5.0 kΩ
150 nF	150 nF	330 Ω	1200 Ω	15.0 kΩ
200 nF	200 nF	390 Ω	2200 Ω	50.0 kΩ
300 nF	300 nF	-	-	150 kΩ

Tab. 3

We found plenty of combinations of components' values that cause the origin of quasi-periodic behaviour in circuit. We found combinations components' values that cause the unbounded rise of voltages on capacitors and combinations of components' values that cause the attenuation and expiration of initial oscillations (open loop gain lesser than 1). But we did not find any chaotic behaviour.

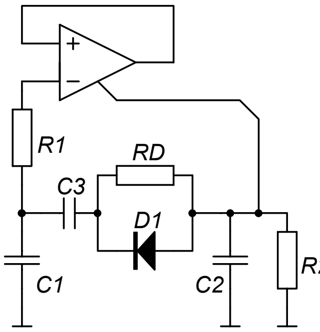

Fig. 8 Fourth investigated circuit

As a following step we investigated the circuit shown in Fig. 8. The obtained results are like to results obtained by analysis of previous circuit. We did not find any chaotic behaviour by MC6 simulation program.

C_1	C_2	R_1	R_2	R_D
50 nF	50 nF	120 Ω	390 Ω	1.0 kΩ
80 nF	80 nF	180 Ω	470 Ω	2.2 kΩ
100 nF	100 nF	220 Ω	560 Ω	5.0 kΩ
150 nF	150 nF	270 Ω	820 Ω	15.0 kΩ
200 nF	200 nF	330 Ω	1800 Ω	50.0 kΩ
300 nF	300 nF	-	-	150.0 kΩ

Tab. 4

We described the circuit by the set of state eqns. (6) and investigated the circuit for all possible combinations of the components' values that are shown in Table 4. The value of capacitor $C_3 = 100$ nF was constant during simulations. For mentioned values of components the open loop gain was in interval <0.14, 4.1>. We did not investigated the cases with open loop gain lesser than 1.

We found only quasi-periodic behaviours, unbounded rises of voltages on capacitors and expirations of initial oscillations (open loop gain lesser than 1). We did not find any chaotic behaviour.

The same manner, we tried to examine circuit in Fig. 2 with parallel combination of diode and resistor as non-linear component. We tried 6480 various combinations of components' values without finding any chaotic behaviour.

$$\frac{dv_{C1}}{dt} = \left[-\frac{1}{(R_{CFOA}+R_1)C_1} - \frac{1}{R_3 C_1}\right] v_{C1} + \left[\frac{1}{(R_{CFOA}+R_1)C_1} + \frac{1}{R_3 C_1}\right] v_{C2} - \frac{1}{R_3 C_1} v_{C3} + \frac{f(v_{C2} - v_{C1} - v_{C3})}{C_1}$$

$$\frac{dv_{C2}}{dt} = \left[-\frac{1}{(R_{CFOA}+R_1)C_2} + \frac{1}{R_3 C_2}\right] v_{C1} + \left[\frac{1}{(R_{CFOA}+R_1)C_2} - \frac{1}{R_2 C_2} - \frac{1}{R_3 C_2}\right] v_{C2} + \frac{1}{R_3 C_2} v_{C3} - \frac{f(v_{C2} - v_{C1} - v_{C3})}{C_2}$$

$$\frac{dv_{C3}}{dt} = \frac{f\left(v_{C2} - v_{C1} - v_{C3}\right)}{C_3} + \frac{v_{C2} - v_{C1} - v_{C3}}{R_3\, C_3} \tag{6}$$

4. Conclusion

We examined design the simple RC autonomous circuits with chaotic behaviour by"Semi-systematic procedure for producing the chaos from sinusoidal oscillators". Firstly we tried to use the advised non-linearity introduced by JFET. Although mentioned procedure is not straightforward to the acquisition of circuit with chaotic behaviour, it is not complicated to design such circuit by usage of this procedure. The main problem is to choose a right position for connecting of JFET. Procedure, however, does not concern with the attributes of designed system like Lyapunov exponents, basis of attraction, statistical attributes of generated signals, power spectrums of generated signals etc. If designed autonomous circuit has to fulfil some of that attributes procedure cannot be used. It does not also concern with design of hyper-chaotic systems.

We investigated the possibility to replace the advised JFET by diode. The modification of quasi-sinusoidal oscillator to circuit with chaotic behaviour by means of one diode has not been presented yet. But none of investigated circuits behaved chaotically. It could be caused by several reasons like usage improper topology of investigated circuits, incorrect values of components (we investigated only finite number of combinations of components' values), incorrect initial condition (all simulation was done for one initial condition) etc. This possible modification is further investigated.

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About authors...

Peter BERNÁT was born in 1975 in Piešťany. He received Ing. degree in electrical engineering from Faculty of Electrical Engineering and Information Technology, Slovak Technical University in Bratislava in 2000. Currently he is Ph.D. student at the Dept. of Radio and Electronics. He is interested in non-linear electronic circuits.

Igor BALÁŽ was born in 1939. He received the Ph.D. degree and DrSc. degree both in electrical engineering in 1973 respectively 1988. Currently he is Full Professor at the Department of Radio and Electronics at Faculty of Electrical Engineering and Information Technology, Slovak Technical University in Bratislava.