BIQUAD BASED ON A GENERALIZED DIVIDER STRUCTURE

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Abstract

The paper deals with the biquad based on a generalized divider structure. This principle can be considered as an effective way of universal filter and equalizers synthesis, which easily realizes any general transfer function. An attention is devoted to the derivation of optimum design conditions. As shown, these conditions strongly depend on the amplifier type used. The results obtained are illustrated by some design examples.

Keywords

Active filters, universal filter, biquad, equalizers, generalized divider, immitance converters, sensitivity

1. Introduction

The generalized divider principle can be considered as an effective way of universal filter and equalizers synthesis. It allows easily realization of any general transfer function. The basic idea of this method was published in [1,2] and it is the original contribution in general to the universal filters synthesis theory, and respectively the selective circuits.

With respect to the practical realization conditions, the circuit configuration using a cascade of the generalized immitance converters (GIC), characterized by the 1st-order conversion function, was found as the most effecttive. General structure arrangement is shown in Fig. 1.

The presented circuit can be described by the transfer function (1)

$$H(s)=\frac{Y_{20}+k_1(s)[Y_{21}+k_2(s)[Y_{22}+\dots}{(Y_{10}+Y_{20})+k_1(s)[(Y_{11}+Y_{21})+k_2(s)[(Y_{12}+Y_{22})+\dots}\dots$$

$$\dots\frac{\dots+k_n(s)[Y_{2n}]\dots]]}{\dots+k_n(s)[(Y_{1n}+Y_{2n}]\dots]]}\Rightarrow\ =\frac{a_0+a_1s+a_2s^2+\dots+a_ns^n}{b_0+b_1s+b_2s^2+\dots+b_ns^n}. \quad (1)$$

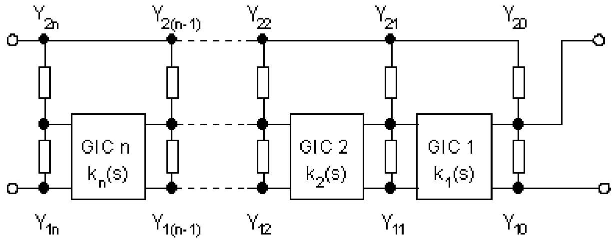

Fig. 1 The structure of the generalized divider using 1st-order GICs

A comparison of the general transfer function coefficients to (1) shows the definite (outright) expression of the general transfer function coefficients by divider admittances in the form

$$a_i=Y_{2i}\prod_{k=1}^{i}k_k,$$
$$b_i=(Y_{1i}+Y_{2i})\prod_{k=1}^{i}k_k \quad i=0,1,2,\dots,n. \quad (2)$$

Note that the shown arrangement of the circuit structure based on a generalized divider principle is not unique, other configurations were presented, for example, in [3], [4].

2. Biquad Circuit

The universal character of a generalized divider structure and good sensitivity properties, especially in the case of the 2nd-order filters, make it a good prospect for general biquad design and application.

Let us consider the biquad transfer function expressed in the form (3)

$$H_B(s)=\frac{a_2s^2+a_1s+a_0}{s^2+\frac{\omega_p}{Q_p}s+\omega_p^2}=h\frac{s^2+\frac{\omega_n}{Q_n}s+\omega_n^2}{s^2+\frac{\omega_p}{Q_p}s+\omega_p^2}. \quad (3)$$

Compared (3) to the transfer function of a general structure (1) leads to the basic arrangement of the biquad circuit shown in Fig. 2.

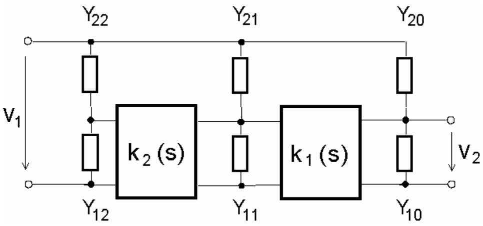

Fig. 2 The basic configuration of the biquad

The corresponding transfer function is expressed as (4)

$$H(s)=\frac{u_2}{u_1}=\frac{Y_{20}+k_1 s[Y_{21}+k_2 sY_{22}]}{(Y_{10}+Y_{20})+k_1 s[(Y_{11}+Y_{21})+k_2 s(Y_{12}+Y_{22})]}\ .\quad(4)$$

Using (2) and the relationship between (3) and (4) make it possible to easily derive the general form of design equations, as shown in Tab. 1.

The general design equations	
$Y_{10}=k_1k_2Y_{12}\dfrac{\omega_p-a_1Q_p}{a_2-1}$	$Y_{20}=k_1k_2Y_{12}\dfrac{a_0}{a_2-1}$
$Y_{11}=k_2Y_{12}\dfrac{\omega_p-a_1Q_p}{(a_2-1)Q_p}$	$Y_{21}=k_2Y_{12}\dfrac{a_1}{a_2-1}$
Y_{12} optional parameter	$Y_{22}=Y_{12}\dfrac{a_2}{a_2-1}$

Tab. 1 The general design equations

As evident, the advantage of the introduced circuit structure is mainly its easy realization of the all standard types of the 2nd-order transfer functions. At the same time, the arbitrary biquadratic functions can be obtained by a suitable choice of the divider admittances Y_{ik}, as shown in the following Tab. 2.

Divider branch choice for standard transfer functions				
LP:	$a_0=h.\omega_p^2$	$a_1=0$	$a_2=0$	$Y_{21}=0$ $Y_{22}=0$
BP:	$a_0=0$	$a_1=h\dfrac{\omega_p}{Q_p}$	$a_2=0$	$Y_{20}=0$ $Y_{22}=0$
HP:	$a_0=0$	$a_1=0$	$a_2=h$	$Y_{20}=0$ $Y_{21}=0$
ES:	$a_0=h.\omega_p^2$	$a_1=0$	$a_2=\dfrac{a_0}{\omega_n^2}$	$Y_{21}=0$
abbreviation ES denotes elliptic (notch) section				

Tab. 2 Branch admittances simplification conditions

From the practical point of view, the important question concerns the sensitivity properties of the designed structure. As is known, the biquad sensitivities can be characterized by transfer function coefficient- or pole- / zero-sensitivities to the circuit elements. Starting from the coefficient expressions (2), it is easy to derive relative sensitivities of numerator transfer function coefficients to divider branch admittances in the form (5)

$$\begin{aligned}&S_{Y_{2m}}^{a_i}=1 && for\quad m=i,\\ &S_{Y_{2m}}^{a_i}=0 && for\quad m\neq i,\\ &S_{k_k}^{a_i}=1 && for\quad k=1,...,i\,.\end{aligned}\qquad(5)$$

Similarly, the denominator relative sensitivities is possible to be obtained as

$$\left.\begin{aligned}&S_{Y_{1m}}^{b_i}=\frac{Y_{1m}}{Y_{2m}+Y_{1m}}\\ &S_{Y_{2m}}^{b_i}=\frac{Y_{2m}}{Y_{2m}+Y_{1m}}\end{aligned}\right\}\ for\quad m=i\,,\qquad(6)$$

$$\begin{aligned}&S_{Y_{1m}}^{b_i}=0\qquad S_{Y_{2m}}^{b_i}=0 && for\quad m\neq 1,\\ &S_{k_k}^{b_i}=1 && for\quad k=1,...,i\,.\end{aligned}\qquad(7)$$

Pole- / zero-sensitivities can be expressed rather than by relative sensitivities of the corresponding parameters ω_n, Q_n, ω_p, Q_p to circuit elements. Eqns. (8) summarize the evaluated results

$$\begin{aligned}&S_{Y_{10}}^{\omega_p}=0.5\frac{Y_{10}}{Y_{20}+Y_{10}},\quad S_{Y_{12}}^{\omega_p}=-0.5\frac{Y_{12}}{Y_{12}+Y_{22}},\\ &S_{Y_{20}}^{\omega_p}=0.5\frac{Y_{20}}{Y_{20}+Y_{10}},\quad S_{Y_{22}}^{\omega_p}=-0.5\frac{Y_{22}}{Y_{22}+Y_{12}},\\ &S_{k_1}^{\omega_p}=S_{k_2}^{\omega_p}=-0.5,\\ &S_{Y_{10}}^{Q_p}=0.5\frac{Y_{10}}{Y_{20}+Y_{10}},\quad S_{Y_{12}}^{Q_p}=0.5\frac{Y_{12}}{Y_{22}+Y_{12}},\\ &S_{Y_{11}}^{Q_p}=-\frac{Y_{11}}{Y_{21}+Y_{11}},\\ &S_{Y_{20}}^{Q_p}=0.5\frac{Y_{20}}{Y_{20}+Y_{10}},\quad S_{Y_{21}}^{Q_p}=-\frac{Y_{21}}{Y_{21}+Y_{11}},\\ &S_{Y_{22}}^{Q_p}=0.5\frac{Y_{22}}{Y_{22}+Y_{12}},\quad S_{k_1}^{Q_p}=-S_{k_2}^{Q_p}=0.5\,.\end{aligned}\qquad(8)$$

Comparing evaluated sensitivities to the ones of known biquads, we have found the presented structure as good as the preferred low-sensitivity circuits, e.g. two-integrator feedback configurations.

The relatively high output impedance of the basic circuit in Fig. 2 presents a relevant disadvantage, particularly in the cascade filter synthesis and other applications expecting voltage output. Fortunately, when we use the known Antoniou's circuit to realize GICs, original output terminal can be replaced by some GIC's amplifier output. The same possibility is in the output modification when simplified single-amplifier GIC is used for SC circuit implementation – see Ref. [7]. Using a modified output, the numerator of biquad transfer function changes (denominator remains identical) and the design equations are necessary to re-arrange, in dependence, on the converter's circuit type and from the particular type of amplifier in the resulting circuit structure.

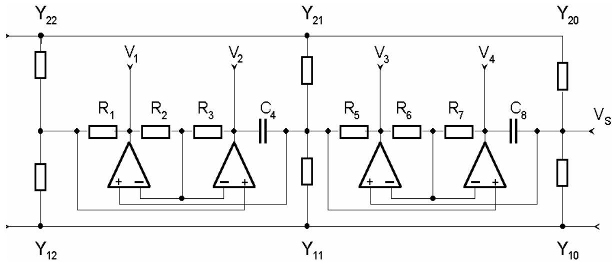

Fig. 3 Biquad with Antoniou's GIC and possible output modifications

To achieve the optimum choice of modified output we appoint, at first, the transfer functions of ideal circuits, considering outputs of all four amplifiers V_1...V_4. In the following, these functions were compared to the original transfer, corresponding to the standard output V_s – see circuit diagram in Fig.3.

Under this assumption, the transfer function denominator is unchanged and the numerator differs under the choice of the modified output. With respect to this, in all modifications it is evidently possible to find design conditions, either for general transfer function or standard 2nd-order transfer functions.

To gain the optimum result, we take the following requests into consideration:

1. The simplest design equations.
2. A minimum occurrence of single converter's elements in the design equations (except for the standard expression of the conversion function).
3. Minimum terms containing the difference of divider admittances or GICs elements.
4. Simple application of optimization conditions.

With respect to the first three criteria, the best results give circuits applying a modified output V_1 and V_2. But the second choice complicates application of the optimization conditions. The design conditions and optimization conditions interact against and can lead up to results giving negative values of circuit's elements. For this reason this option was rejected. As the most suitable the option V_1 was chosen (biquad output identical with the first amplifier output in circuit diagram shown in Fig. 3).

The general design equations corresponding to the particular type of functions are shown in Tab. 3. For simplification, the unity constants of conversion functions of both converters were considered.

Note that the case of allpass, e.g. the 2nd-order phase equalizer, is not introduced into the set of the transfer functions. Realization of this transfer function requires a modification of the original circuit structure and we will deal with it in another paper.

Relationships for divider admittances calculation for standard transfer functions and modified output V_1
LP: $Y_{10}=Y_{12}\dfrac{\left(1-h+R_1Y_{12}\right)}{1+R_1Y_{12}}\omega_p^2$ $Y_{11}=Y_{12}\dfrac{\omega_p}{Q_p}$ $Y_{20}=h\dfrac{Y_{12}}{1+R_1Y_{12}}\omega_p^2$ $Y_{21}=0$ $Y_{22}=0$
HP: $Y_{10}=Y_{12}\dfrac{\left(1+R_1Y_{12}\right)}{1-h+R_1Y_{12}}\omega_p^2$ $Y_{11}=Y_{12}\dfrac{1+R_1Y_{12}}{1-h+R_1Y_{12}}\cdot\dfrac{\omega_p}{Q_p}$ $Y_{20}=-h\dfrac{R_1Y_{12}^2}{(h-1)R_1Y_{12}+h(2-h)-1}\omega_p^2$ $Y_{21}=-h\dfrac{R_1Y_{12}^2}{(h-1)R_1Y_{12}+h(2-h)-1}\cdot\dfrac{\omega_p}{Q_p}$ $Y_{22}=-Y_{12}\dfrac{h}{h-1}$
BP: $Y_{10}=Y_{12}\omega_p^2$ $Y_{11}=Y_{12}\dfrac{1-h+R_1Y_{12}}{1+R_1Y_{12}}$ $Y_{20}=0$ $Y_{21}=h\dfrac{Y_{12}}{1+R_1Y_{12}}\cdot\dfrac{\omega_p}{Q_p}$ $Y_{22}=0$
ES: $Y_{10}=Y_{12}\dfrac{\omega_p^2\left(1+R_1Y_{12}\right)-h\omega_n^2}{1-h+R_1Y_{12}}$ $Y_{11}=Y_{12}\dfrac{1+R_1Y_{12}}{1-h+R_1Y_{12}}\cdot\dfrac{\omega_p}{Q_p}$ $Y_{20}=-hY_{12}\dfrac{R_1Y_{12}\omega_p^2+\omega_n^2(1-h)}{(h-1)R_1Y_{12}+h(2-h)-1}$ $Y_{21}=-h\dfrac{R_1Y_{12}^2}{(h-1)R_1Y_{12}+h(2-h)-1}\cdot\dfrac{\omega_p}{Q_p}$ $Y_{22}=-Y_{12}\dfrac{h}{h-1}$

Tab. 3 Modified design equations

2.1 An Optimum Design of Biquad Using Two-Amplifier GICs

The design rules shown in Tab. 3 provide sufficient freedom for obtaining circuit optimization. This optimization procedure concerns:

- A minimization of non-ideal circuit element behavior influence to final transfer function parameters.

- An equalization of maximum voltage levels at amplifier outputs, i.e. optimization of circuit internal dynamic properties. Note that dynamics optimization is absolute in the case of modified biquad output, because it is included into the set of optimized amplifier output voltages.

The first criterion includes the minimization of parameters ω_p, Q_p errors, caused by finite frequency dependent gain of GIC amplifiers. It is important to say the procedure strongly depends on the amplifier type, as was described and explained in [5]. The best results were gained for GIC circuits using current Op-Amps or transimpedance amplifiers (TIA). In the case of conventional Op-Amp or OTA optimization conditions are more complicated and lead to an unacceptable design result in many cases. This consideration is fully valid, when voltage-mode circuits are designed. Current-mode optimization conditions have to be modified under the rules derived in Ref. [5]. Note that current-mode biquad realization is limited by request of "floating output" amplifiers necessary for transformed GIC circuitry.

The optimization algorithm, solving the aforementioned procedure, is formed with respect to keeping a symbolic character of all the computation steps of the circuit design. The reason is in the general solution level, saved to obtain, as possible, symbolic solution. The basic steps can be specified as follows:

1. Symbolic analysis of a non-ideal circuit: → symbolic transfer function $H_s(s)$.
2. A simplification of an evaluated transfer function by neglecting higher-order errors, caused by finite amplifier gain → simplified 3rd-order transfer function $H_3(s)$.
3. Expression of $H_3(s)$ denominator as $D(s)=(s+\sigma_e)(s^2+x_e s+\Omega_e)$ and equivalent parameters σ_e, x_e, Ω_e evaluation.
4. Evaluation of x_e, Ω_e errors with respect to the corresponding parameters of the ideal circuit transfer function.
5. Creation of a set of design equations for circuit elements, based on:
   - a comparison of symbolic transfer function coefficients to corresponding ones of given transfer function (3),
   - an acceptance of the optimization conditions leading to the zeroing of parameter x_e, Ω_e errors,
   - acceptance of dynamic optimization conditions.

An optimized design procedure using the described algorithm has been developed for biquad design in the case of conventional Op-Amps and TIA as well. To simplify the original 6th-order circuit symbolic transfer function to the 3rd-order function and to evaluate the equivalent parameters σ_e, x_e, Ω_e in symbolic form, the original algorithm was developed and implemented in MAPLE software [7]. This algorithm is applicable for both original and modified output circuit configurations. Optimization results gained for both circuit configurations showed only a negligible difference, related to the transfer function numerator parameters. The most relevant problem was found in the transfer zeroes Q-factor decreasing, when a conventional Op-Amp was considered. Fortunately - this disadvantage is important in the case of an elliptic section transfer and can be additionally corrected. A careful design of an elliptic section using TIA can avoid this problem.

2.2 Optimization Conditions for Biquad Using TIA

As shown in [5] and [7], transimpedance amplifier application brings new features into the non-ideal circuit behavior. When simplified TIA model in Fig. 4 ($R_{IN} = 0$) is considered, the resulting "real" transfer function is of the 4th-order only. This fact, of course, significantly simplifies the procedure of optimum GIC operation conditions search, when a frequency dependent TIA transimpedance is taken into account. At the same time, it makes possible to simplify the resulting design equations.

A critical point of optimum biquad design using TIA is the strong request to keep a sufficient ratio of a circuit resistive element impedance level - to - amplifier transresistance R_T.

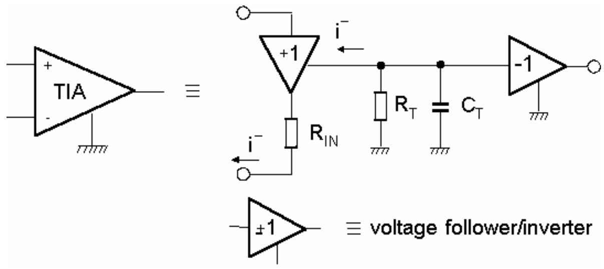

Fig. 4 Transimpedance amplifier (TIA) model

The advantage of TIA-based GICs used in biquad structure is the possibility to achieve a "complex" minimization (or zeroing) of transfer function pole- and zero-errors simultaneously.

Two ways of transfer function parameter optimization have been tested:

A. *Low-frequency* case:

Transimpedance is considered to be frequency independent. The optimum design conditions are derived from zeroing of „static“ errors of „real“ transfer function coefficients. The coefficient-error character allows to set to zero errors of three parameters. A combination (9) was chosen as the preferred case:

$$\delta d_0 = 0\,,\ \ \delta d_1 = 0\,,\ \ \delta a_1 = 0\ \ \text{and}\ \ \partial \delta a_0 / \partial R_6 = 0\,. \qquad (9)$$

Here δd_0 denotes a relative error of the zero-order coefficient of the "real" symbolic transfer function denominator, and δd_1 a relative error of the first-order denominator coefficient. Similarly δa_0, δa_1 denote relative errors of the zero- and the first-order coefficients of the "real" symbolic transfer function numerator.

Set of equations (9) gives the solution for optimum values of GIC elements R_1, R_2, R_5, R_7

$$R_1 = 2\frac{R_T R_2}{R_6(Y_{22}+Y_{12})(R_T+R_2)}, \quad (10)$$

$$R_5 = 2\frac{R_T(R_T+R_6)}{Y_{11}+Y_{21}}, \quad (11)$$

$$R_7 = \frac{R_6(2R_T+R_6)}{R_T}, \quad (12)$$

$$R_3 = R_2 \,. \quad (13)$$

At the same time, additional conditions $R_2 = R_1$, $R_6 = R_5$, $C_8 = 1/(\omega_P R_7)$ were applied. The C_4 value is optional.

Design equations are completed by adding a set of formulae given by comparison of coefficients of general transfer function (3) to the corresponding coefficients of ideal circuit transfer. The final set of design rules was derived under the MAPLE program in the form:

$$Y_{10} = 2\frac{R_T Q_p(\omega_p^2 - h\omega_n^2)}{R_6\omega_p(R_T+R_6)}, \qquad Y_{20} = 2\frac{h\omega_n^2 Q_p R_T}{R_6\omega_p(R_T+R_6)},$$

$$Y_{11} = \frac{2R_T(\omega_p Q_n - h\omega_n Q_p)}{R_6\omega_p Q_n(R_T+R_6)}, \qquad Y_{21} = 2\frac{h\omega_n Q_p R_T}{R_6\omega_p Q_n(R_T+R_6)},$$

$$Y_{12} = \frac{1-h}{h}, \quad (14)$$

$$R_2 = \frac{R_T(2h-R_6)}{R_6},$$

$$C_4 = \frac{2hQ_p}{\omega_p(R_T+R_6)(2h-R_6)} \qquad C_8 = \frac{R_T}{R_6(2R_T+R_6)}.$$

B. *High-frequency* case:

From the "practical" point-of-view, the second case respecting frequency-dependent transimpedance is more important. Transimpedance Z_T is considered as the frequency-dependent parameter. Note that the correct expression $Z_T(s) = R_T\,\omega_x / (s+\omega_x)$ can be substituted by "integrator approximation" as $Z_T(s) = R_T\,\omega_x/s$, $\omega_x = 1/(R_T C_T)$ without a relevant loss of accuracy. In this case, the optimization procedure starts from the simplified 3rd-order transfer function $H_3(s)$ and its equivalent parameters σ_e, x_e, Ω_e for both the numerator and denominator $H_3(s)$. The basic set of equations for the optimized converter element values has the form

$$dd_0 = 0,\; dd_1 = 0,\; dd_{11} = 0 \text{ and } \partial dd_{10}/\partial C_8 = 0\,, \quad (15)$$

where:

dd_0 … means error of denominator parameter Ω_e,
dd_1 … means error of denominator parameter x_e,
dd_{10} … means error of numerator parameter Ω_e,
dd_{11} … means error of numerator parameter x_e.

Adding the basic set of equations for transfer function coefficient comparison, the four circuit elements remain optional. This fact provides more combinations in "free" element choice. Note that some of them are rejected as an unacceptable solution. From the practical point-of-view, the simplest choice was found to be a combination

$$R_5 = R_6\,,\; R_7 = R_2\,,\; R_2 = R_1\,,\; R_3 = 1/(\omega_p C_4). \quad (16)$$

Conditions (16) give 3 numerical solutions, from which only one is applicable.

A practical example of results gained by using the mentioned optimization procedures is given in Tab. 4. General biquadratic function (3) with $\omega_p = 1.0$, $Q_p = 4.0$, $\omega_n = 2.0$, $Q_n = 12.0$ was applied as the testing function. The first column contains simulation results corresponding to the use of simplified TIA model ($R_{IN} = 0$). The second corresponds to the simulation using the "full" TIA model with non-zero R_{IN}.

As can be seen, influence of non-zero R_{IN} is not relevant, when a sufficient ratio between R_T and the circuit element impedance level is preserved.

Evaluated circuit elements		
$R_1 = R_2 = 2.046563$	$R_3 = 2.031563$	$C_4 = 0.492232$
$R_5 = R_6 = 2.026562$	$R_7 = 2.046562$	$C_8 = 0.488378$
$Y_{10} = 0.0$	$Y_{11} = 0.833333$	$Y_{12} = 3.0$
$Y_{20} = 4.0$	$Y_{21} = 0.166667$	$Y_{22} = 1.0$
Simulation results		
TIA model: main parameters: $R_T = 100.0$ $C_T = 0.010$		
Simplified $R_{IN} = 0.0$	**Full** $R_{IN} = 0.001250$	
$\omega_{dp} = 0.998076$	$\omega_{dp} = 0.996008$	
$Q_{dp} = 4.087039$	$Q_{dp} = 4.007557$	
	parasitic pole - 40.404713	
$\omega_{np} = 2.000029$	$\omega_{np} = 1.999976$	
$Q_{np} = 11.985614$	$Q_{np} = 11.865251$	
	parasitic zero - 49.795870	

Tab. 4 Biquad simulation results

2.3 Dynamic Optimization

Design optimization oriented to the minimization of transfer function errors need not guarantee the achievement of the optimal internal dynamic properties of the resulting circuit. It means maximum amplifier output voltages equalization, and, as possible, near to the maximum output voltage of the whole biquad.

Such optimization requires to know the output voltage maxima of individual amplifiers, i.e. to find extremes of partial module frequency responses of the auxiliary transfer functions $H_{Vi} = V_{0(Vi)}/V_1$, where $i = 1, \ldots, 2n$, and n denotes a transfer function order.

To simplify the task, the following conditions can be taken into account:

a. An evaluation of auxiliary transfer function can be worked out under circuit simulation with ideal op-amps. Optimized design assumes only a small difference in behavior of an ideal and correctly designed real circuit.

b. Extremes of the 2^{nd}-order module frequency response appear near to the natural pole-frequency ω_p. With respect to this, search of a "pure" extreme can be replaced by selection of module value for $\omega = \omega_p$.

c. When point a. is accepted, then dynamic optimization, in its general form, is valid for all the types of converter amplifiers, because it is solved for an idealized circuit.

As evident, the optimization procedure itself includes evaluation of the auxiliary transfer function modules at the frequency of module extremes, especially in the 2^{nd}-order function case, at the frequency $\omega = \omega_p$, and a comparison of evaluated values. This task is not correctly solvable in a "pure" symbolic form because compared expressions contain absolute values and the condition of equal values of all the compared modules give more solutions which cannot be effectively processed. For this reason, the dynamic optimization procedure was incorporated into a complex design algorithm working with numerically expressed parameters of the given transfer function.

In conclusion we can say: The greater part of optimization criteria related to error minimization of transfer function parameters gives acceptable solution of dynamic ratios as well. On the contrary, when optimization of dynamic properties is preferred to achieve a maximum dynamic ratio, the dynamic optimization procedure provides an acceptable level of transfer function parameter errors. It is important to point out the results of dynamic optimization, in sense of circuit output voltage-to-amplifier maximum voltage ratio, strongly depend on appropriate correlation between converter element impedance-to divider branch impedance levels. When converter circuit element impedances are too high in comparison to the divider branch admittances, then the amplifier output voltages exceed biquad output voltage by about tens of dB. Such a situation is unacceptable from a practical point-of-view. Optimum results can be achieved only equalizing impedance levels of converter elements and divider branches (admittances Y_{ik}, $i = 1, 2$, $k = 0, 1, 2$). Impedance level equalization has to respect the condition of the sufficient distance (separation) of amplifier trans-resistance R_T to the circuit element impedance level as mentioned before. Infringement of this rule leads to essentially worse design results.

3. Design Examples

To demonstrate the aforementioned design procedures, some examples of optimized LP-, BP- and ES-sections are presented.

LP

Testing LP transfer function parameters (under eqn. 3): $\omega_p = 1.0$, $Q_p = 5.0$, $h = 1.0$, $a_0 = h\omega_p = 1.0$.

Evaluated circuit elements:

$Y_{10} = 0.475062$, $Y_{11} = Y_{12} = 1.0$,
$Y_{20} = 0.524938$, $Y_{21} = Y_{22} = 0.0$,
$R_1 = R_2 = R_3 = 0.904988$, $C_4 = 5.524938$,
$R_5 = R_6 = R_7 = 0.20$, $C_8 = 1.0$.

The design procedure used is valid for conventional op-amps. The basic optimization condition $R_2 = R_3$, $R_6 = R_7$ is considered together with the unity conversion function multiplicative constant. Dynamic optimization is not applied in this case, but, as can be observed in Fig. 5, the results are fully acceptable. Modified output V_1 is approximately 1 dB higher than remaining amplifier ones.

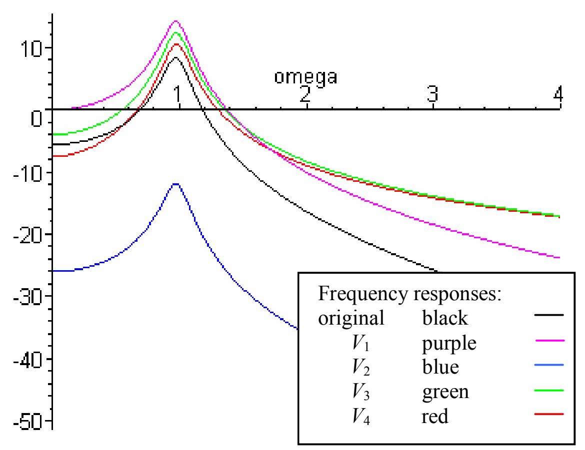

Fig. 5 Designed LP frequency response

Non-ideal circuit simulation results show appropriate errors of ω_p and Q parameters, corresponding to the optimization conditions used; for $B = \omega_T = 100$ were evaluated $\omega_{dp} = 0.980883$, $Q_{dp} = 5.086242$. B denotes gain-bandwidth product, ω_T means unity-gain frequency.

BP

Testing BP transfer function parameters (under eqn. 3): $\omega_p = 1.0$, $Q_p = 25.0$, $h = 1.0$, $a_1 = h\omega_p/Q_p = 0.040$.

Evaluated circuit elements:

$Y_{10} = 1.0$, $Y_{11} = 0.4950$, $Y_{12} = 1.0$,
$Y_{20} = 0.0$, $Y_{21} = 0.5050$, $Y_{22} = 0$,
$R_1 = R_2 = R_3 = 0.98020$, $C_4 = 25.5050$,
$R_5 = R_6 = R_7 = 0.040$, $C_8 = 1.0$.

Design conditions are similar to the first example. Non-ideal bandpass simulation (Amplifier Gain Bandwidth product equal to the LP circuit) gives ω_{dp} = 0.980581, Q_{dp} = 25.242814. Filter frequency response is shown in Fig. 6.

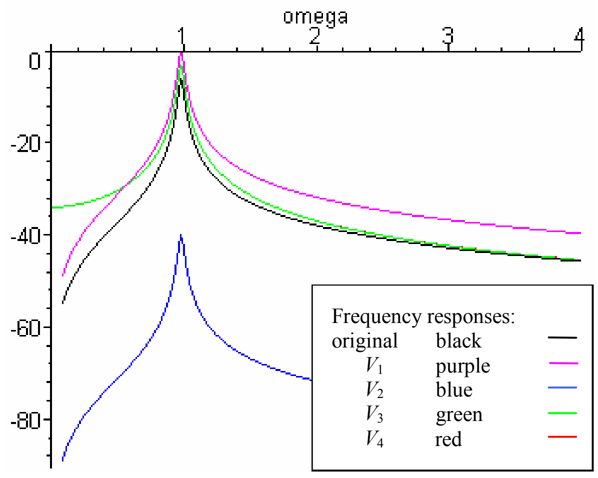

Fig. 6 BP frequency response

ES

Testing ES transfer function parameters (under eqn. 3): ω_p = 1.0, Q_p = 16.0, h = 0.444444, ω_n = 1.50.

Evaluated circuit elements:

$Y_{10} = 0.492189$, $Y_{11} = 1.406248$, $Y_{12} = 1.0$,
$Y_{20} = 1.307811$, $Y_{21} = 0.393752$, $Y_{22} = 0.80$,
$R_1 = R_2 = R_3 = 0.538466$, $C_4 = 29.714060$,
$R_5 = R_6 = R_7 = 0.034722$, $C_8 = 1.80$.

This example is the third from the set of "op-amp type" designs. The op-amps used are the same as the previous. Simulation results show small errors of ω_p and Q_p, similar to the LP and BP design:

$$\omega_p = 0.980594\,,\;\; Q_p = 16.2110\,,\;\; \omega_n = 1.49910\,.$$

The negligible error of ω_n is achieved by the design condition $R_2 = R_3$, $R_7 = R_6$. With respect to the properties of non-ideal GIC using conventional op-amps, transfer zero Q-factor significantly decreases to value Q_n = 23.3070. This effect causes distortion of frequency response (Fig. 7) and it needs designed parameters prewarping. Dynamic behavior is similar to the previous circuits (Fig. 8.

4. Conclusion

In this paper we have tried to present the prospective universal biquad arrangement based on a generalized divider principle. The proposed circuit allows easy realization of all the "standard" biquad transfer functions or arbitrary biquadratic functions by the appropriate choice of divider branch admittances. At the same time it saves an excellent sensitivity properties, fully comparable to preferred known low-sensitivity biquads.

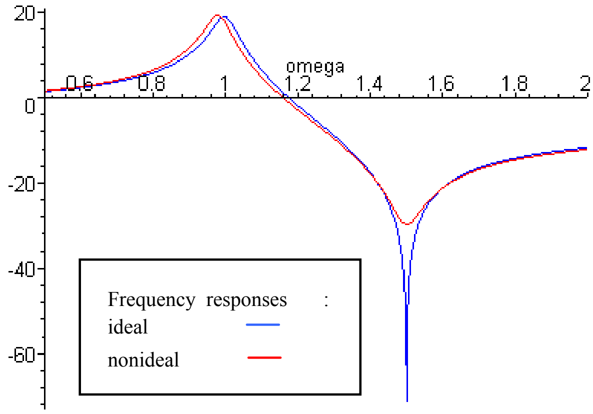

Fig. 7 Frequency response of the ES example

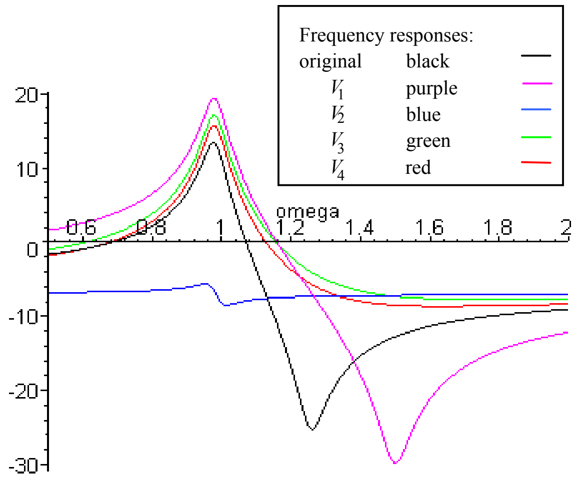

Fig. 8 Dynamic behavior of the ES

Great attention was devoted to circuit implementation containing transimpedance amplifiers as the main part of Antoniou's GIC circuitry. The reason is in their wide frequency band and the possibility to minimize errors of transfer function pole/zeroes caused by finite frequency dependent amplifier gain. Developed optimum-design algorithms were tested at the wide set of testing examples. With respect to the practical application aspects, TIA-based biquads should be suitable for filter implementation in relatively high frequency ranges (approximately to tens of MHz).

Apart from the fact that we did not explicitly mention the current-mode implementation, this topic was very carefully studied as well. The practical limitation of the current-mode option is in the necessity of floating-output amplifiers to realize transformed GIC circuitry. On the other hand, derived relationships between voltage-mode and current-mode generalized divider circuits published in [7] offer easy way how to realize current-mode biquads using developed optimum design algorithms.

With respect to a possible biquad implementation in CMOS technology, sampled-data switched-capacitor version of the basic circuit was studied and designed. The results are published in detail in [7] and they were partially presented in the Proceedings of ECCTD'99 [6].

References

[1] MARTINEK, P., BOREŠ, P., MATZNER, I. Selective circuits containing s^n immitance (in Czech). In Proceedings. of the workshop New ways in signal processing. Liptovský Mikuláš (Slovak Republic) 1990, vol. I., p. 14 - 17.

[2] TICHÁ, D., MARTINEK, P., BOREŠ, P. Equalizers Based On A Generalized Divider Principle (in Czech). In the Proceedings. of the International Conference New ways in signal processing. Liptovský Mikuláš (Slovak Republic), 1996, vol. II., p. 8 - 11.

[3] VRBA, K., ČAJKA, J. Special Nth-order One Ports Using Current Conveyors. In Proceedings of the International Conference CSS '96. Brno (Czech republic), 1996, vol. 2, p. 377-380.

[4] VRBA, K., ČAJKA, J., VRBA, R. Special Transform Blocks for Higher Order Immitance Element Realization. In Proc. Int. Conf. ITHURS '96. León, 1996, vol. 2, p. 411-416.

[5] MARTINEK, P. An Optimized design of circuits containing GICs. In Proceedings of the 8th International Czech-Slovak Scientific Conference Radioelektronika '98. Brno, 1998, p. 42-45.

[6] TICHÁ, D., MARTINEK, P. Biquad Based on a Generalized Divider Structure. In Proceedings of the 1999 European Conference on Circuit Theory and Design. (ECCTD '99). Torino: Politecnico di Torino, 1999. p. 229-232.

[7] TICHÁ, D. Integrated filters for telecommunications systems (in Slovak). PhD thesis, Žilina, 2000.

[8] TOUMAZOU, C., LIDGEY,F.J., HAIGH,D.G. Analogue IC Design: The Current-Mode Approach. Peter Peregrinus Ltd. 1993.

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