MIMO Spatial Diversity Communications — Signal Processing and Channel Capacity

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Abstract

This paper derives an equivalent discrete channel model for MIMO spatial diversity communications generally considering multidimensional spatial branch symbols and arbitrary path delays. This model is subsequently used for the information capacity evaluation under various special cases.

Keywords

MIMO spatial diversity, channel capacity, signal processing

1. Introduction

A purpose of this paper is to build a firm ground for the investigation of MIMO (Multiple Input Multiple Output) spatial diversity communications. The communication system uses wireless propagation medium which is inherently a continuous time waveform medium. However for the purpose of channel capacity investigation, code design, etc., it is useful to work with a discrete time model. This is done by deriving equivalent discrete model based on the signal space decomposition and the sufficient statistic principle. This somewhat appears to be an overlooked issue in most of the papers which traditionally start with the equivalent model not paying attention to its relation to the underlying continuous channel. Moreover we extend the results by considering arbitrary delay in the signal spatial paths and also by considering generally multidimensional spatial branch symbols. This allows us to consider also nonlinear modulation schemes unlike the most of other papers. By proper addressing of the issue of multidimensional symbols and arbitrary delays, we will see their influence on the MIMO space-time signal processing and overall properties of the system.

2. Spatial Diversity Systems

2.1 System model

Spatial diversity communication system is generally a communication system using a multidimensional channel where the channel dimensionality is physically resolved in spatial dimensions. This is usually achieved by a multi-element antenna arrays having a capability of distinguishing signals to/from various spatial angles.

We will assume a system with N_T dimensional channel input and N_R dimensional output. Such system will be denoted by (N_T, N_R). The modulated signal on the channel input is a N_T dimensional vector signal

$$\mathbf{s}(t) = \sum_n \mathbf{h}'(\mathbf{q}'_n, t - nT_S) \tag{1}$$

where $\mathbf{q}'_n$ are N_q dimensional channel symbols (codewords) and $\mathbf{h}'(\mathbf{q}'_n, t)$ are modulation functions describing the expansion part of the modulator. The discrete part of the modulator (the coder) is described by the output equation

$$\mathbf{q}'_n = \mathbf{q}'(d_n, \sigma_n), \quad \mathbf{q}'_n \in \{\mathbf{q}'^{(i)}\}_{i=1}^{M_q} \tag{2}$$

where $d_n \in \{d^{(i)}\}_{i=1}^{M_d}$ are data symbols and $\sigma_n \in \{\sigma^{(i)}\}_{i=1}^{M_\sigma}$ are the modulator states. The modulator states are ruled by the state equation

$$\sigma_{n+1} = \sigma(d_n, \sigma_n)\ . \tag{3}$$

An arbitrary modulated signal (including nonlinear modulations) can be expressed with a linear (however generally multidimensional) expansion part (for details see [1]). Thus we can write

$$\mathbf{s}(t) = \sum_n \mathbf{Q}_n^T \mathbf{h}(t - nT_S) \tag{4}$$

where

$$\mathbf{Q}_n = [\mathbf{q}_{n,1}, \ldots, \mathbf{q}_{n,N_T}] = \mathbf{Q}(d_n, \sigma_n) \in \{\mathbf{Q}^{(i)}\}_{i=1}^{M_q} \tag{5}$$

is $N_q \times N_T$ space-time channel symbol (codeword). N_q is a dimension of the channel symbol in one branch. In a special case of *linear modulation*, it becomes $N_q = 1$. Impulse $\mathbf{h}(t)$ is N_q dimensional modulation impulse which is assumed to be shared by all spatial dimensions. The impulse is further assumed to be a complex Nyquist one with orthonormal components (see [1]). Its energy time-domain correlation function is

$$\mathcal{R}_{\mathbf{h}}^{\mathcal{E}}[m] = \mathcal{R}_{\mathbf{h}}^{\mathcal{E}}(mT_S) = \delta[m]\mathbf{I} \,. \tag{6}$$

The modulated signal passes through (N_T, N_R). channel. The k-th element of the channel output vector $\mathbf{x}$ is a superposition of contributions from all channel inputs with additive white Gaussian noise (AWGN)

$$x_k(t) = u_k(t) + w_k(t) = \sum_{i=1}^{N_T} \mathcal{G}_{ki}[s_i(t)] + w_k(t) \,. \tag{7}$$

$u_k(t)$ is the useful signal at k-th receiver branch. Complex noise components in individual branches are assumed to be IID circularly symmetric Gaussian processes with correlation function $R_{w_k}(\tau) = 2N_0\delta(\tau)$. An individual contribution of the signal from i-th input to k-th output is generally described by the operator $\mathcal{G}_{ki}[.]$. In a special case of *frequency flat linear* channel this operator takes a form of

$$\mathcal{G}_{ki}[s_i(t)] = g_{ki} s_i(t - \tau_{ki}) \tag{8}$$

where g_{ki} are channel transfer coefficients and τ_{ki} are path delays. We also define a matrix $[\mathbf{G}]_{ki} = g_{ki}$. We consider a block-constant fading channel which has coefficients constant during the channel observation (typically a data frame). This will be assumed to hold in the following treatment.

2.2 Equivalent Signal Space Channel

Up to this point we treated the channel in the continuous time domain. However it is very useful to find its equivalent signal space model. This model uses a suitably defined signal vector expansions obtained by the orthonormal expansion. The orthonormality assumption guarantees the mutual numerical equivalence of the inner product operation in both domains. The signal space (contrary to the sampling) expansion is necessary in order to relate the equivalent model directly to the channel symbols (codewords).

The situation on the transmitter side is easy to handle since we assumed Nyquist modulation impulses with orthonormal components. Therefore the signal space expansion for n-th symbol is easily recognized as $\mathbf{s}_n = \mathbf{Q}_n$ with the expansion basis $\mathbf{h}(t - nT_S)$. Because of the Nyquist modulation impulse assumption the whole signal expansion $\mathbf{s} = [\ldots, \mathbf{s}_n, \ldots]$ has orthonormal basis $\mathcal{A} = \{\mathbf{h}(t - nT_S)\}_n$.

Contrary to the transmitter side, the situation at the receiver side is somewhat more complicated. This is because of the channel parameters. Particularly we allowed arbitrary delays τ_{ki} on individual branches. The expansion basis and the corresponding signal with a simple relation to transmitted codewords (it does not have to be directly the received signal) cannot be identified by a similar simplistic approach. We need to adopt a systematic procedure based on the information theory—namely using the principle of sufficient statistic. For a good background see e.g. [2] or [3].

We will assume that the receiver has a *perfect knowledge* of the channel state information (CSI), i.e. the gains g_{ki} and delays τ_{ki}. A conditional probability density function (PDF) of the received signal conditioned by the transmitted signal is

$$\begin{aligned} p(\mathbf{x}\,|\,\mathbf{s}) &= c \exp\left(-\frac{1}{2N_0} \sum_{k=1}^{N_R} \int_{-\infty}^{\infty} |\, x_k - u_k(\mathbf{s})\,|^2 \, dt \right) \\ &= c \exp\left(-\frac{\sum_{k=1}^{N_R} \int_{-\infty}^{\infty} |\, x_k\,|^2 - 2\,\mathrm{Re}[x_k u_k^*(\mathbf{s})] + |\, u_k(\mathbf{s})\,|^2 \, dt}{2N_0} \right). \end{aligned} \tag{9}$$

We utilized the equivalence of inner product values in the Hilbert space of continuous signals and their signal space vector representations. The basis for the received signal $x(t)$ is not important and can be get by an extension of the basis $\mathcal{A}$ in order to be complete. The sufficient statistic will be found (based on the Neyman-Fisher factorization theorem) inside the inner product evaluation

$$\begin{aligned} &\sum_{k=1}^{N_R} \int_{-\infty}^{\infty} x_k u_k^* dt = \\ &= \sum_{k=1}^{N_R} \int_{-\infty}^{\infty} x_k \sum_{i=1}^{N_T} g_{ki}^* \sum_n \mathbf{q}_{n,i}^H \mathbf{h}^*(t - \tau_{ki} - nT_S) dt \,. \end{aligned} \tag{10}$$

Clearly, on condition of perfect CSI knowledge at receiver the sufficient statistic is

$$\begin{aligned} \mathbf{y}_{n,k,i} &= \mathbf{T}_k(\mathbf{x}) = \int_{-\infty}^{\infty} x_k \mathbf{h}^*(t - \tau_{ki} - nT_S) dt, \\ & i \in \{1, \ldots, N_T\}, k \in \{1, \ldots, N_R\}. \end{aligned} \tag{11}$$

The crosscorrelation part of the receiver detector metric is then get from the sufficient statistic

$$\sum_{k=1}^{N_R} \int_{-\infty}^{\infty} x_k u_k^* dt = \sum_{k=1}^{N_R} \sum_{i=1}^{N_T} g_{ki}^* \sum_n \mathbf{q}_{n,i}^H \mathbf{y}_{n,k,i} \,. \tag{12}$$

Based on this statistic, we can equivalently investigate the signal space channel with the input for n-th symbol

$$\mathbf{s}_n = \mathbf{Q}_n = [\mathbf{q}_{n,1}, \ldots, \mathbf{q}_{n,N_T}] \tag{13}$$

and the output $\mathbf{y}_{n,k,i}$ related to the received signal $x_k(t)$ by the equation above. It is important to stress that the perfect CSI knowledge was necessary.

The properties analysis of the new equivalent channel output will reveal its relation to transmitted signal and channel gains—which was the goal of our effort. We substitute for the received signal to get

$$\mathbf{y}_{n,k,i} = \int_{-\infty}^{\infty} \left(\sum_{i'=1}^{N_T} g_{ki'} \sum_{n'} \mathbf{q}_{n',i'}^T \mathbf{h}(t - \tau_{ki'} - n'T_S) + w_k(t) \right) \times \mathbf{h}^*(t - \tau_{ki} - nT_S) dt. \tag{14}$$

Skipping some algebra we get

$$\mathbf{y}_{n,k,i} = \sum_{i'=1}^{N_T} g_{ki'} \sum_{n'} \mathcal{R}_{\mathbf{h}}^{\mathcal{E}*} \left((n-n')T_S + \tau_{ki} - \tau_{ki'} \right) \mathbf{q}_{n',i'} + \mathbf{z}_{n,k,i}. \tag{15}$$

The noise component can be easily verified to be complex circularly symmetric Gaussian with covariance matrix

$$\mathbf{C}_{\mathbf{z}_{n,k,i}} = \sigma_z^2 \mathbf{I} = 2N_0 \mathbf{I} . \tag{16}$$

In a general case of arbitrary and *mutually unequal* paths delays, the expression for $\mathbf{y}_{n,k,i}$ suffers from the inter-symbol interference from all transmitter branches. Amount of the interfering symbols increases with the dimensionality of the MIMO system and therefore have bigger impact than in the case of SISO system. As we observe, the problem is caused by existence of T_S-fractional delay differences, which is something that we have to consider in the real system deployment. Unfortunately, the consequences of this fact and efficient counter measures are still far from being well understood and they are widely ignored. For some results mapping this area see e.g. [4]. This problem needs to be much more focused in the future research.

On the other side, in the case of *mutually equal* delays τ_{ki}=const., we get using the properties of modulation impulse

$$\mathbf{y}_{n,k,i} = \sum_{i'=1}^{N_T} g_{ki'} \mathbf{q}_{n',i'} + \mathbf{z}_{n,k,i} . \tag{17}$$

We can simplify previous results into a simple matrix notation (dropping now superfluous dependence on i and by stacking the vectors)

$$\mathbf{y}_n = \mathbf{G}\mathbf{q}_n + \mathbf{z}_n \tag{18}$$

where the stacked vectors are $\mathbf{y}_n = [\mathbf{y}_{n,1}, \ldots, \mathbf{y}_{n,N_R}]^T$, $\mathbf{q}_n = \mathbf{Q}_n^T = [\mathbf{q}_{n,1}, \ldots, \mathbf{q}_{n,N_R}]^T$, $\mathbf{z}_n = [\mathbf{z}_{n,1}, \ldots, \mathbf{z}_{n,N_R}]^T$. The transposition of stacked vectors and also the multiplication by $\mathbf{G}$ is understood in the vector-wise manner, i.e. $\mathbf{q}_n = \mathbf{Q}_n^T = [\mathbf{q}_{n,1}, \ldots, \mathbf{q}_{n,N_T}]^T$ is N_T×1 vector of N_q×1 vectors. It is also very useful to realize that the dimensionality of individual transmitter branches (which is greater than one for nonlinear or block coded modulations) in fact multiplies with the spatial dimensionality to the overall dimensionality of the equivalent channel.

From the results above, we can conclude that there exists the discrete signal space equivalent representation of general MIMO system with multidimensional branch symbols. Multidimensionality of the branch symbol allows the model to be used for general nonlinear and block coded modulations. The output of this discrete system corresponds to the output signal $y_k(t)$ which forms the sufficient statistic, Perfect CSI knowledge is required. A necessary condition for avoiding ISI is the modulation impulse being Nyquist one with orthonormal components and mutually equal branch delays. This model will be assumed in the subsequent treatment.

2.3 Channel Eigenmode

We start our investigation from the equivalent channel model derived above, i.e.

$$\mathbf{y} = \mathbf{G}\mathbf{q} + \mathbf{z} . \tag{19}$$

We dropped the explicit dependence on the index n for the brevity of the notation. From now on, everything relates to the n-th channel symbol by default. The channel matrix $\mathbf{G}$ can be expressed using Singular Value Decomposition (SVD) as a product of two unitary matrices $\mathbf{U}$,($N_R \times N_R$) and $\mathbf{V}$,($N_T \times N_T$) (orthonormal columns $\mathbf{U}^H\mathbf{U}=\mathbf{I}$) and a diagonal matrix $\mathbf{D}$

$$\mathbf{G} = \mathbf{U}\mathbf{D}\mathbf{V}^H . \tag{20}$$

Matrix $\mathbf{D}$ is a diagonal $N_R \times N_T$ matrix with nonzero values

$$\sqrt{\lambda_1}, \ldots, \sqrt{\lambda_{N_G}} \tag{21}$$

at first N_G main diagonal positions. These are the nonzero eigenvalues of the matrix $\mathbf{G}\mathbf{G}^H$. N_G is the rank of the same matrix

$$N_G = \mathrm{rank}(\mathbf{G}\mathbf{G}^H) . \tag{22}$$

This value is called a rank of the MIMO system.

Substituting the SVD into the channel model and multiplying the result from the left-hand side by $\mathbf{U}^H$, we get

$$\mathbf{y}' = \mathbf{D}\mathbf{q}' + \mathbf{z}' \tag{23}$$

where

$$\mathbf{y}' = \mathbf{U}^H \mathbf{y}, \quad \mathbf{q}' = \mathbf{V}^H \mathbf{q}, \quad \mathbf{z}' = \mathbf{U}^H \mathbf{z} . \tag{24}$$

From the point of view of these new transformed channel input and output, the channel appears to be separated (multiplexed) into N_G independent (orthogonal) channels. The unitary linear transformations preserve the mutual information. This will be later used when evaluating the channel information capacity.

Utilizing properties of unitary matrices, we get

$$\mathbf{q} = \mathbf{V}\mathbf{q}' . \tag{25}$$

Using this result and substituting back into the channel model we get

$$\mathbf{y}' = \mathbf{U}^H \underbrace{\left(\mathbf{G}\underbrace{(\mathbf{V}\mathbf{q}')}_{\mathbf{q}} + \mathbf{z} \right)}_{\mathbf{y}} . \tag{26}$$

From the input signal **q’** and output signal **y’** point of view, the channel appears to be multiplexed into independent channels. This expression is in fact a serial concatenation of the following operations.

1. The real signal **q** is get from **q’** by unitary transformation.
2. This signal is passed through the real channel.
3. At the output of the real channel, we use another unitary transformation to obtain signal **y’** which is the output of virtual multiplexed channel.

We see that the unitary transformations were able to transform the input and output signal into virtual multiplexed channel. The corresponding transformations depend on the particular channel matrix **G** and they are called the channel *eigenmode*.

3. Information Capacity

The idea of the information capacity evaluation is based on the equivalent channel decomposition into independent scalar channels [5]. The capacity of scalar channel is well know Shannon formula. We extend these results for generally multidimensional transmitter branch channel symbols which allows an application of the results also to nonlinear or block-coded schemes. We start with the capacity of deterministic channel and we proceed with the case of random Rayleigh IID (Independent and Identically Distributed) fading channel. In the following, we will use these two auxiliary theorems.

Lemma 1. Let **a,b** be two arbitrary size column random vectors. Let **U,V** be two properly sized unitary matrices. We define $\mathbf{a}'=\mathbf{U}^H\mathbf{a}$ and $\mathbf{b}'=\mathbf{V}^H\mathbf{b}$. Then the following holds for mutual information

$$I(\mathbf{a};\mathbf{b}) = I(\mathbf{a}';\mathbf{b}') . \tag{27}$$

This means that the unitary transformation does not change the mutual information.

Proof of this lemma easily follows from the definition of mutual information

$$I(\mathbf{a};\mathbf{b}) = E\left[\log_2 \frac{p(\mathbf{b}\,|\,\mathbf{a})}{p(\mathbf{b})} \right] . \tag{28}$$

Then we utilize the expression for the probability density function transformation and the fact that the determinant and also Jacobian of any unitary transformation is equal to unity.

Lemma 2. Let **a** be a random vector with diagonal covariance matrix $\mathbf{C_a}=\sigma^2\mathbf{I}$. Let **U** be an unitary matrix. The unitary transformation $\mathbf{a}'=\mathbf{U}^H\mathbf{a}$ preserves the covariance

$$\mathbf{C}_{\mathbf{a}'} = \mathbf{C}_{\mathbf{a}} = \sigma^2 \mathbf{I} . \tag{29}$$

Proof trivially follows from the definition of covariance matrix and unitary matrix properties.

3.1 Deterministic Channel

The lemma 1 and 2 above imply that we can investigate equivalent channel $\mathbf{y}'=\mathbf{D}\mathbf{q}'+\mathbf{z}'$ for the capacity evaluation instead of the original one. The equivalent channel is however formed as N_G mutually independent N_q dimensional channels. For a given spatial dimension branch, each channel branch symbol dimension has the common scalar gain $\sqrt{\lambda_i}$ and IID noise.

The previous statement can be formalized in two ways. The first possibility is to split stacked vector notation into N_q individual vector component wise equations, each for the particular dimension within the spatial branch. These individual equations will have the same channel matrix and noise properties—hence producing N_q equivalent independent channels. The second possible way is to rewrite the stacked vector equation as one large component wise equation. Clearly the component wise channel matrix will be $(N_q N_R) \times (N_q N_T)$ matrix

$$\tilde{\mathbf{G}} = \mathbf{G} \otimes \mathbf{I}_{N_q} \tag{30}$$

where $\otimes$ is the Kronecker matrix product. The SVD decomposition reveals, as expected, the N_q multiple presence of each singular value of the original **G** matrix.

For these independent channels, the overall capacity is a sum of individual capacities which can be obtained from the standard Shannon formula. The capacity per one channel (spatial) symbol is

$$C = \max_{\mathbf{C_{q'}}:\mathcal{V}} \sum_{m=1}^{N_q} \sum_{i=1}^{N_G} \log_2 \left(1 + \frac{\lambda_i E\left[|\, q'_{i,m} \,|^2 \right]}{\sigma_z^2} \right) . \tag{31}$$

The maximization is performed over all symbol covariance matrices $\mathbf{C_{q'}}$ subject to the condition $\mathcal{V}$. The maximum is achieved for symbols with *zero mean Gaussian distribution independent over the spatial branches and symbol dimensions*. From the lemma 2, it follows that also **q** will be zero mean Gaussian with the same diagonal covariance matrix. The condition can have various particular forms, however the two most important ones follow.

The *CSI is known* at the transmitter side and the total mean complex envelope symbol energy is constrained by $\mathcal{E}_m$

$$\mathcal{E}_S = E\left[\|\mathbf{q}'\|^2\right] \le \mathcal{E}_m . \tag{32}$$

We utilized the fact that the modulation impulse is the Nyquist one with orthonormal components. In this case the optimal symbol energy distribution is given by the constrained optimization theorem (Khun-Tucker conditions, see e.g. [6]). The solution is known as a "water-filling" theorem [3]

$$\mathcal{E}_i = \left(\mu - \frac{\sigma_z^2}{\lambda_i}\right)^+ \tag{33}$$

where $\mathcal{E}_i$ is the mean complex envelope spatial branch symbol energy, $(a)^+ = \max(0,a)$ and the value μ is chosen such that the total energy criterion is fulfilled

$$\mathcal{E}_m = \sum_{i=1}^{N_G} \mathcal{E}_i = \sum_{i=1}^{N_G} \left(\mu - \frac{\sigma_z^2}{\lambda_i}\right)^+ . \tag{34}$$

The energy per spatial branch $\mathcal{E}_i$ is evenly split across the branch symbol dimensions N_q

$$E\left[|q'_{i,m}|^2\right] = \frac{\mathcal{E}_i}{N_q} . \tag{35}$$

The resulting capacity is

$$C_{\mathrm{CSI}} = N_q \sum_{i=1}^{N_G} \log_2\left(1 + \frac{\lambda_i \mathcal{E}_i}{N_q \sigma_z^2}\right). \tag{36}$$

Notice that the capacity per symbol is expressed in terms of the *mean symbol energy*. This is a direct consequence of the mean value $E[\|\mathbf{q}'\|^2]$ being the symbol energy. An appearance of the channel symbol here follows from our previous transformation to the discrete channel equivalent model based on *signal space* expansion. Notice that some authors consider the equivalent model to be rather defined on the basis of signal samples. This would imply that the mean quantity above has physical interpretation as a power of the ergodic process.

The second typical maximization constrain is the one where the *CSI is not available* at the transmitter and only total symbol energy constrain is put in effect. In this case the maximizing solution is to split the energy evenly over all spatial branches as well as over dimensions within the branch

$$\mathcal{E}_i = \frac{\mathcal{E}_m}{N_T}, \qquad E\left[|q'_{i,m}|^2\right] = \frac{\mathcal{E}_i}{N_q} . \tag{37}$$

Notice that since there is no CSI at the transmitter side we must distribute the energy uniformly over *all* spatial dimensions N_T unlike the case of known CSI. The resulting capacity is

$$C = N_q \sum_{i=1}^{N_G} \log_2\left(1 + \frac{\lambda_i \mathcal{E}_m}{N_q N_T \sigma_z^2}\right). \tag{38}$$

Very often, the following formal form of the capacity equation is useful

$$C = N_q \log_2 \det\left(\mathbf{I} + \frac{\mathcal{E}_m}{N_q N_T \sigma_z^2} \mathbf{G}\mathbf{G}^H\right). \tag{39}$$

We obtain this expression utilizing the following matrix algebra identities

$$\begin{aligned} \det \mathbf{A} &= \prod_i \operatorname{eig}_i(\mathbf{A}), \\ \operatorname{eig}_i(\mathbf{I}+\mathbf{A}) &= 1 + \operatorname{eig}_i(\mathbf{A}), \\ \operatorname{eig}_i(\alpha\mathbf{A}) &= \alpha \operatorname{eig}_i(\mathbf{A}). \end{aligned} \tag{40}$$

An essential observation that can be drawn from the both cases is that the channel capacity scales *linearly with the rank of the channel* and *dimensionality of symbols*. This means that for a given fixed energy per symbol to noise variance, the MIMO system has great capacity advantage over a scalar system.

3.2 Rayleigh IID Fading Channel

In the previous text, we treated an instantaneous channel capacity for a given channel matrix **G**. However, for a random fading channel, we are rather interested in the average capacity. Let us assume that the channel is ergodic, i.e. its randomness exhibits itself entirely within the channel observation frame. On this condition, the ergodic average capacity (no CSI at the transmitter) is get by averaging over all channel states

$$\begin{aligned} \overline{C} &= E_{\mathbf{G}}[C] \\ &= E_{\mathbf{G}}\left[N_q \log_2 \det\left(\mathbf{I} + \frac{\mathcal{E}_m}{N_q N_T \sigma_z^2} \mathbf{G}\mathbf{G}^H\right)\right]. \end{aligned} \tag{41}$$

In the case of nonergodic channel, we must resort to the outage capacity which is in fact a probability distribution of instantaneous capacity.

The capacity averaging for a general case of arbitrary N_T, N_R is mathematically rather involved task. Details of the derivation can be found in [5]. The resulting expression for IID zero mean complex Gaussian channel transfer coefficients with unity variance is

$$\overline{C} = N_q \int_0^\infty \log_2\left(1 + \frac{\mathcal{E}_m \lambda}{N_q N_T \sigma_z^2}\right) \times \sum_{k=0}^{m-1} \frac{k!}{(k+n-m)!} \left(L_k^{n-m}(\lambda)\right)^2 \lambda^{n-m} e^{-\lambda} d\lambda \qquad (42)$$

where m=min(N_T, N_R), n=max(N_T, N_R) and $L^i_j(.)$is Laguerre polynomial.

It can be shown [5] that for a special, however significant, case of large $N_T = N_R$ the average capacity per symbol again linearly scales with the dimensionality of the channel

$$\overline{C} = N_q N_R \int_0^4 \log_2\left(1 + \frac{\mathcal{E}_m \nu}{N_q \sigma_z^2}\right) \frac{1}{\pi} \sqrt{\frac{1}{\nu} - \frac{1}{4}} d\nu \,. \qquad (43)$$

A graphical representation of this equation is on Fig. 1 and Fig. 2. It is very important to notice a limiting behavior of its dependence on the symbol dimensionality for a constant mean symbol energy

$$\lim_{N_q \to \infty} \overline{C} = N_r \frac{\mathcal{E}_m}{\sigma_z^2 \ln 2} \,. \qquad (44)$$

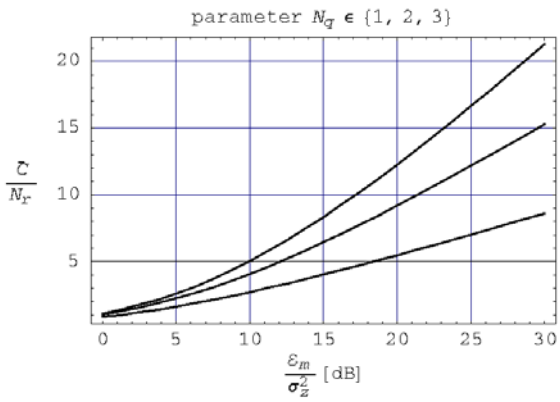

Fig. 1 Channel capacity (43) as a function of symbol energy to noise variance ratio.

4. Conclusions

We developed an equivalent discrete channel model for MIMO space-time communications generally allowing a multidimensional channel symbols and arbitrary path delays. Subsequently we analyzed information capacity of the channel under several special cases. The most important observation is that the capacity of MIMO system linearly scales with the dimensionality of the channel. This creates a substantial advantage over the scalar system with the same energy per symbol and the same noise.

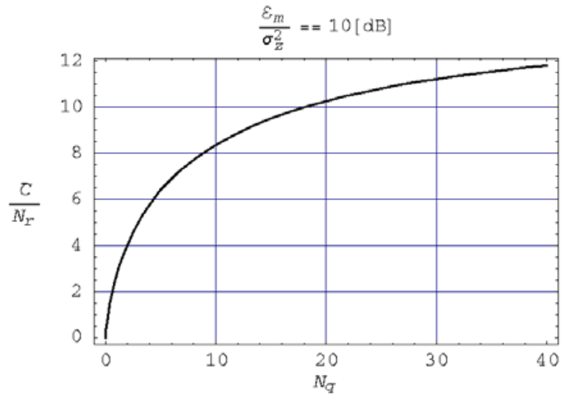

Fig. 2 Channel capacity (43) as a function of symbol dimensionality.

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