New Second-Order Optimized Filter Design

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Abstract. *Starting from the piecewise-linear (PWL) autonomous dynamical system optimized from the eigenvalue sensitivities viewpoint the corresponding optimized non-autonomous linear (single-input single-output) system is derived. Such a design procedure gives the possibility to obtain minimum eigenvalue sensitivities with respect to the change of the individual model parameters also for non-autonomous linear systems. Two examples of the system having the complex conjugate poles and zeros, i.e. the optimized second-order band-reject and all-pass filter design, are shown.*

Keywords

Dynamical systems, second-order systems, state models, sensitivity properties, optimized design.

1. Introduction

In the recent papers [1], [2], some new results in the field of linear and piecewise-linear (PWL) dynamical systems have been published. It is especially:

(i) Generalized mutual relation between non-autonomous linear and autonomous PWL systems.

(ii) State model of autonomous PWL systems with optimized eigenvalue sensitivities.

These two results give a natural possibility to convey optimized properties of the autonomous PWL system to transfer function of the linear non-autonomous system, i.e. to obtain the new optimized design procedure for linear systems. As an examples two typical systems with complex conjugate poles and zeros of the corresponding transfer function, i.e. the second-order band-reject and all-pass filter, are introduced.

2. Relation between PWL Autonomous and Linear Non-Autonomous Dynamical Systems

Autonomous PWL systems of Class C [3], [4] can be described by the general state matrix form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}\ \mathrm{h}\ (\mathbf{w}^{\mathrm{T}}\mathbf{x}) \tag{1}$$

where the elementary PWL feedback function (Fig. 1)

$$\mathrm{h}\ (\mathbf{w}^{\mathrm{T}}\mathbf{x}) = \frac{1}{2}\left(\left|\mathbf{w}^{\mathrm{T}}\mathbf{x}+1\right| - \left|\mathbf{w}^{\mathrm{T}}\mathbf{x}-1\right|\right) \tag{2}$$

contains the regions D_0 and D_{+1} (D_{-1}). General block diagram corresponding to basic eqn. (1) is shown in Fig. 2.

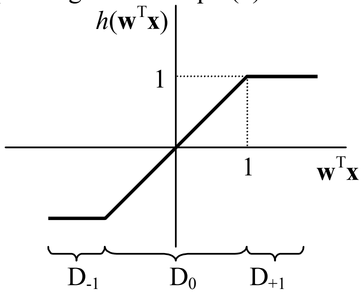

Fig. 1. Simple memoryless PWL feedback function.

The dynamical behavior of this system is determined by two characteristic polynomials associated to the individual regions, i.e.

$$D_0:\ P(s) = (s-\mu_1)(s-\mu_2)\dots(s-\mu_n) = \det(s\mathbf{1}-\mathbf{A}) =$$
$$= s^n - p_1 s^{n-1} + p_2 s^{n-2} - \dots + (-1)^{n+1} p_{n-1} s + (-1)^n p_n, \tag{3}$$

$$D_{+1}, D_{-1}:\ Q(s) = (s-\upsilon_1)(s-\upsilon_2)\dots(s-\upsilon_n) = \det(s\mathbf{1}-\mathbf{A}) =$$
$$= s^n - q_1 s^{n-1} + q_2 s^{n-2} - \dots + (-1)^{n+1} q_{n-1} s + (-1)^n q_n \tag{4}$$

where $\mathbf{1}$ is the unity matrix. Their roots represent the eigenvalues of the corresponding state matrices and their coefficients are the so-called equivalent eigenvalue parameters [4], [5].

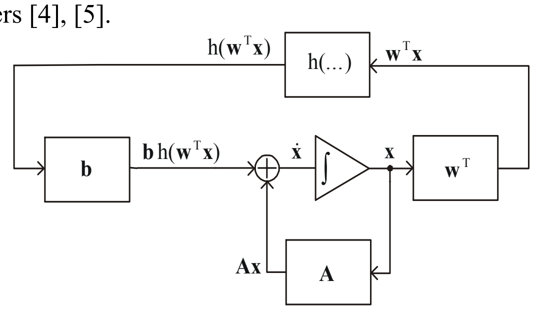

Fig. 2. General block diagram of an autonomous PWL dynamical system described by eqn. (1).

Any non-autonomous linear system with single input (v variable) and single output (y variable) can be described by the general state matrix equations

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\,v \tag{5}$$

$$y = \mathbf{C}\mathbf{x} + \mathrm{D}\,v \quad . \tag{6}$$

The corresponding general block diagram is shown in Fig. 3. Using the Laplace transform the transfer function of the non-autonomous linear system is generally given [2] as

$$\mathrm{K}(s) = \frac{Y(s)}{V(s)} = \mathrm{D} + \mathbf{C}(s\mathbf{1} - \mathbf{A})^{-1}\mathbf{B} = \ldots = -\frac{P(s)}{Q(s)} \tag{7}$$

if the following conditions are valid

$$\mathbf{B} = \mathbf{b} \ , \ \mathbf{C} = \mathbf{w}^{\mathrm{T}} \ \text{ and } \ \mathrm{D} = -1 \ . \tag{8}$$

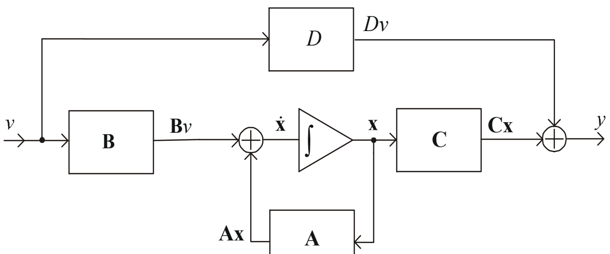

Fig. 3. General block diagram of a non-autonomous linear dynamical system described by eqns. (5) and (6).

3. Optimized State Model of the 2nd-Order Dynamical Systems

3.1 PWL Autonomous System

Considering the complex conjugate eigenvalues in the outer regions D_{+1}, D_{-1} ($\nu_{1,2}=\nu'\pm j\nu''$) as well as in the inner region D_0 ($\mu_{1,2} = \mu'\pm j\mu''$), the optimized state matrices corresponding to the outer and inner regions can be chosen in simplified and decomposed complex form [3], [7], i.e.

$$\mathbf{A} = \begin{bmatrix} \nu' & -\nu'' \\ \nu'' & \nu' \end{bmatrix} \ \text{ and } \ \mathbf{A}_0 = \begin{bmatrix} \mu' & -\mu''\mathrm{K} \\ \mu''\mathrm{K}^{-1} & \mu' \end{bmatrix}, \tag{9a,b}$$

respectively. These state matrices can mutually be expressed by the relation [4]

$$\mathbf{A}_0 = \mathbf{A} + \mathbf{b}\mathbf{w}^{\mathrm{T}} \ , \tag{10}$$

where $\mathbf{b} = \begin{bmatrix} \mathrm{b}_1 \\ \mathrm{b}_2 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} \mathrm{w}_1 \\ \mathrm{w}_2 \end{bmatrix}$.

The optimizing coefficient K in eqn. (9b) is expressed as the real root of the quadratic equation

$$\mathrm{K}^2 - 2\mathrm{K}(\mathrm{M}+1) + 1 = 0 \ \text{, i.e. } \ \mathrm{K} = 1 + \mathrm{M} \pm \sqrt{\mathrm{M}(\mathrm{M}+2)}$$

and the auxiliary parameter M is given in the form

$$\mathrm{M} = \frac{(\mu'-\nu')^2 + (\mu''-\nu'')^2}{2\mu''\nu''} > 0 \ , \quad (\mu'',\nu''\neq 0).$$

Choosing $w_1 = 1$, the other parameters are obtained as

$$b_1 = \mu' - \nu' \ , \quad b_2 = \frac{(\mu'-\nu')^2}{\nu''-\mu''\mathrm{K}} \ , \quad w_2 = \frac{\nu''-\mu''\mathrm{K}}{\mu'-\nu'} \tag{11a,b,c}$$

and the complete state equations of the optimized second-order PWL autonomous system can be written in the form

$$\dot{x}_1 = \nu'[x_1 - \mathrm{h}\,(x_1 + w_2 x_2)] - \nu'' x_2 + \mu'\,\mathrm{h}\,(x_1 + w_2 x_2) \ , \tag{12}$$

$$\dot{x}_2 = \nu'' x_1 + \nu' x_2 + b_2\,\mathrm{h}\,(x_1 + w_2 x_2) \tag{13}$$

where the parameters b_2 and w_2 are given by the formulas (11b,c). The corresponding integrator-based circuit block diagram, suitable as the prototype for the practical realization, is shown in Fig. 4.

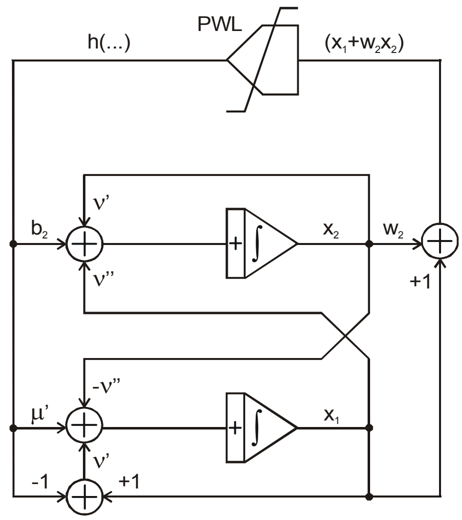

Fig. 4. Integrator-based circuit structure of the 2nd-order PWL state model with minimized sensitivities.

3.2 Linear Non-Autonomous System

Starting from optimized PWL autonomous system with complex conjugate eigenvalues, utilizing the conditions (8), the complete optimized form of the state equations (5) and (6) for the corresponding linear system is obtained, i.e.

$$\dot{x}_1 = \nu'(x_1 - v) - \nu'' x_2 + \mu' v \ , \tag{15}$$

$$\dot{x}_2 = \nu'' x_1 + \nu' x_2 + b_2 v \ , \tag{16}$$

$$y = x_1 + w_2 x_2 - v \ . \tag{17}$$

The corresponding transfer function (7) is (18a)

$$\mathrm{K}(s) = \frac{\mathrm{Y}(s)}{\mathrm{V}(s)} = -\frac{\mathrm{P}(s)}{\mathrm{Q}(s)} = -\frac{s^2 - (\mu_1 + \mu_2)s + \mu_1\mu_2}{s^2 - (\nu_1 + \nu_2)s + \nu_1\nu_2}$$

that corresponds to general second-order form

$$K(s) = K_\infty \frac{s^2 + [\omega_z/Q_z]s + \omega_z^2}{s^2 + [\omega_0/Q_0]s + \omega_0^2} \tag{18b}$$

where the individual parameters are

$$\omega_0 = \sqrt{\nu'^2 + \nu''^2}\,, \quad Q_0 = \frac{1}{-2\nu'}\sqrt{\nu'^2 + \nu''^2}\,, \tag{19a,b}$$

$$\omega_z = \sqrt{\mu'^2 + \mu''^2}\,, \quad Q_z = \frac{1}{-2\mu'}\sqrt{\mu'^2 + \mu''^2}\,. \tag{20a,b}$$

The corresponding integrator-based circuit block diagram, suitable as the prototype for the practical realization, is shown in Fig. 5.

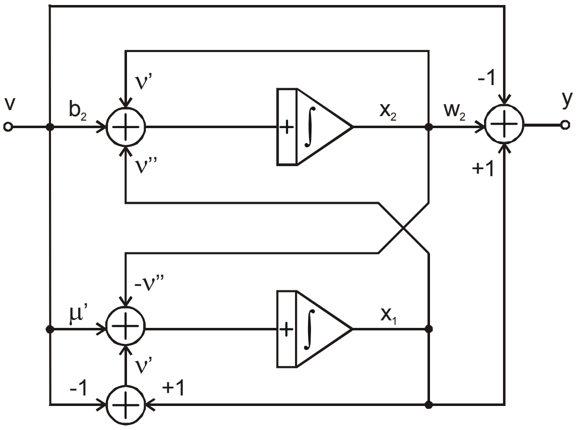

Fig. 5. Integrator-based circuit structure of the 2nd-order linear state model with minimized sensitivities.

4. Applications to 2nd-Order Filters with Optimized Sensitivities

4.1 Band-Reject Filters

As the first example the second-order band-reject filter with complex conjugate poles and imaginary conjugate zeros of the transfer function (18a) is introduced. Here the parameter $Q_z \to \infty$, i.e.

$$\mu' = 0\,, \text{ and } \quad \mu_{1,2} = \pm\, j\mu'' = \pm\, j\omega_z \tag{21}$$

as follows from eqns. (20a,b). It is well known that three different cases can exist as shown in Fig. 6, i.e.

a) $\omega_z > \omega_0$ - (Fig. 6a),

b) $\omega_z = \omega_0$ - (Fig. 6b),

c) $\omega_z < \omega_0$ - (Fig. 6c) .

Then the complete optimized form of the state equations is

$$\dot{x}_1 = \nu'(x_1 - v) - \nu'' x_2\,, \tag{22}$$

$$\dot{x}_2 = \nu'' x_1 + \nu' x_2 + b_2 v\,, \tag{23}$$

$$y = x_1 + w_2 x_2 - v \tag{24}$$

where the parameters b_2 and w_2 can be expressed as

$$b_2 = \frac{-\nu'}{w_2}\,, \quad w_2 = \frac{\mu'' K - \nu''}{\nu'}\,. \tag{25}$$

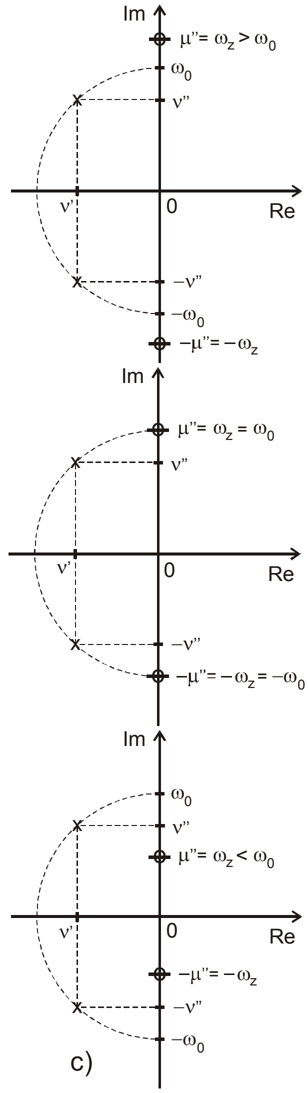

Fig. 6. Zeros and poles of the 2nd-order band-reject filter. a) $\omega_z > \omega_0$, b) $\omega_z = \omega_0$, c) $\omega_z < \omega_0$.

The corresponding integrator-based circuit block diagram, suitable also as the prototype for the practical band-reject filter realization, is shown in Fig. 7. Its transfer function (18) has the following special form

$$\mathrm{K}(s) = \frac{\mathrm{Y}(s)}{\mathrm{V}(s)} = -\frac{\mathrm{P}(s)}{\mathrm{Q}(s)} = -\frac{s^2 + \mu''^2}{s^2 - 2\nu' s + (\nu'^2 + \nu''^2)}$$

which corresponds to general form

$$K(s) = K_\infty \frac{s^2 + \omega_z^2}{s^2 + [\omega_0/Q_0]s + \omega_0^2} \tag{26b}$$

where the individual parameters are:

$$K_\infty = -1,\ \omega_z = \mu'',\ \omega_0 = \sqrt{\nu'^2 + \nu''^2},\ Q_0 = \frac{1}{-2\nu'}\sqrt{\nu'^2 + \nu''^2}$$

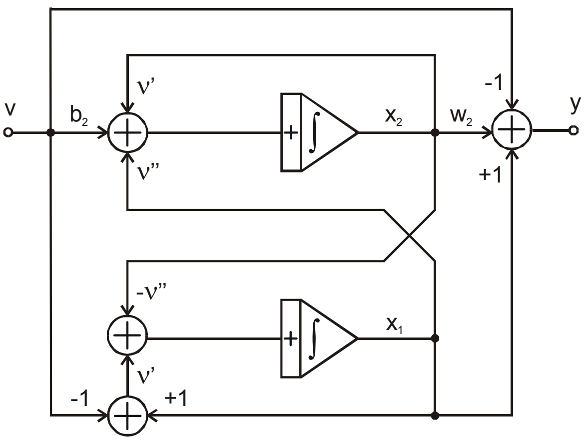

Fig. 7. Integrator-based circuit structure of the 2nd-order band-reject filter with minimized sensitivities.

Typical magnitude characteristics in frequency domain for all three relations between ω_0 and ω_z (Fig 6a, b, c) are introduced in Fig. 8.

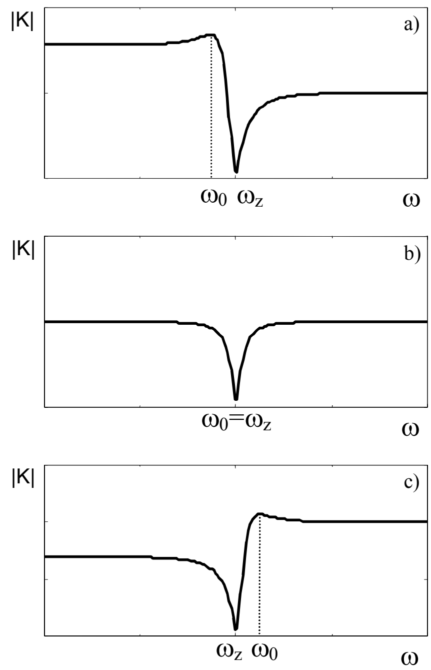

Fig. 8. Computer simulated magnitude characteristics of the optimized 2nd-order band-reject filter.
a) $\omega_z > \omega_0$, b) $\omega_z = \omega_0$, c) $\omega_z < \omega_0$.

4.2 All-Pass Filters

As another example the second-order all-pass filter with symmetric complex conjugate poles and zeros of the transfer function (18) is introduced. Here, in accordance with Fig. 9, the complete state model can be described by using of the real and imaginary parts of zeros only (μ'>0, μ''>0). Considering the root symmetry

$$\nu' = -\mu', \ \nu'' = \mu'', \tag{27}$$

the complete optimized form of the state equations is

$$\dot{x}_1 = -\mu' x_1 - \mu'' x_2 + 2\mu' v, \tag{28}$$

$$\dot{x}_2 = \mu'' x_1 - \mu' x_2 + b_2 v, \tag{29}$$

$$y = x_1 + w_2 x_2 - v \tag{30}$$

where the parameters b_2 and w_2 can be expressed as

$$b_2 = 2\mu'/w_2, \quad w_2 = -\left(m \pm \sqrt{1+m^2}\right), \quad m = \mu'/\mu'' .$$

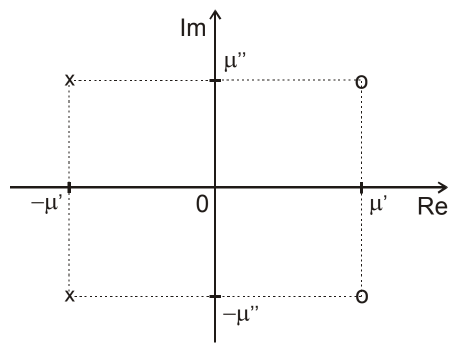

Fig. 9. Zeros and poles of the 2nd-order all-pass filter.

The corresponding integrator-based circuit block diagram, suitable also as the prototype for the practical all-pass filter realization, is shown in Fig. 10. Its transfer function (18)

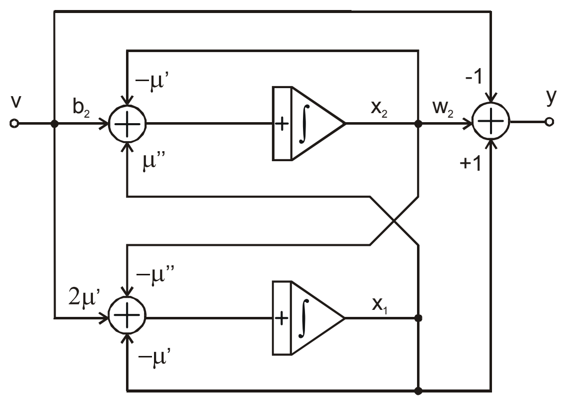

Fig. 10. Integrator-based circuit structure of the 2nd-order all-pass filter with minimized sensitivities.

has the following special form

$$\mathrm{K}(s) = \frac{\mathrm{Y}(s)}{\mathrm{V}(s)} = -\frac{\mathrm{P}(s)}{\mathrm{Q}(s)} = -\frac{s^2 - 2\mu' s + (\mu'^2 + \mu''^2)}{s^2 + 2\mu' s + (\mu'^2 + \mu''^2)}$$

which corresponds to general form

$$K(s) = K_\infty \frac{s^2 - [\omega_0/Q]s + \omega_0^2}{s^2 + [\omega_0/Q]s + \omega_0^2} \tag{32b}$$

where the individual parameters are

$$K_\infty = -1, \quad \omega_0 = \sqrt{\mu'^2 + \mu''^2}, \quad Q = \frac{1}{2\mu'}\sqrt{\mu'^2 + \mu''^2} . \tag{33}$$

The phase characteristic in frequency domain is introduced in Fig. 11.

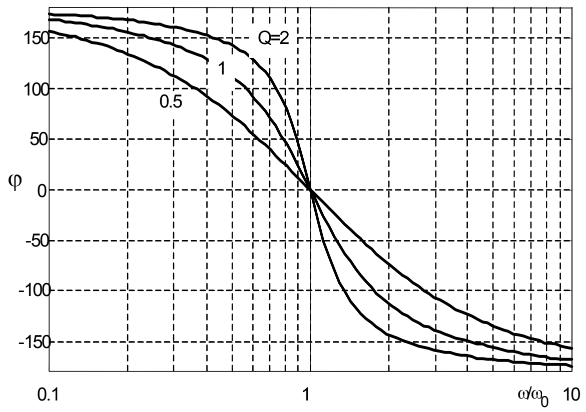

Fig. 11. Computer simulated phase characteristics of the 2nd-order all-pass filter with optimized sensitivities.

5. Conclusion

The general optimization condition for the second-order autonomous PWL dynamical system is utilized for the design of their state models with low eigenvalue sensitivities [7]. Then the state model of the corresponding second-order non-autonomous linear dynamical system is derived. The results achieved are applied to optimized band-reject and all-pass filter design that can be realized in the form of simple electronic circuits having separately adjustable parameters. It has been also proved numerically by simulation and also by the starting laboratory experiments.

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