

# Noise Figure Measurement of Highly Mismatched DUT

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**Abstract.** A device mismatch seriously degrades the accuracy of its noise figure characterization. A new second stage correction technique for highly mismatched device under test is proposed and compared to the standard technique. The presented method is based on additional vector measurement. It takes into account measuring receiver noise figure dependence on the DUT output mismatch besides an available gain correction. Significant accuracy improvement of measured data and decreased error variation is demonstrated. The suggested method is in principle able to eliminate all systematic errors in noise figure measurement.

## Keywords

Microwave measurements, noise figure measurement, mismatched DUT.

## 1. Introduction

The increasing need for high-performance components in mobile communications, calls for an accurate measurement of the noise figure. The most common Y-factor technique is accurate only in case that all components are well matched (noise source, device under test and noise receiver). In most cases, the noise source and the receiver are relatively well matched, and their effect can be neglected. There are increasing demands for mismatched devices to be measured, especially discrete active components (FETs, BJTs, etc.). Therefore, DUT mismatch becomes a critical issue in the noise figure characterization. Recently, a specific technique that combines the classical Y-factor method with scattering parameter measurements has been proposed [1]. The additional vector measurement provides correct information about available gain of the DUT. However, solely available gain correction is useful only for low-mismatch DUTs while it may lead to even greater error for highly mismatched devices [2]. The aim of this contribution is to outline improved second stage correction technique that takes into account not only available gain but also the receiver noise figure dependence on output reflection coefficient of the DUT. A generalization of the approach is able to eliminate all systematic errors of the noise figure measurement.

## 2. Second-Stage Correction

The Y- factor method is the most widely used procedure to measure the noise figure [3]. It requires measurement of the noise power at the output of the DUT for two different temperatures of the noise source ( $T_h$  – hot state and  $T_c$  – cold state). The ratio  $Y = N_h / N_c$  of these two noise power levels is used to calculate the noise figure. However, in any real characterization setup, the measurement system also adds its own noise to the total output measured noise power. A typical configuration for noise figure measurement is depicted in Fig.1. Then, the noise figure of the DUT can be de-embedded using the Friis formula for cascade of two stages:

$$F_{DUT} = F_{sys}(\Gamma_s, \Gamma_{in}) - \frac{F_{rec}(\Gamma_{out}) - 1}{G_a}, \quad (1)$$

where  $\Gamma_{in}$ ,  $\Gamma_{out}$  are reflection coefficients of the DUT,  $\Gamma_s$  is the reflection coefficient of the noise source,  $G_a$  denotes DUT available gain,  $F_{rec}$  is the noise figure of the receiver, and  $F_{sys}$  denotes the global noise figure of the cascaded system comprising a DUT followed by a real receiver.

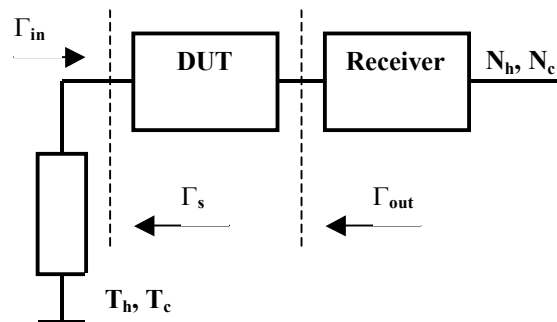


Fig. 1. Block diagram for noise figure measurement.

It is important to notice that the noise figure of the DUT is expressed as a function of three terms:

- The noise figure of the receiver (when the DUT is connected to its input);
- The measured global noise figure of the system made up of the cascade of DUT and receiver;
- The available gain of the DUT.

## 2.1 Standard Correction Method

The standard correction technique replaces in (1) an available gain with an insertion gain and real  $F_{rec}(\Gamma_{out})$  value is replaced with  $F_{rec}(\Gamma_s)$  value obtained in calibration step [2]. In case of highly mismatched DUT, the output reflection coefficient  $\Gamma_{out}$  differs greatly from  $\Gamma_s$ , and significant discrepancies between  $F_{rec}(\Gamma_s)$  and  $F_{rec}(\Gamma_{out})$  have to be expected. Also the insertion gain is significantly different from the available gain in this case. Only when the DUT is well matched the receiver noise figure calculated during the calibration step corresponds to the receiver noise figure applied during the second-stage correction step. Thus the standard correction method is not able to provide the true correction and therefore a measured noise figure value of a highly mismatched DUT may be wrong.

## 2.2 New Correction Method

In order to remove all systematic errors of the second-stage correction, correct values instead of approximated values of the following quantities used in (1) have to be used simultaneously.

- Available gain of the DUT.
- Noise figure of the receiver as a function of the output reflection coefficient of the DUT.

The available gain can be computed directly from S-parameters of the DUT and known  $\Gamma_s$ , obtained by additional vector measurement [2]

$$G_a = \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2} |S_{21}|^2 \frac{1}{1 - |\Gamma_{out}|^2}, \quad (2)$$

where

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s}. \quad (3)$$

$S_{ij}$  are S-parameters of the DUT,  $\Gamma_s$  is the reflection coefficient of the noise source connected at the input of the DUT, and  $\Gamma_{out}$  is the output reflection coefficient of DUT.

The receiver noise figure can be expressed as

$$F_{rec}(\Gamma_{out}) = F_{min} + 4 \frac{R_n}{Z_0} \frac{|\Gamma_{out} - \Gamma_{opt}|^2}{|1 + \Gamma_{opt}|^2 (1 - |\Gamma_{out}|^2)} \quad (4)$$

where  $F_{min}$ ,  $R_n$ ,  $\Gamma_{opt}$  represent the four real classical noise parameters.  $Z_0$  is the reference impedance. Thus the receiver noise parameters  $F_{min}$ ,  $R_n$ ,  $\Gamma_{opt}$  can be obtained by solving a set of nonlinear equations based on (4). This new calibration step can be done by noise figure measurements of at least four appropriate passive two-ports, where the available gain, noise figure and output reflection are known from previous vector measurements of their S-parameters. The available gain of a passive two-port is a reciprocal value of its noise figure. Therefore the noise figure of passive two-port can be computed from its scattering parameters.

The final form of the set of equations is given by substitution of rearranged (1) into (4)

$$G_a^i F_{sys}^i(\Gamma_s, \Gamma_{in}) = F_{min} + 4 \frac{R_n}{Z_0} \frac{|\Gamma_{out}^i - \Gamma_{opt}|^2}{|1 + \Gamma_{opt}|^2 (1 - |\Gamma_{out}^i|^2)} \quad (5)$$

where  $i$  is the index of a calibration element. Backward substitution of (4) and (2) into (1) gives an expression for the new true second stage correction.

Nevertheless, there is still remaining degradation of  $F_{sys}$  measurement accuracy caused by nonzero input reflection coefficient  $\Gamma_{in}$ . In other words the noise source calibration is related to 50- $\Omega$  systems. When the noise source is loaded by impedance different from  $Z_0$  (i.e.  $\Gamma_{in} \neq 0$ ) then also ENR (*excess noise ratio*) of the noise source becomes different from its tabular value. In order to avoid this additional systematic error the input reflection coefficient of calibration elements applied should be kept as low as possible. Other systematic errors, such as changes in  $\Gamma_s$  from the hot to the cold state, were omitted in this paper because of their negligible influence on the resultant accuracy.

## 3. Experimental Results

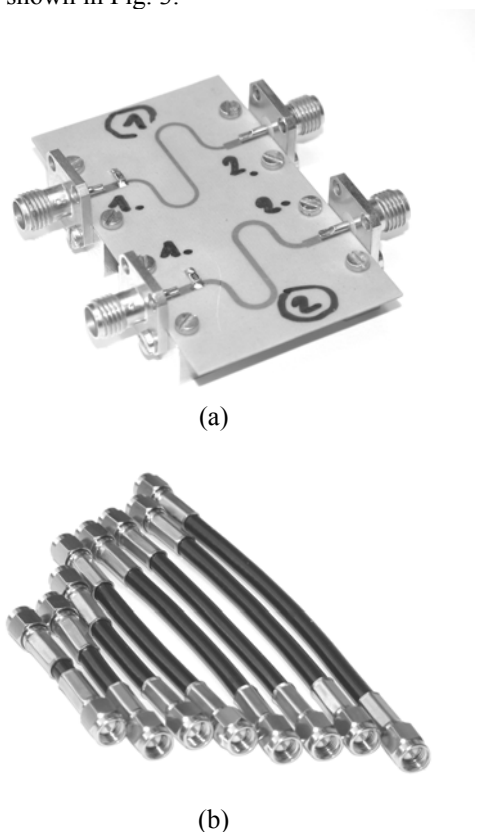
The new correction method described above was verified at frequency 1 GHz. A simplifying assumption  $\Gamma_s = 0$  was considered. HP8970A receiver was used in the experiments as noise figure meter and PNA vector network analyzer Agilent 8364A was used for measurement of scattering parameters.

An essential task of the method is proper choice of calibration two-ports. The output reflection coefficients should lay as far as possible from centre of the Smith chart with angles spread equally along its circumference. It is strongly recommended to append the calibration set with one well-matched standard and thus to overdetermine the system of equations.

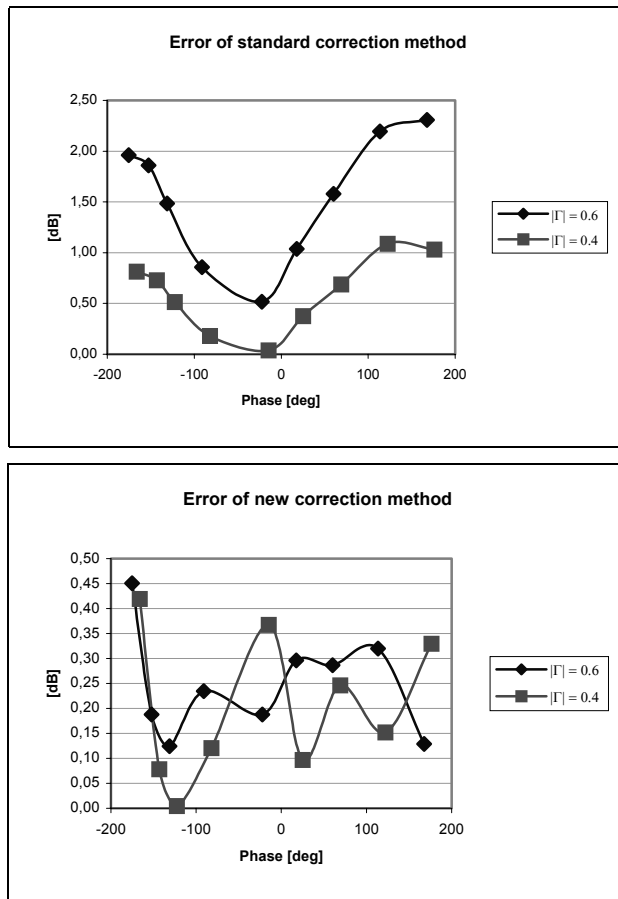
A suitable structure, SMD resistor (82  $\Omega$  resp. 120  $\Omega$ ) connected in parallel to ground in the input of a section of the microstrip line ( $w = 0.63$  mm resp.  $w = 0.94$  mm,  $l = 48$  millimeters fabricated on substrate Arlon 25N with SMA connectors attached) followed 50- $\Omega$  coaxial cables with different length – was chosen, see Fig. 2.

It makes it possible to minimize the input reflection coefficient  $\Gamma_{in}$  and, at the same time, to set a suitable phase and magnitude of the output reflection coefficient  $\Gamma_{opt}$ . Eighteen passive two-ports were obtained by combination of two passive 2-ports and set of coaxial cables. Some of them were used as calibration elements the rest as measured devices. The worst-case measured input reflection coefficient of calibration elements and DUTs was equal to -28 dB. Thus changes in ENR of the noise source are negligible. Output reflection coefficients of calibration elements

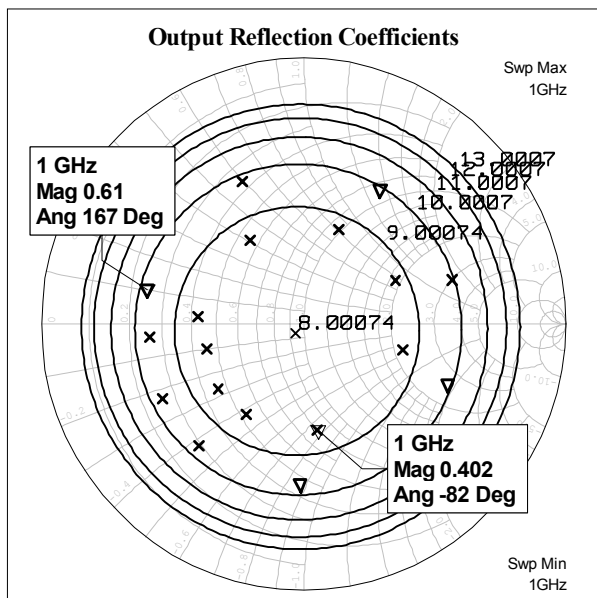
and measured two-ports along with noise circles of the receiver are shown in Fig. 3.



**Fig. 2.** Realization of DUTs and calibrating elements (a) Two-ports well-matched at the input and mismatched at the output. (b) Set of 50-Ω coaxial cables used as phase shifters.



**Fig. 4.** Dependence of measurement error on phase of DUT output reflection coefficient for two magnitudes 0.6 and 0.4 (top - standard method, bottom - new correction method);  $f = 1$  GHz.



**Fig. 3.** Output reflection coefficients of DUTs and circles of constant noise figure of the receiver @ 1GHz (triangles mark calibration elements).

Noise parameters of the receiver at 1 GHz,  $R_N = 69 \Omega$ ,  $F_{min} = 8$  dB,  $\Gamma_{opt} = 0.045 \angle -133^\circ$ , were obtained by solving a set of complex nonlinear equations given by (5). Standard built-in functions of MATHCAD<sup>®</sup> were used for this purpose. Numerical results of the standard correction method and the new one are compared in Tab. 1. All errors were computed as an absolute value of the difference between the result of the corresponding correction method and the value computed directly from measured scattering parameters. The last mentioned value of the noise figure based on the measurement is considered to be true. A significant reduction of the maximum noise figure error of about 2 dB can be observed in the new method. Fig. 4. shows that the greater magnitude of the reflection coefficient causes the greater error of the standard correction method in contrast to the new one.

Consequently, the new correction method is effective namely for strongly mismatched DUTs. The method also offers smaller and quasi-random error variations with respect to the phase of the output reflection coefficient of the DUT.

DUTs	NF <sub>comp</sub> [dB]	NF <sub>cor</sub> [dB]	NF <sub>std</sub> [dB]	Err <sub>std</sub> [dB]	Err <sub>cor</sub> [dB]
calib. 1	2.16	2.03	4.47	2.31	0.13
DUT 02	1.50	1.17	2.53	1.03	0.33
calib. 2	2.38	2.67	3.96	1.58	0.29
DUT 12	1.62	1.37	2.30	0.69	0.25
DUT 21	2.37	2.66	3.40	1.04	0.30
DUT 22	1.61	1.71	1.99	0.38	0.10
calib. 3	2.45	2.26	2.97	0.52	0.19
DUT 32	1.66	1.30	1.70	0.04	0.37
calib. 4	2.39	2.62	3.24	0.86	0.23
DUT 42	1.64	1.76	1.82	0.18	0.12
DUT 51	2.28	2.40	3.77	1.49	0.12
DUT 52	1.58	1.58	2.10	0.51	0.00
DUT 61	2.27	2.46	4.13	1.86	0.19
DUT 62	1.58	1.50	2.30	0.73	0.08
DUT 71	2.34	1.88	4.30	1.96	0.45
DUT 72	1.62	1.20	2.43	0.81	0.42
DUT 81	2.34	2.66	4.53	2.20	0.32
DUT 82	1.16	1.45	2.70	1.09	0.15
Max. error				2.31	0.45

Tab. 1. Comparison of standard and new correction method @ 1 GHz. NF comp-noise figure computed from S – parameters of passive DUTs, NF cor- noise figure of new correction method, NF std- noise figure of standard correction method, Err std- absolute error of standard method, Err cor-absolute error of new correction method.

## 4. Conclusion

The new noise figure measurement correction method for device under test that are highly mismatched at the output exploiting additional vector measurement was proposed, experimentally verified and compared with the standard technique. The method makes possible significant reduction of maximum measurement error. The approach suggested can be generalized for  $\Gamma_{in} \neq 0$ . A generalization of the approach is able to eliminate all systematic errors of the noise figure measurement.

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