Mixed Noise Suppression in Color Images by Signal-Dependent LMS L-Filters

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Abstract. *The paper is devoted to the signal-dependent (SD) design of adaptive LMS L-filters with marginal data ordering for color images. The same stem of SD processing of noised grayscale images was applied on noisy color images. Component-wise and multichannel modifications of SD LMS L-filter in R'G'B' (gamma corrected RGB signals) color space were developed. Both modifications for filtering two-dimensional static color images degraded by mixed noise consisting of additive Gaussian white noise and impulsive noise were used. Moreover, single-channel spatial impulse detectors as detectors of impulses and details were used, too. Considering experimental results, SD modifications of L-filters for noisy color images can be concluded to yield the best results.*

Keywords

Order-statistics, signal-dependent filter, color image filtering, L-filter, marginal data ordering.

1. Introduction

Many applications in telecommunications employ filtration of signals that are corrupted by various types of noise. If noises are of a nonlinear nature, nonlinear filters have to be used to execute their effective suppression. On the other hand, linear filters have to be exploited when additive noise is to be cancelled. If additive mixing of nonlinear and additive noises appear, the question *what type of a filter is optimal* arises. The appropriate solution offers the linear combination of order statistics. This class of filters is called L-filters, and they approximate filter coefficients considering to the noise model well [1] – [4], [7].

In the paper, two modifications of adaptive SD LMS L-F (Signal-Dependent Least Mean Square L-filter) for noised color images are designed using single- or multichannel versions of adaptive L-filters and SID (Spatial Impulse Detector). Signal-dependent (SD) processing processes homogeneous input observations apart from detailed ones separately. The SMD detector (Spatial Median Detector) is used as one of SID detectors and serves as a switch between outputs of partial filters [4].

In Section 2, the simple theory of the single- and multi-channel adaptive LMS L-filters is introduced. Section 3 is devoted to the design of adaptive component wise and multichannel signal-dependent LMS L-filters with SMD. The experimental results are presented in Section 4. In Section 5, obtained results and further development that can improve the filtration results are discussed.

2. Adaptive LMS L-Filters

2.1 Single-Channel Adaptive LMS L-Filter

The component-wise filtration of noised color images is a method that employs filters designed for processing noised grayscale static images [1], [2]. The disadvantage of this method is hidden in the fact that it doesn't exploit correlation between channels. On the other hand, this feature is advantageous if color images degraded by non-correlated noise are processed. In this case, the filtration by multichannel filters inexactly estimates the reference signal.

Filters based on linear combination of order statistic (L-filters) designed for grayscale images can be used as partial filters for component-wise processing of color images. The output of a L-filter is given by

$$y_i = \mathbf{w}^{\mathrm{T}} \mathbf{x}\,\mathbf{r}_i \tag{1}$$

where $\mathbf{w} = [w_1, \ldots, w_N]^{\mathrm{T}}$ is a vector filter coefficients and $\mathbf{xr}_i$ is a vector of ordered input pixels (ascending or descending order)

$$\mathbf{xr}_i = \left({}_1x_i, {}_2x_i, \ldots, {}_{N-1}x_i, {}_Nx_i\right)^{\mathrm{T}}. \tag{2}$$

Inverting autocorrelation matrix for non-stationary signals is computationally difficult (it is time varying), and therefore, the adaptive algorithm can offer good estimation of filter coefficients. Adaptive algorithms minimize a criterion function (Mean Absolute Error, Mean Square Error, total power, …). LMS algorithm that is based on noise gradient and Steepest Descent method belongs to frequently used adaptive algorithms:

$$\overline{\mathbf{w}}_{i+1} = \overline{\mathbf{w}}_i + 2\mu\,\varepsilon_i \mathbf{xr}_i. \tag{3}$$

Eqn. (3) is equivalent to the well-known linear LMS algorithm (Widrow one), which is very popular in linear adaptive filtering. Whereas eqn. 3 uses the vector of ordered observations to update the adaptive L-filter coefficients, LMS algorithm exploits non-ordered ones. Adaptive component-wise LMS L-filter (LMS^C L-filter) arises by using adaptive single channel LMS L-filters in the component-wise filtration of color images.

2.2 Multichannel Adaptive LMS L-Filter

Recently, attention has been given to the non-linear processing of vector-valued signals. It is known that there is no universally accepted method to order multivariate data. Therefore, the marginal (M-ordering), reduced (R-ordering), partial, conditional ordering etc. as sub-ordering principles were developed [3], [7]. Filters based on these principles employ correlation between channels, and hence, they have more information about processing signals.

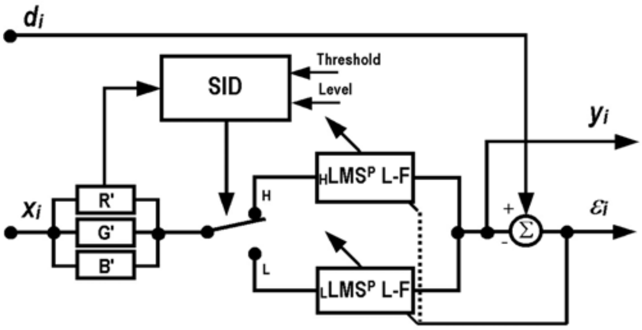


Fig. 1. The adaptive signal-dependent LMSP L-filter.

Let $_1\mathbf{x}, \ldots, {}_N\mathbf{x}$ be a random sample of N observations of a p-dimensional random variable $\mathbf{X}$. Then each input observation

$$_j\mathbf{x} = \left({}_{j,1}x, {}_{j,2}x, \ldots, {}_{j,p}x\right)^{\mathrm{T}} \tag{4}$$

belongs to the p-dimensional space denoted as R^P [3]. M-ordering orders components of observations separately by the next law

$$_{k,1}x \leq {}_{k,2}x \leq \cdots \leq {}_{k,N}x \qquad k = 1, \ldots, p\,. \tag{5}$$

Output of p-dimensional L-filter that processes p-dimensional vectors $\mathbf{x}_i$ is given as

$$\mathbf{y}_i = \sum_{k=1}^{p} {}_k\mathbf{A}\ {}_k\tilde{\mathbf{x}}_i \tag{6}$$

where $_k\mathbf{A}$ are $p \times N$ matrices of filter coefficients and $_k\tilde{\mathbf{x}}_i$ is vector of order statistics in k-th channel of $N \times 1$ dimension

$$_k\tilde{\mathbf{x}}_i = \left({}_{k,1}x_i, {}_{k,2}x_i, \ldots, {}_{k,N}x_i\right)^{\mathrm{T}}. \tag{7}$$

Let $_{k,l}\mathbf{w}^{\mathrm{T}}$, $l = 1, \ldots, p$ denotes the l-th row of the matrix $_k\mathbf{A}$. Then filter coefficients $_k\mathbf{w}$, $k = 1, \ldots, p$ that minimize MSE

$$_k\mathbf{w} = \left[{}_{1,k}\mathbf{w}^{\mathrm{T}} \middle| {}_{2,k}\mathbf{w}^{\mathrm{T}} \middle| \ldots \middle| {}_{p,k}\mathbf{w}^{\mathrm{T}}\right]^{\mathrm{T}} \tag{8}$$

can be recursively computed by the steepest descent algorithm as follows

$$\begin{aligned} _k\overline{\mathbf{w}}_{i+1} &= {}_k\overline{\mathbf{w}}_i + 2\mu\left[{}_k d_i - \tilde{\mathbf{X}}_i^{\mathrm{T}}\ {}_k\overline{\mathbf{w}}_i\right]\tilde{\mathbf{X}}_i \\ &= {}_k\overline{\mathbf{w}}_i + 2\mu\ e_i\,\tilde{\mathbf{X}}_i\,. \end{aligned} \tag{9}$$

Eqn. (9) is an adaptation equation of an adaptive multichannel LMS L-filter (LMS^P L-F) [3], [7]. LMS algorithm offers simple recursive equation of L-filter coefficients updating. Convergence properties of general LMS L-filter depend on eigenvalues of autocorrelation matrix distribution.

3. Adaptive Signal-Dependent LMS L-Filters

Very good filtration results were achieved by signal-dependent processing of noised grayscale images redound to their application on color images. The principle of SD processing was applied in conjunction with single- and multi-channel adaptive LMS L-filters.

Color images in R'G'B' color space are defined by three components, and impulse detection can be performed for any of them. This case, spatial impulse detectors (SID) developed for noisy grayscale images can be used [4].

3.1 SID for Grayscale Images

The SD principle is based on a separate processing of detailed and homogeneous input observations. SID serves for the determination of the input type. The determination result defines the filter, which will process input data.

The decision rule for the general SD L-filter in combination with SID is given by

IF

$$\sum_{k=1}^{N} {}_kD_i \geq Level \tag{10}$$

THEN $\quad$ $_H$*L-filter*
ELSE $\quad$ $_L$*L-filter*

Here, $_kD_i$ is the result of impulse detection for k-th image pixel in the i-th input observation. The value of the level defines the number of detected impulses in the observed samples.

The $_H$*L-filter* processes the detailed input observations and $_L$*L-filter* the homogeneous ones.

The spatial impulse detectors were derived from impulse detectors [8], [9], which detect impulses or image details in all image pixels of input observations.

The spatial median order statistics detector (SMD) [4] belongs to the most popular SID. The decision rule of SMD is given by

IF

$$\left|\,\mathrm{med}\{x_i\} - {}_kx_i\right| \geq Threshold \tag{11}$$

THEN $\quad$ $_kD_i = 1$
ELSE $\quad$ $_kD_i = 0$

If the difference between the k-th input sample magnitude and the median one is more than the threshold value, the input sample is marked as the detected impulse.

3.2 Adaptive Multichannel Signal-Dependent LMS L-Filter

Adaptive signal-dependent LMS^P L-filter (SD LMS^P L-F), which is shown in Fig. 1, utilizes two LMS^P L-filters to process homogeneous or detailed image areas.

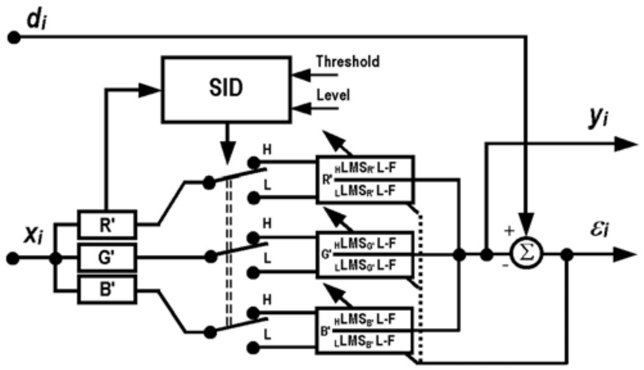


Fig. 2. The adaptive signal-dependent LMSC L-filter.

Input color data in i-th observations $\mathbf{X}_i$ are divided to the color components. Since the detector designed for grayscale images is used as SID, the impulse detection has to be done for one of color components. Considering SID decision, the processed i-th input observation serves for adapting one of partial filters. Thus, $_H$LMS^P L-F is the partial filter that processes detailed input observations, and $_L$LMS^P L-F processes homogeneous input observations. The output of SD LMS^P L-F is given as a combination of the partial L-filter outputs and it can be expressed as

$$y_i = \begin{cases} y_{_L LMS^P\, L-F} & if \quad \sum_{i=1}^{N} {}_k D_i < Level \\ y_{_H LMS^P\, L-F} & if \quad \sum_{i=1}^{N} {}_k D_i \geq Level \end{cases} \tag{12}$$

SD LMS^P L-F is advantageous in employing correlation among color channels and their joint processing. On the other hand, higher computation complexity (operation with matrices) and higher number of filter coefficients (162 for 3×3 square filter window) are disadvantages of SD LMS^P L-F. Moreover, the shape of the filter window can be changed for the whole partial LMS^P L-filter only.

3.3 Adaptive Component-wise Signal-Dependent LMS L-Filter

Adaptive signal-dependent LMS^C L-filter (SD LMS^C L-F) is shown in Fig.2. SD LMS^C L-filter is based on assumption that filter coefficients of partial filters can be adapted jointly by suppressing the correlated noise (color pixel is degraded in each channel). Moreover, impulse detection probability at the same position in every component is high.

SD LMS^C L-F is based on the same principle as SD LMS^P L-F, and can employ any modification of SID. The usage of partial filters is only difference. The output of SD LMS^C L-filter is given by

$$y_i = \begin{cases} \left.\begin{matrix} y_{_L LMS_{R'}\ L-F} \\ y_{_L LMS_{G'}\ L-F} \\ y_{_L LMS_{B'}\ L-F} \end{matrix}\right\} & if \quad \sum_{i=1}^{N} {}_k D_i < Level \\ \left.\begin{matrix} y_{_H LMS_{R'}\ L-F} \\ y_{_H LMS_{G'}\ L-F} \\ y_{_H LMS_{B'}\ L-F} \end{matrix}\right\} & if \quad \sum_{i=1}^{N} {}_k D_i \geq Level \end{cases} \tag{13}$$

The filter is advantageous in the smaller number of filter coefficients (54 for 3×3 square filter window), in simple updating and independent shape of filter mask of partial filters. The filter does not exploit correlation among channels, which is its main disadvantage.

4. Experimental Results

For experiments, reference color images Lena and Mandrill were used (Fig. 4a, d). They were corrupted by a mixed noise (Fig. 4b, e) consisting of additive Gaussian white noise with the standard deviation $\sigma = 20$ and a correlated impulsive noise with the probability $p = 10\%$ (degraded pixel at the same position in each channel). This mixed noise was denotes as G20CI10.

Filtering results were evaluated by MAE (Mean Absolute Error), MSE (Mean Square Error), NR (Noise Reduction), MAER (Mean Absolute Error Reduction) and CD (Color Difference) criteria, respectively [1], [5]. NR and MAER criteria are in dB scale.

We used the SMD detector as SID in experiments [4]. Results of adaptive L-filters were compared with VMF$_{L2}$ filter (Vector Median Filter with L2 norm) and some are shown in the Table 1 [6].

SD processing improved suppression of the mixed noise in comparison with conventional component-wise or multichannel filters. The above-mentioned impulse detector for grayscale static images was used for impulse detection in each color channel. Best results were achieved by detection in *R'* channel. Input parameter for SMD detector was determined to *Level* = 1 and its optimal threshold was obtained experimentally. Detailed NR and CD dependencies from threshold are shown in Fig. 3.

Filtering noised Lena image (less composite image), better results were achieved compared to $_{SMD}SD_{R'}$ LMS^P L-filter. For Mandrill (more composite image) the $_{SMD}SD_{R'}$ LMS^C L-filter is preferable. Optimal thresholds for SMD detector *Level* = 1 for both images are shown in the Tab. 2.

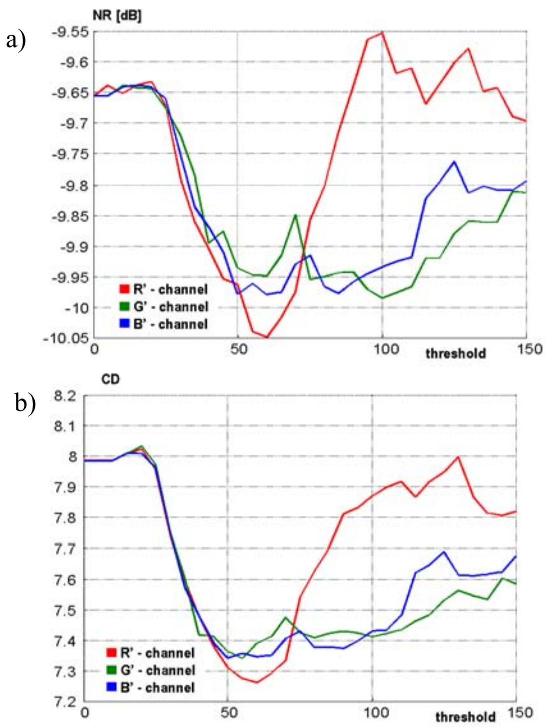
Filter	Lena				
	MAE	MSE	NR	MAER	CD
Noised	21.16	1197	-	-	20.54
VMF_{L2}	10.77	204.3	-7.67	-5.87	11.18
LMS^P L-F	8.42	129.4	-9.66	-8.00	7.98
LMS^C L-F	8.47	133.9	-9.51	-7.95	8.17
$_{SMD}SD_{R'}$ LMS^P L-F	7.91	118.4	-10.05	-8.55	7.25
$_{SMD}SD_{R'}$ LMS^C L-F	8.27	129.2	-9.68	-8.17	7.97
	Mandrill				
Noised	21.06	1160.3	-	-	48.21
VMF_{L2}	14.55	391.2	-4.75	-3.24	7.85
LMS^P L-F	17.12	469.2	-3.94	-1.80	8.21
LMS^C L-F	14.32	363.1	-5.07	-3.36	7.55
$_{SMD}SD_{R'}$ LMS^P L-F	14.56	373.9	-4.95	-3.23	7.51
$_{SMD}SD_{R'}$ LMS^C L-F	13.72	345.9	-5.30	-3.75	7.40

Tab. 1. The filter performance indices achieved for color images Lena and Mandrill corrupted by G20CI10 noise.

Filter	Lena	Mandrill
$_{SCOSD2}SD_{R'}$ LMS^P L-F	60	85
$_{SCOSD2}SD_{R'}$ LMS^C L-F	65	70

Tab. 2. Optimal thresholds for SMD detector (*Level*=1, G20CI10 noise).

In case of a different definition of the optimal threshold for NR and CD criterions, the optimal value is determined by NR parameter (NR is derived by minimizing the MSE criterion function of LMS algorithm).



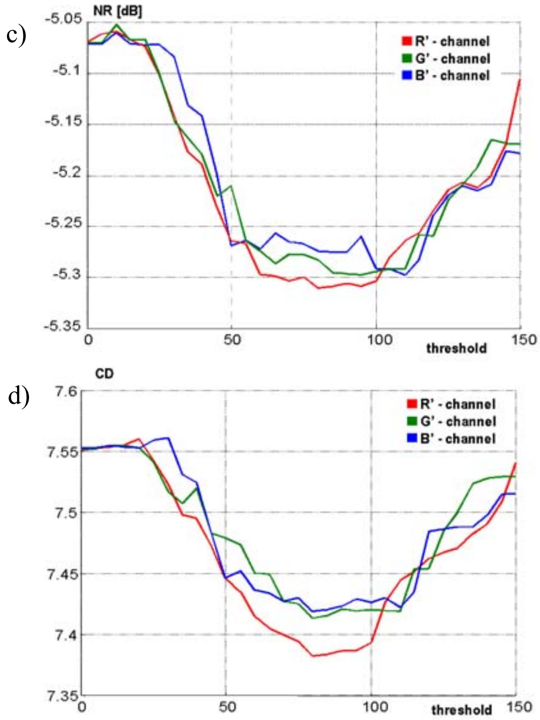


Fig. 3. NR and CD dependencies from threshold, a-b) Lena filtered by $_{SMD}SD_{R'}$ LMS^P L-filter, c-d) Mandrill filtered by $_{SMD}SD_{R'}$ LMS^C L-filter.

In the R'–G'–B' sense, the original image of Lena contains around 47% – 26% – 27% and image of Mandrill 36% – 34% – 30% of the total color power. Obviously, the R'-channel is dominant, and therefore, the best results were achieved in this color channel (but they are not robust). For robustness in adjusting filters, thresholds from 60 to 90 have to be chosen by detection in G' or B' channels. Generally, the detection has to be done in a non-dominant channel.

Tab. 2 shows that a higher value of threshold will be used for filtration of details in images, and vice versa. Since perception of details and impulses is the same, subjective evaluation of filtered images founds better detailed images. Filtered color images are shown in Fig. 4c, f. Improvement of a smoothing parameter by SD L-filters is visible in the face detail of Lena (Fig. 5). Adaptation step of LMS algorithm was $\mu = 1 \cdot 10^{-7}$.

5. Conclusion

In the paper, two adaptive signal-dependent LMS L-filters were described: the adaptive SD component-wise one and SD multichannel one. Both filters employ spatial impulse detectors designed for grayscale images. The described adaptive SD filters were designed for a mixed noise removed from color images. Moreover, the paper contains optimal thresholds for different composite images, and recommendations for signal-dependent processing of color images by adaptive L-filters. Due to their high computational complexity, they can be used in offline applications.

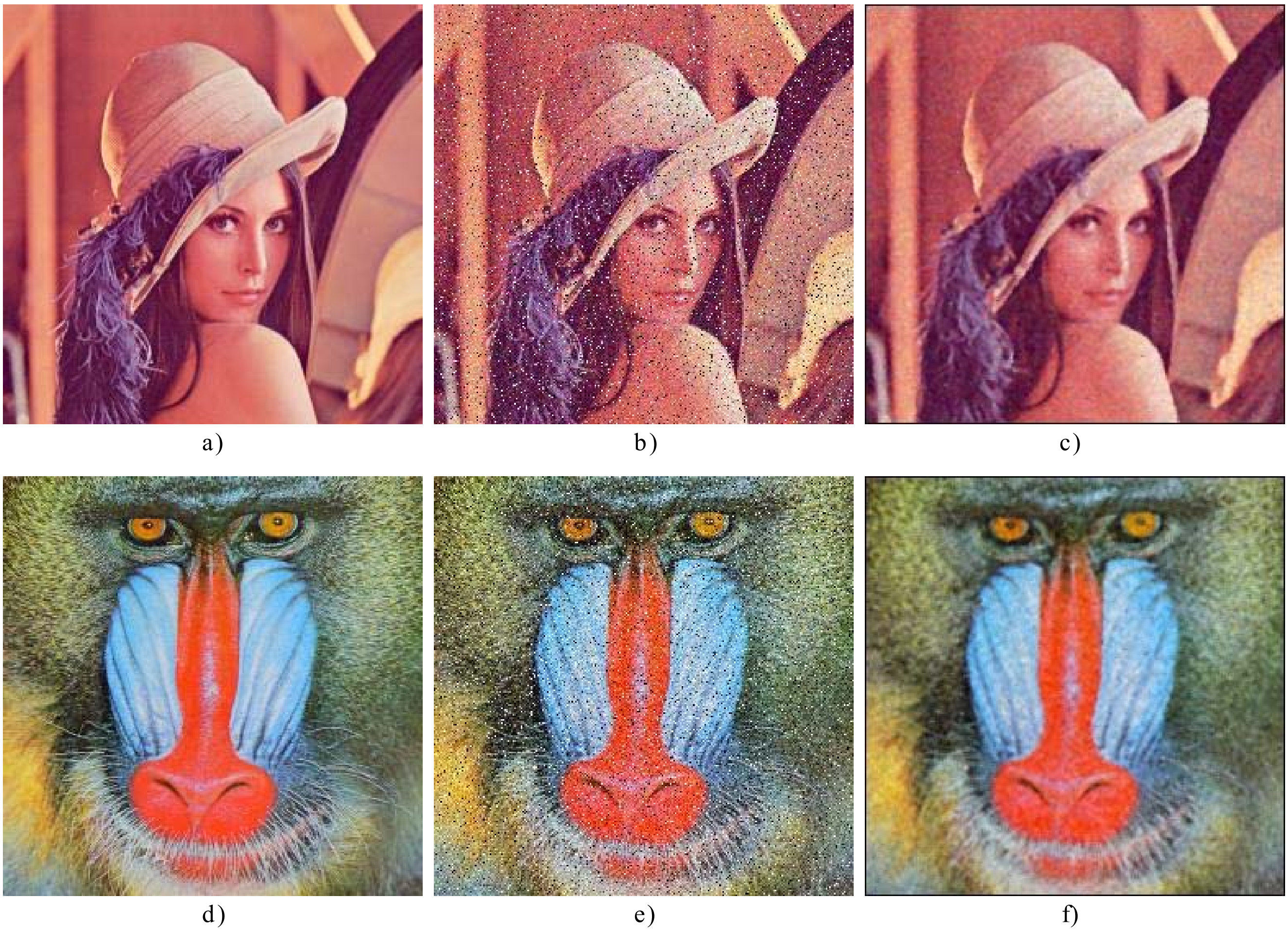


Fig. 4. Color images. (a) Original Lena, (b) Lena corrupted by mixed G20CI10 noise, (c) Lena filtered by $_{SMD}SD_{R'}$ LMS^{P} L-filter, (d) Original Mandrill, (e) Mandrill corrupted by mixed G20CI10 noise, (f) Mandrill filtered by $_{SMD}SD_{R'}$ LMS^{C} L-filter.

The adaptive SD L-filters were proven that their noise-suppression characteristics are able to reduce well the mixed noise in color images. Exploiting vector detection of impulses and details can improve the filtration results in the future modifications.

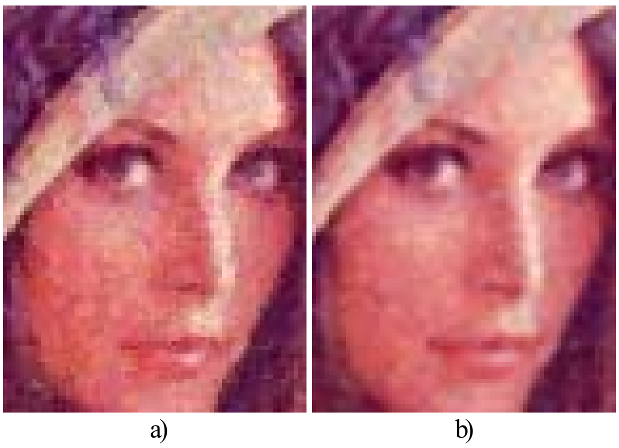


Fig. 5. Filtered images of Lena's face by (a) VMF$_{L2}$, (b) $_{SMD}SD_{R'}$ LMS^{P} L-filter.

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