Exploitation of Higher Order Moments Increase the Tracking Aircraft by the Extended Alpha-Beta Filter

Zdeněk DOSTÁL 1, Igor MOKRIŠ 2

1 Dept. of Navigation Systems, Military Academy, P.O.Box 45, 031 01 Liptovsky Mikulas, Slovak Republic
2 Dept. of Information Systems, Matej Bel University, 097 04 Banska Bystrica, Slovak Republic
dostal@valm.sk, mokris@financ.umb.sk

Abstract. The paper analyzes the possibility of exploitation of higher order moments for increasing the precision of tracking of a flying aircraft by the α-β filter. For tracking of a flying aircraft by the α-β filter the 3rd and 4th order moments in 3D space are used.

Keywords
Aircraft track moving, α-β filter.

1. Introduction

Up to nowadays, the tracking of flying aircraft by the α-β filters has been used for computation of a position of a flying aircraft using moments of the 1st and 2nd order. These moments represent speed and acceleration of a flying aircraft. This paper describes the tracking of a flying aircraft by extended algorithm for its position generation using the moments of higher orders. For the analysis of flying aircraft positions the 3rd and 4th order moments are used in 3D space by α-β filter [2 - 5, 7, 8].

For the aircraft flight modeling, the track including four sections of flight, i.e. straightforward uniform track, a course maneuver track, speed maneuver track and a height maneuver track were used. The modeling of a flight of an aircraft corresponds to a real aircraft flight.

2. Basic Terms

A utilization of the α-β filter for aircraft tracking in the basic form [2 - 8] uses the 1st and 2nd order moments of a flying aircraft track in a 3-dimension space which represent its speed and acceleration. This filter uses a prediction-correction principle and is expressed by the following terms:

Equation of prediction position
\[ x(k + 1, k) = F(k + 1, k) . x(k) \]  (1)

Equation of correction position
\[ x(k + 1, k + 1) = x(k + 1, k) + K(k + 1) [ s_x(k + 1) - x(k + 1, k) ] \]  (2)

where, \( x(k) = [ s_x(k), s_y(k), s_z(k), v_x(k), v_y(k), v_z(k) ]^T \) is a status vector; \( F(k+1,k) \) is the system transmission matrix of system of time interval \( t(k) \) to \( t(k+1) \); \( K(k+1) \) is the matrix of tracking filter gain; \( s_x(k), s_y(k), s_z(k) \) are coordinates of the aircraft position measured by radar in \( t(k) \); \( v_x(k), v_y(k), v_z(k) \) are speed elements of the aircraft in \( t(k) \); \( x(k) \) is the state vector in \( t(k) \); \( x(k+1,k) \) is the prediction position vector in \( t(k+1) \); \( x(k+1,k+1) \) is the correction position vector in \( t(k+1) \) and \( s_x(k+1,k+1) \) is the measurement vector of the aircraft position coordinates in \( t(k+1) \).

For the system transmission matrix, the following equation holds
\[
F(k+1,k) = \begin{bmatrix}
1 & 0 & 0 & \Delta t & 0 & 0 \\
0 & 1 & 0 & 0 & \Delta t & 0 \\
0 & 0 & 1 & 0 & 0 & \Delta t \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 
\end{bmatrix}
\]  (3)

where, \( \Delta t \) is a time difference between two samples of the aircraft positions and the system transmission matrix uses 1st order moments.

For the analysis of a higher-order moment effect for prediction of the aircraft’s position based on formula (1), the state vector \( x(k) \) can be analyzed and due to the system transmission matrix \( F(k+1,k) \) (3) of a tracking filter have to be extended by higher-order moments of a position coordinates. Position coordinates with higher-order moments are represented by the 2nd, 3rd and 4th order moments. Three further models of tracking filter have been created in this case. They have a different form of state vector \( x(k) \) and a different form of system transmission matrix \( F(k+1,k) \), as follows
\[
x(k) = [ s_x(k), s_y(k), s_z(k), v_x(k), v_y(k), v_z(k), a_x(k), a_y(k), a_z(k) ]^T
\]  (4)
moments and

\[ \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 & \frac{\Delta^2 t}{2} & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & \frac{\Delta^2 t}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & \frac{\Delta^2 t}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

which, for the position prediction, uses the 1st and 2nd order moments

\[ x(k) = \left[ x_m(k), y_m(k), z_m(k), v_x(k), v_y(k), v_z(k), a_x(k), a_y(k), a_z(k) \right] \]

\[ F(k + 1,k) = \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 & \frac{\Delta^2 t}{2} & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & \frac{\Delta^2 t}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & \frac{\Delta^2 t}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

The precise track \( Y_P \) of an aircraft flight was generated according to the higher demands and it represents a real aircraft flight. The measured track \( Y_M \) of the aircraft flight was created from the precise track using noise by a noise generator. The tracking of the aircraft flight was obtained from measured positions \( Y_M \) based on filtration using the prediction – correction tracking method as filtered track \( Y_i \), where \( i \) is the number of the used order moment.

Expressions of the coordinate difference \( \Delta x_{io} \) for all coordinates, \( x, y, z \), for determination of the positions of the flying aircraft track based on from 1st to 4th order moments may be defined as

\[ \Delta x_{io} = v_i(k) \Delta t \]

\[ \Delta x_{io} = 0.5 a_i(k) \Delta t^2 \]

\[ \Delta x_{io} = 0.166 w_i(k) \Delta t^3 \]

\[ \Delta x_{io} = 0.042 u_i(k) \Delta t^4 \]

where, \( v(k) \) is speed, \( a(k) \) is acceleration, \( w(k) \) and \( u(k) \) present the dynamic properties of the flying aircraft in time interval \( t(k) \).

3.1 Analysis of Tracking Precision for Track Including Section of Straightforward Uniform Flight by Extended \( \alpha-\beta \) Filter

For the analysis of aircraft track, the moments of the 1st \( (Y_{io}) \), 2nd \( (Y_{io}) \), 3rd \( (Y_{io}) \) and 4th \( (Y_{io}) \) order by the extended \( \alpha-\beta \) filter were used. On the basis of the \( \alpha-\beta \) filter with regard to the sowed moments of higher order there were obtained the results of the aircraft tracking for the track including a straightforward uniform flight section and coordinate difference which are depicted in Tab. 1 and Fig. 1.

The additional contribution of moments from 1st to 4th orders for determination of the flying aircraft positions is sowed in Fig. 1 in meters. Values of 1st order moments express the coordinate imprecision error between two positions of the track and they are in interval approximately to 7000 meters. Values of 2nd order moments express the
coordinate imprecision error between two positions of the track and they are approximately in interval to 500 meters. Values of 3rd order moments express the coordinate imprecision error between two positions of track and are approximately in interval to 350 meters. Values of 4th order moments express the coordinate imprecision error between two positions of track and are approximately in interval to 200 meters.

<table>
<thead>
<tr>
<th>Straightforward uniform flight of aircraft</th>
<th>x - coordinate [m]</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Δx₁₀</td>
<td>6990</td>
<td>7070</td>
</tr>
<tr>
<td>Δx₁₀</td>
<td>13</td>
<td>42</td>
</tr>
<tr>
<td>Δx₃₀</td>
<td>52.788</td>
<td>9.628</td>
</tr>
<tr>
<td>Δx₄₀</td>
<td>62.58</td>
<td>-10.92</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>y - coordinate [m]</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Δy₁₀</td>
<td>3300</td>
</tr>
<tr>
<td>Δy₂₀</td>
<td>-71.712</td>
</tr>
<tr>
<td>Δy₃₀</td>
<td>-98.28</td>
</tr>
<tr>
<td>Δy₄₀</td>
<td>-98.28</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>z - coordinate [m]</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Δz₁₀</td>
<td>20</td>
</tr>
<tr>
<td>Δz₂₀</td>
<td>17.9</td>
</tr>
<tr>
<td>Δz₃₀</td>
<td>4.98</td>
</tr>
<tr>
<td>Δz₄₀</td>
<td>-0.42</td>
</tr>
</tbody>
</table>

Tab. 1. Coordinate difference for the track including straightforward uniform flight section of aircraft with regard to higher - order moments.

3.2 Analysis of Tracking Precision for Track Including Section of Course Manoeuvre Flight by Extended α-β Filter

By modeling the tracking filter, results were obtained for the section of a flight of an aircraft with a left loop at 360°. These are given in Tab. 2 and expressed in Fig. 2.

The obtained values difference and corresponding coordinates of tracking a flying aircraft with moments from the 1st to 4th order correspond to the results, which were obtained in the section of direct, steady flight of an aircraft.

Tab. 2. Coordinate order difference for section of tracking flight of aircraft with left loop at 360° with regard to the higher order moments.

3.3 Analysis of Tracking Precision for Track Including Section of Height Manoeuvre Flight by Extended α-β Filter

By modeling the tracking filter, results, which are given in Tab. 3 and expressed in Fig. 3, were obtained for track including the section of height manoeuvre flight by extended α-β filter of track.

The obtained moments of the first to the fourth order correspond to the results that were obtained in the previous case.
3.4 Analysis of Tracking Precision for Track Including Section of Speed Manoeuvre Flight by Extended $\alpha$-$\beta$ Filter

By modeling the tracking filter, the results were obtained for the section of track including the direct unsteady movement of an air object. These are given in Tab. 4 and expressed in Fig. 4.

Moments of the 1$^{\text{st}}$ to 4$^{\text{th}}$ order obtained by modeling of this case confirm the results in the previous case.

4. Conclusion

By analysis of the presented $\alpha$-$\beta$ filter with regard to the 1$^{\text{st}}$ to 4$^{\text{th}}$ order moments there were obtained results, which were used for prediction of the position of a moving air object during processing of radar data. These results confirm that the accuracy of tracking does not depend on the character of tracks and movement of flying air object. It depends on the order of the used moment. For individual moments, results were achieved for coordinate tracking positions in ranges:
- for the 1$^{\text{st}}$ moments up to 7000 m,
- for the 2$^{\text{nd}}$ moments up to 500 m,
- for the 3$^{\text{rd}}$ moments up to 350 m,
- for the 4$^{\text{th}}$ moments up to 200 m,

whereby it is confirmed that their application is useful due to the increase of tracking accuracy.

![High manoeuvre flight](image)

Fig. 3. Results of modeling of track of moving air object for section of tracking transition to higher-flying level.

![Speed manoeuvre flight](image)

Fig. 4. Results of modeling the track of moving air object for section of track with direct unsteady movement.

References


**About Authors...**

Zdeněk DOSTÁL received Ing (M.Sc.) degree in electrical engineering from the Military Technical University, Liptovsky Mikulas, and CSc. (PhD) degree in military technical from the same university. He was Associate Professor in military technical with the Dept. of Air Defense Military, TU, Liptovsky Mikulas. His research is focused on radar following of aircrafts.

Igor MOKRIŠ received Ing (MSc) degree in technical cybernetics from the Technical University, Kosice in 1972 and CSc. (PhD) degree from the Slovak Technical University in Bratislava in 1980. From 1985 he was Associate Professor and from 1997 Professor in technical cybernetics and artificial intelligence with the Dept. of Informatics and Computers, Military Academy, Liptovsky Mikulas. From 1998 he is with the Dept. of Information Systems, Matej Bel University, Banska Bystrica, as Professor. His research interests include signal and image processing, pattern recognition and networks.

---

**Radioengineering Reviewers**

September 2003, Volume 12, Number 3

- BIOLEK, D., Military Academy, Brno
- BILÍK, V., Slovak Univ. of Technology, Bratislava
- CZARNECKI, M., University of Lodz, Lodz
- ČERNOCKÝ, J., Brno Univ. of Technology, Brno
- ČERNOHORSKÝ, D., Brno Univ. of Technol., Brno
- DOBOŠ, L., Technical University of Košice, Košice
- DOSTÁL, T., Brno University of Technology, Brno
- HALÁMEK, J., Czech Academy of Sciences, Brno
- HANUS, S., Brno University of Technology, Brno
- KOZUMPLÍK, J., Brno Univ. of Technology, Brno
- KVIČERA, V., TESTCOM, Prague
- LÉDL, P., Czech Technical University, Prague
- LEVICKÝ, D., Technical Univ. of Košice, Košice
- MOHYLOVÁ, J., Techn. Univ. of Ostrava, Ostrava
- POLLÁK, P., Czech Technical University, Prague
- PROVAZNÍK, I., Brno Univ. of Technology, Brno
- SMÉKAL, Z., Brno University of Technology, Brno
- SVAČINA, J., Brno University of Technology, Brno
- ŠEBESTA, V., Brno University of Technology, Brno
- VLČEK, K., Technical Univ. of Ostrava, Ostrava
- VRBATA, J., Czech Technical University, Prague
- WIESER, V., University of Žilina, Žilina