Exploitation of Higher Order Moments Increase the Tracking Aircraft by the Extended Alpha-Beta Filter

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Abstract. *The paper analyzes the possibility of exploitation of higher order moments for increasing the precision of tracking of a flying aircraft by the α-β filter. For tracking of a flying aircraft by the α-β filter the 3rd and 4th order moments in 3D space are used.*

Keywords

Aircraft track moving, α-β filter.

1. Introduction

Up to nowadays, the tracking of flying aircraft by the α-β filters has been used for computation of a position of a flying aircraft using moments of the 1st and 2nd order. These moments represent speed and acceleration of a flying aircraft. This paper describes the tracking of a flying aircraft by extended algorithm for its position generation using the moments of higher orders. For the analysis of flying aircraft positions the 3rd and 4th order moments are used in 3D space by α-β filter [2 - 5, 7, 8].

For the aircraft flight modeling, the track including four sections of flight, i.e. straightforward uniform track, a course maneuver track, speed maneuver track and a height maneuver track were used. The modeling of a flight of an aircraft corresponds to a real aircraft flight.

2. Basic Terms

A utilization of the α-β filter for aircraft tracking in the basic form [2 - 8] uses the 1st and 2nd order moments of a flying aircraft track in a 3-dimension space which represent its speed and acceleration. This filter uses a prediction-correction principle and is expressed by the following terms:

Equation of prediction position

$$x(k+1,k) = \mathbf{F}(k+1,k).x(k) \tag{1}$$

Equation of correction position

$$x(k+1,k+1) = x(k+1,k) + \mathbf{K}(k+1)\left[x_m(k+1) - x(k+1,k)\right] \tag{2}$$

where, $x(k) = [s_{xm}(k), s_{ym}(k), s_{zm}(k), v_x(k), v_y(k), v_z(k)]^T$ is a status vector; $\mathbf{F}(k+1,k)$ is the system transmission matrix of system of time interval $t(k)$ to $t(k+1)$; $\mathbf{K}(k+1)$ is the matrix of tracking filter gain; $s_{xm}(k)$, $s_{ym}(k)$, $s_{zm}(k)$ are coordinates of the aircraft position measured by radar in $t(k)$; $v_x(k)$, $v_y(k)$, $v_z(k)$ are speed elements of the aircraft in $t(k)$; $x(k)$ is the state vector in $t(k)$; $x(k+1,k)$ is the prediction position vector in $t(k+1)$; $x(k+1,k+1)$ is the correction position vector in $t(k+1)$ and $x_m(k+1)$ is the measurement vector of the aircraft position coordinates in $t(k+1)$.

For the system transmission matrix, the following equation holds

$$\mathbf{F}(k+1,k) = \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

where, Δt is a time difference between two samples of the aircraft positions and the system transmission matrix uses 1st order moments.

For the analysis of a higher-order moment effect for prediction of the aircraft's position based on formula (1), the state vector $x(k)$ can be analyzed and due to the system transmission matrix $\mathbf{F}(k+1,k)$ (3) of a tracking filter have to be extended by higher-order moments of a position coordinates. Position coordinates with higher-order moments are represented by the 2nd, 3rd and 4th order moments. Three further models of tracking filter have been created in this case. They have a different form of state vector $x(k)$ and a different form of system transmission matrix $\mathbf{F}(k+1,k)$, as follows

$$x(k) = \left[s_{xm}(k), s_{ym}(k), s_{zm}(k), v_x(k), v_y(k), v_z(k), a_x(k), a_y(k), a_z(k)\right]^T \tag{4}$$

$$\mathbf{F}(k+1,k)=\begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 & \frac{\Delta t^2}{2} & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & \frac{\Delta t^2}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & \frac{\Delta t^2}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (5)$$

which, for the position prediction, uses the 1st and 2nd order moments

$$x(k)=\lfloor s_{xm}(k), s_{ym}(k), s_{zm}(k), v_x(k), v_y(k), v_z(k), a_x(k), a_y(k), a_z(k) \\ w_x(k), w_y(k), w_z(k)\rfloor \quad (6)$$

$$\mathbf{F}(k+1,k)=\begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 & \frac{\Delta t^2}{2} & 0 & 0 & \frac{\Delta t^3}{6} & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & \frac{\Delta t^2}{2} & 0 & 0 & \frac{\Delta t^3}{6} & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & \frac{\Delta t^2}{2} & 0 & 0 & \frac{\Delta t^3}{6} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (7)$$

which for the position prediction uses from 1st to 3rd order moments and

$$x(k)=\lfloor s_{xm}(k), s_{ym}(k), s_{zm}(k), v_x(k), v_y(k), v_z(k), a_x(k), a_y(k), a_z(k) \\ w_x(k), w_y(k), w_z(k), u_x(k), u_y(k), u_z(k)\rfloor \quad (8)$$

$$\mathbf{F}(k+1,k)= \\ =\begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 & \frac{\Delta t^2}{2} & 0 & 0 & \frac{\Delta t^3}{6} & 0 & 0 & \frac{\Delta t^4}{24} & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & \frac{\Delta t^2}{2} & 0 & 0 & \frac{\Delta t^3}{6} & 0 & 0 & \frac{\Delta t^4}{24} & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & \frac{\Delta t^2}{2} & 0 & 0 & \frac{\Delta t^3}{6} & 0 & 0 & \frac{\Delta t^4}{24} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (9)$$

which, for the position prediction, uses from the 1st to 4th order moments. All tracking filters have been analyzed for identical track of an aircraft flight.

3. An Analysis of the Extended α-β Filter

A track of aircraft flight includes, for the needs of analysis, the following sections:

- a straightforward uniform flight
- a course manoeuvre flight
- a height manoeuvre flight
- a speed manoeuvre flight.

The precise track Y_P of an aircraft flight was generated according to the higher demands and it represents a real aircraft flight. The measured track Y_M of the aircraft flight was created from the precise track using noise by a noise generator. The tracking of the aircraft flight was obtained from measured positions Y_M based on filtration using the prediction – correction tracking method as filtered track Y_{io}, where i is the number of the used order moment.

Expressions of the coordinate difference Δx_{io} for all coordinates, x, y, z, for determination of the positions of the flying aircraft track based on from 1st to 4th order moments may be defined as [1]

$$\Delta x_{1o}(k) = v_x(k).\Delta t \quad (10)$$

$$\Delta x_{2o}(k) = 0{,}5.a_x(k).\Delta t^2 \quad (11)$$

$$\Delta x_{3o}(k) = 0{,}166.w_x(k).\Delta t^3 \quad (12)$$

$$\Delta x_{4o}(k) = 0{,}042.u_x(k).\Delta t^4 \quad (13)$$

where, $v(k)$ is speed, $a(k)$ is acceleration, $w(k)$ and $u(k)$ present the dynamic properties of the flying aircraft in time interval $t(k)$.

3.1 Analysis of Tracking Precision for Track Including Section of Straightforward Uniform Flight by Extended α-β Filter

For the analysis of aircraft track, the moments of the 1st (Y_{1o}), 1st to 2nd (Y_{2o}), 1st. to 3rd (Y_{3o}) and 1st to 4th (Y_{4o}) order by the extended α-β filter were used. On the basis of the α-β filter with regard to the sowed moments of higher - order there were obtained the results of the aircraft tracking for the track including a straightforward uniform flight section and coordinate difference which are depicted in Tab. 1 and Fig 1.

The additional contribution of moments from 1st to 4th orders for determination of the flying aircraft positions is sowed in Fig. 1 in meters. Values of 1st order moments express the coordinate imprecision error between two positions of the track and they are in interval approximately to 7000 meters. Values of 2nd order moments express the coordinate imprecision error between two positions of the track and they are approximately in interval to 500 meters. Values of 3rd order moments express the coordinate imprecision error between two positions of track and are approximately in interval to 350 meters. Values of 4th order moments express the coordinate imprecision error between two positions of track and are approximately in interval to 200 meters.

Straightforward uniform flight of aircraft					
	x - coordinate [m]				
Δx_{1o}	6990	7070	6100	6450	6470
Δx_{2o}	13	42	-484.9	172.75	10.5
Δx_{3o}	52.788	9.628	-176.964	218.29	-53.95
Δx_{4o}	62.58	-10.92	-46.62	99.54	-68.88
	y - coordinate [m]				
Δy_{1o}	3300	2950	4520	4060	4260
Δy_{2o}	52.15	-178.15	486.35	-229.65	202.7
Δy_{3o}	-71.712	-76.526	320.214	-337.312	110.39
Δy_{4o}	-98.28	-1.26	100.38	-166.3	113-4
	z - coordinate [m]				
Δz_{1o}	20	20	50	10	-40
Δz_{2o}	17.9	-0.05	14.25	-18.1	-27.7
Δz_{3o}	4.98	-5.976	4.814	-10.79	-3.154
Δz_{4o}	-0.42	-2.94	2.52	-3.78	2.1

Tab. 1. Coordinate difference for the track including straightforward uniform flight section of aircraft with regard to higher - order moments.

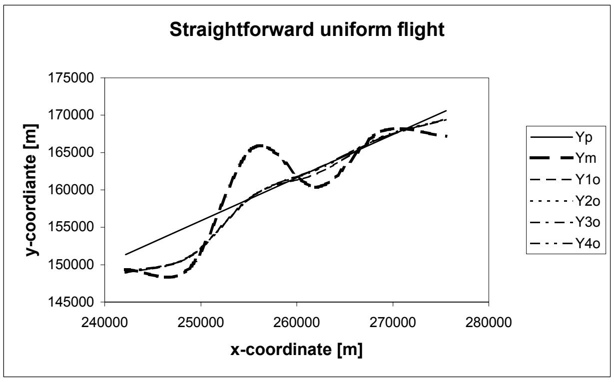

Fig. 1. Results of model tracking aircraft flight for section of rectilinear equal flying of aircraft (Direct, steady flight of aircraft).

3.2 Analysis of Tracking Precision for Track Including Section of Course Manoeuvre Flight by Extended α-β Filter

By modeling the tracking filter, results were obtained for the section of a flight of an aircraft with a left loop at 360°. These are given in Tab. 2 and expressed in Fig. 2.

The obtained values difference and corresponding coordinates of tracking a flying aircraft with moments from the 1st to 4th order correspond to the results, which were obtained in the section of direct, steady flight of an aircraft.

Course manoeuvre flight of aircraft					
	x-coordinate [m]				
Δx_{1o}	2040	1540	1200	180	-430
Δx_{2o}	-127.15	-253.45	-169.5	-507.45	-304.85
Δx_{3o}	-60.922	-41.998	27.888	-112.216	67.23
Δx_{4o}	-15.12	4.62	17.64	-35.28	45.36
	y-coordinate [m]				
Δy_{1o}	1540	2010	1960	2100	1570
Δy_{2o}	60.9	233.7	-24.7	71.5	-265.55
Δy_{3o}	29.216	57.436	-85.822	31.872	-111.884
Δy_{4o}	2.94	7.14	-36.12	29.82	-36.54
	z-coordinate [m]				
Δz_{1o}	-50	-30	-50	-10	-50
Δz_{2o}	-48.65	9.6	-7.8	20.55	-24.65
Δz_{3o}	-21.082	19.422	-5.81	9.462	-14.94
Δz_{4o}	-7.14	10.08	-6.3	3.78	-6.3

Tab. 2. Coordinate order difference for section of tracking flight of aircraft with left loop at 360° with regard to the higher order moments.

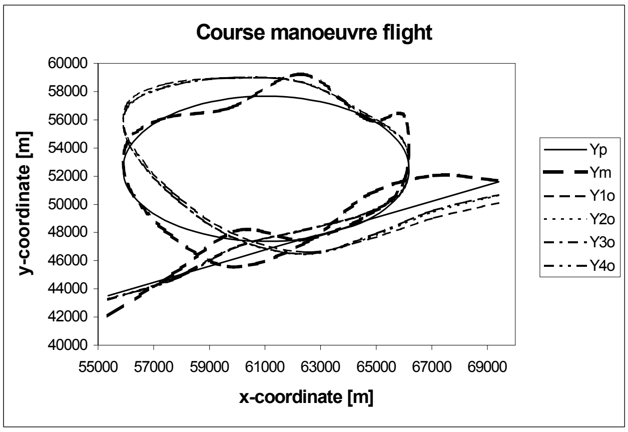

Fig. 2. Results of modeling track of flying aircraft for the section of tracking flight of the aircraft with left loop at 360°.

3.3 Analysis of Tracking Precision for Track Including Section of Height Manoeuvre Flight by Extended α-β Filter

By modeling the tracking filter, results, which are given in Tab. 3 and expressed in Fig. 3, were obtained for track including the section of height manoeuvre flight by extended α-β filter of track.

The obtained moments of the first to the fourth order correspond to the results that were obtained in the previous case.

Height manoeuvre flight of aircraft					
	x-coordinate [m]				
Δx_{1o}	990	1340	1800	2100	2200
Δx_{2o}	243.55	175.2	232.8	150.7	42.25
Δx_{3o}	-44.488	-22.742	19.09	-27.224	-34.694
Δx_{4o}	-13.02	5.46	10.5	-11.76	-1.68
	y-coordinate [m]				
Δy_{1o}	-1000	-250	140	490	1210
Δy_{2o}	397.05	372.9	19645	175.1	360.3
Δy_{3o}	106.406	-7.968	-58.598	-7.138	61.42
Δy_{4o}	-5.04	-28.98	-12.6	13.02	17.22
	z-coordinate [m]				
Δz_{1o}	180	260	380	420	470
Δz_{2o}	93.9	42.15	59.15	21.95	26.1
Δz_{3o}	30.046	-16.098	5.644	-12.284	1.328
Δz_{4o}	1.68	-11.76	5.88	-4.62	3.36

Tab. 3. Differences of coordinates for track including the section of the height manoeuvre flight of an air object to a higher flying level with regard to higher order moments.

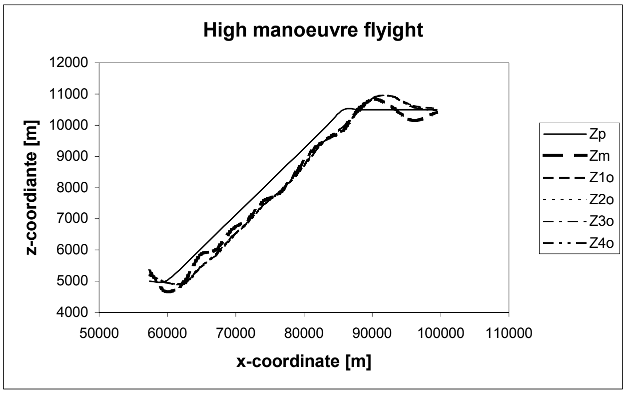

Fig. 3. Results of modeling of track of moving air object for section of tracking transition to higher-flying level.

3.4 Analysis of Tracking Precision for Track Including Section of Speed Manoeuvre Flight by Extended α-β Filter

By modeling the tracking filter, the results were obtained for the section of track including the direct unsteady movement of an air object. These are given in Tab. 4 and expressed in Fig. 4.

Moments of the 1st to 4th order obtained by modeling of this case confirm the results in the previous case.

4. Conclusion

By analysis of the presented α-β filter with regard to the 1st to 4th order moments there were obtained results, which were used for prediction of the position of a moving air object during processing of radar data. These results confirm that the accuracy of tracking does not depend on the character of tracks and movement of flying air object. It depends on the order of the used moment. For individual moments, results were achieved for coordinate tracking positions in ranges:

- for the 1st moments up to 7000m,
- for the 2nd moments up to 500m,
- for the 3d moments up to 350m,
- for the 4th moments up to 200m,

whereby it is confirmed that their application is useful due to the increase of tracking accuracy.

Speed manoeuvre flight of aircraft					
	x-coordinate [m]				
Δx_{1o}	2540	2660	3190	3440	3880
Δx_{2ov}	-28.4	58.95	261.6	126.4	222.8
Δx_{3o}	-12.948	29.05	67.23	-44.82	32.038
Δx_{4ov}	2.52	10.5	9.66	-28.56	19.32
	y-coordinate [m]				
Δy_{1o}	1060	1570	1440	1670	1720
Δy_{2o}	266.4	-15.4	-66.2	117.35	20.4
Δy_{3o}	121.18	-91624	-16.932	60.922	-32.204
Δy_{4o}	27.72	-54.18	19.32	19.74	-23.52
	z-coordinate [m]				
Δz_{1o}	70	10	120	40	-30
Δz_{2o}	-113.55	-30	53.8	-37.35	-33.15
Δz_{3o}	-9.13	27.722	27.888	-30.212	1.328
Δz_{4o}	3.36	9.24	0	-14.7	7.98

Tab. 4. Differences of coordinates for the section of the track with direct unsteady movement of an air object with regard to higher order moments.

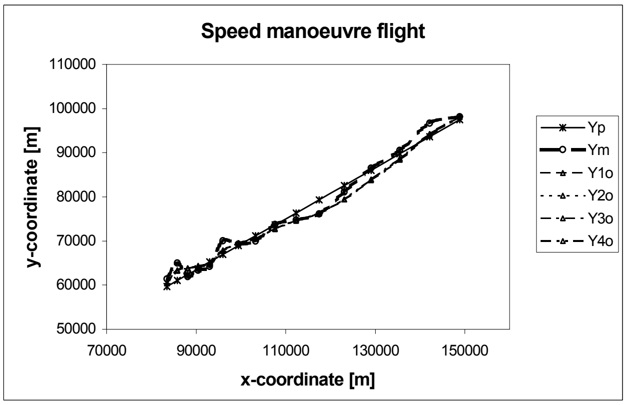

Fig. 4. Results of modeling the track of moving air object for section of track with direct unsteady movement.

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RADIOENGINEERING REVIEWERS

September 2003, Volume 12, Number 3