

An Accuracy Comparison of the Digital Filter Poles-Zeros Analysis With MATLAB and CIA Algorithms

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Abstract. In the paper, a comparison of MATLAB and CIA procedures for computing the poles and zeros of digital filters is performed. The accuracy of both tools is tested using finite impulse response filters of a higher order – the first was designed by “remez” algorithm and the second came from the area of psychoacoustics.

Keywords

MATLAB, digital filter, \mathcal{Z} -transform, finite impulse response, poles-zeros analysis, general eigenvalue problem, sparse-matrix reduction.

1. Introduction

At present, digital signal processing represents mostly used way of constructing the radio communication systems. One of the most important parts of the digital signal processing is the analysis of digital filters. Many sophisticated requests on the digital filters may cause an enormous complexity of the final realization. Therefore, the precision of the algorithms has to be thoroughly checked.

In the poles-zeros analysis of a digital filter transfer function two major procedures are applied:

- In the first one the transfer function is found initially, and then the poles/zeros as roots of the transfer function denominator/numerator polynomials are determined, respectively. This way can be used in tools of the MATLAB type.
- The second procedure is based on using *inverse* \mathcal{Z} -transform, which is known to be more advantageous than the standard one, to formulate a general eigenvalue problem in the z^{-1} plane, to reduce the general eigenvalue problem to a standard one, and, finally, to solve the standard eigenvalue problem by the QR algorithm. This way is utilized in the CIA algorithms.

In both cases, it is convenient to use longer numerical data types together with more precise arithmetic – either as just fully utilizing the given hardware capabilities or by applying a multiple-precision arithmetic library [2].

2. Generally Usable Algorithm

2.1 An Alternative to the Reduction Algorithm for the Digital Filter Poles-Zeros Analysis

A system of linear equations of a digital filter can be written by means of inverse \mathcal{Z} -transform in the matrix form

$$(z^{-1}\mathbf{P} + \mathbf{Q})\mathbf{X} = \mathbf{Y}, \quad (1)$$

where z denotes the \mathcal{Z} -transform operator, \mathbf{X} is the vector of z^{-1} images of the digital circuit variables, and \mathbf{Y} is a vector of the sources that excite the circuit.

The poles of all the transfer functions and the zeros of a transfer function are determined by the equations

$$\begin{aligned} \det(z^{-1}\mathbf{P} + \mathbf{Q}) &= 0 \text{ for poles in } z^{-1} \text{ plane,} \\ \det(z^{-1}\mathbf{P}^{(0j)} + \mathbf{Q}^{(ij)}) &= 0 \text{ for zeros in } z^{-1} \text{ plane,} \end{aligned} \quad (2)$$

where matrices $\mathbf{P}^{(0j)}$ and $\mathbf{Q}^{(ij)}$ arise from the original ones – the first by clearing j^{th} column and the second by replacing j^{th} column with \mathbf{Y} , where all its elements are cleared, with the exception of that corresponding to the i^{th} source.

Solving the general eigenvalue problem defined by (2) is, of course, more difficult task than solving a standard one. Therefore, a systematic reduction [1] is applied for transforming (2) to the standard form, which has been described in more detail in [2]. After the transformation, the determinant can be computed by the classical evolution

$$\begin{aligned} \det(z^{-1}\mathbf{P} + \mathbf{Q}) &= \\ (-1)^{n_{\text{exch}}} \prod_{i=m+1}^n Q_{22,ii} \det(\mathbf{P}_{11}\mathbf{P}_{11}^{-1}(z^{-1}\mathbf{P}_{11} + \mathbf{Q}_{11})) &= \quad (3) \\ (-1)^{n_{\text{exch}}} \prod_{i=1}^m P_{11,ii} \prod_{i=m+1}^n Q_{22,ii} \det(z^{-1}\mathbf{1} + \mathbf{P}_{11}^{-1}\mathbf{Q}_{11}), \end{aligned}$$

where n_{exch} is the total number of all the row and column exchanges during the reduction, \mathbf{P}_{11} and \mathbf{Q}_{22} are the one-diagonal upper-left and lower-right submatrices obtained after the reduction, respectively, and $\mathbf{1}$ is the unity matrix – see [2] where the reduction algorithm is thoroughly defined including the pivots selection.

2.2 Solving the Standard Eigenvalue Problem

The final step for determining the poles and zeros of a transfer function is the same as that in analog circuits, i.e., to compute the eigenvalues of the matrix

$$\mathbf{Q}' = -\mathbf{P}_{11}^{-1}\mathbf{Q}_{11}, \quad (4)$$

where \mathbf{Q}_{11} is the final square matrix $m \times m$, $m < n$ obtained by the reduction. The matrix has irregular structure, and therefore is handled by the full-matrix QR algorithm (double stepped with automatic shifts of the origin).

3. Comparing the Accuracy of Results

3.1 A Filter Designed by “Remez” Algorithm

Let’s consider a 30th-order symmetrical finite impulse response (FIR) digital filter (see Fig. 2) designed by the classical “remez” algorithm [3, 4] with the coefficients a_4 - a_{26} (unspecified in Fig. 2) defined by the expressions

$$\begin{aligned} a_4 &= a_{26} = -0.00559342349260, \\ a_5 &= a_{25} = 0.00624262031546, \\ a_6 &= a_{24} = -0.00684884769613, \\ a_7 &= a_{23} = 0.00897910496988, \\ a_8 &= a_{22} = -0.00897865368875, \\ a_9 &= a_{21} = 0.01750107253210, \\ a_{10} &= a_{20} = -0.00695363594317, \\ a_{11} &= a_{19} = 0.03977449895168, \\ a_{12} &= a_{18} = -0.06465598090179, \\ a_{13} &= a_{17} = 0.08524095225662, \\ a_{14} &= a_{16} = -0.13129215646492, \\ a_{15} &= 0.19514096846301 \end{aligned} \quad (5)$$

(naturally, the relations $a_{29} = a_1$, $a_{28} = a_2$, and $a_{27} = a_3$ are also fulfilled due to the symmetry). A comparison of the zeros of the transfer function “32nd circuit variable / 1st input source” obtained by MATLAB and CIA is shown in Tab. 1 (the genuine 15- and 7-digit prints used). See also the characteristic zeros diagram of this “remez” filter in Fig. 1.

The filter has 28 complex zeros (i.e. 14 pairs) and 2 real zeros. Comparing all the digits in the MATLAB and CIA outputs in Tab. 1, we obtain that 2 pairs have equal 5 valid digits, 3 pairs have equal 6 valid digits, and 2 real zeros and 9 pairs have equal 7 or more valid digits.

3.2 A Filter Designed for the Psychoacoustics – a Part of Physiological Volume Control

Let’s consider a 48th-order symmetrical finite impulse response (FIR) filter of the same circuit structure as the previous one, used as a part of a sequence for the sophisticated physiological volume control [4] – the filter processes the higher frequency part of the acoustic range. All the coefficients of the filter are defined by the expressions

$$\begin{aligned} a_0 &= a_{48} = 0.00012511398639, \\ a_1 &= a_{47} = 0.00001335284427, \\ a_2 &= a_{46} = 0.00016015250121, \\ a_3 &= a_{45} = -0.00000634686622, \\ a_4 &= a_{44} = 0.00026201837991, \\ a_5 &= a_{43} = -0.00007281852105, \\ a_6 &= a_{42} = 0.00045629795460, \\ a_7 &= a_{41} = -0.00022819555936, \\ a_8 &= a_{40} = 0.00071588589103, \\ a_9 &= a_{39} = -0.00047316021190, \\ a_{10} &= a_{38} = 0.00110189764986, \\ a_{11} &= a_{37} = -0.00088563032407, \\ a_{12} &= a_{36} = 0.00184819117706, \\ a_{13} &= a_{35} = -0.00188050116629, \\ a_{14} &= a_{34} = 0.00282392666400, \\ a_{15} &= a_{33} = -0.00363976768981, \\ a_{16} &= a_{32} = 0.00591155524557, \\ a_{17} &= a_{31} = -0.00644303257612, \\ a_{18} &= a_{30} = 0.01406190034797, \\ a_{19} &= a_{29} = -0.00537162176461, \\ a_{20} &= a_{28} = 0.03594691432517, \\ a_{21} &= a_{27} = -0.06164502638211, \\ a_{22} &= a_{26} = 0.08276620944465, \\ a_{23} &= a_{25} = -0.13009560635626, \\ a_{24} &= 0.19452719610477. \end{aligned} \quad (6)$$

A comparison of the zeros of the transfer function “50th circuit variable / 1st input source” obtained by the MATLAB and CIA algorithms is shown in Tab. 2. (See the typical module characteristic for high frequencies in Fig. 3.)

The filter has 48 complex (24 pairs) zeros. Comparing again all the digits in the MATLAB and CIA outputs in Tab. 2, we obtain that 6 pairs have equal 6 valid digits and 18 pairs have equal 7 or more valid digits.

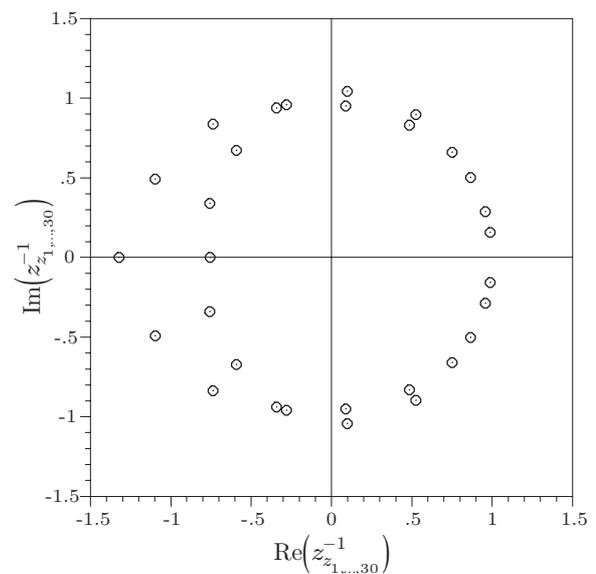


Fig. 1. Diagram of the zeros of the FIR “remez” digital filter transfer function in the z^{-1} plane.

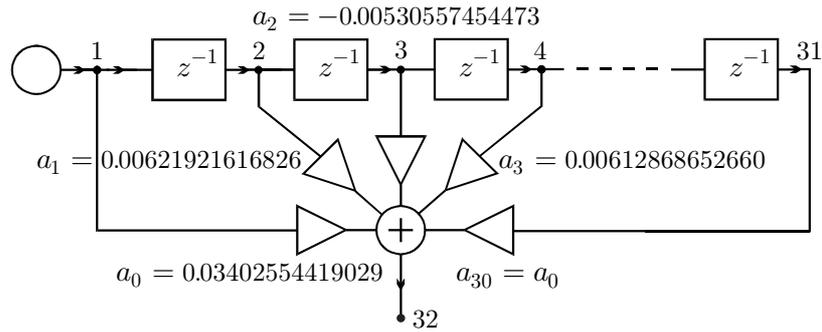


Fig. 2. Digital filter with finite impulse response (FIR) designed by “remez” algorithm.

Algorithm of MATLAB	Full-pivot matrix deflation and QR algorithm of CIA
$0.98762289580755 \pm 0.15684710923928 j$	$0.9876229 \pm 0.1568471 j$
$0.95786444294520 \pm 0.28722066245533 j$	$0.9578645 \pm 0.2872207 j$
$0.86485064857005 \pm 0.50202923786167 j$	$0.8648506 \pm 0.5020292 j$
$0.75053028577891 \pm 0.66083605389584 j$	$0.7505303 \pm 0.6608360 j$
$0.52542838736165 \pm 0.89769090601108 j$	$0.5254284 \pm 0.8976910 j$
$0.48564262387778 \pm 0.82971719365131 j$	$0.4856426 \pm 0.8297172 j$
$0.09771969508857 \pm 1.04311171951640 j$	$0.09771969 \pm 1.043112 j$
$0.08902780391880 \pm 0.95032987512232 j$	$0.08902780 \pm 0.9503298 j$
-1.32239487744419	-1.322395
$-0.28137226477781 \pm 0.95959869144023 j$	$-0.2813723 \pm 0.9595987 j$
$-0.34165819858630 \pm 0.93982427896856 j$	$-0.3416582 \pm 0.9398243 j$
$-1.09765883781769 \pm 0.49181658799519 j$	$-1.097659 \pm 0.4918166 j$
$-0.73860557659139 \pm 0.83694986030126 j$	$-0.7386056 \pm 0.8369499 j$
$-0.59277029289145 \pm 0.67169681566146 j$	$-0.5927703 \pm 0.6716968 j$
$-0.75871268555997 \pm 0.33994850806526 j$	$-0.7587127 \pm 0.3399485 j$
-0.75620377623718	-0.7562038

Tab. 1. Comparison of zeros of the FIR “remez” digital filter transfer function (the unequal digits of the CIA results are underlined).

3.3 An Example of the Irregular Infinite Impulse Filter Designed by “Chained Fractions”

Let’s consider a 3rd-order infinite impulse response (IIR) filter [6] which is designed by the method of “chained fractions”. The filter is irregular – it is shown in Fig. 4. The tools of MATLAB type are able to analyze regular forms (e.g. canonical) only, and thus they cannot determine poles

and zeros of this filter. Generally usable algorithm outlined in Sec. 2 has no problems with the irregularity – the poles and zeros in z^{-1} plane have been easily determined as

$$\begin{aligned} z_{p_{1,2}}^{-1} &= 0.9588639 \pm 0.7240575j, z_{p_3}^{-1} = 1.511628, \\ z_{z_{1,2}}^{-1} &= -0.9049098 \pm 0.1414979j, z_{z_3}^{-1} = -1.192327. \end{aligned} \quad (7)$$

Note that IIR digital filters of very high order may sometimes have problems to reduce “similar” poles/zeros. In that case, a secondary root polishing [5] can be utilized.

Algorithm of MATLAB	Full-pivot matrix deflation and QR algorithm of CIA
$-1.27851808211318 \pm 0.62684824819101 j$	$-1.278518 \pm 0.6268482 j$
$-1.36614299238589 \pm 0.27717114439577 j$	$-1.366143 \pm 0.2771711 j$
$-1.07284006705681 \pm 0.91235620647926 j$	$-1.072840 \pm 0.9123562 j$
$-0.77267104757162 \pm 1.12541933041583 j$	$-0.7726710 \pm 1.125419 j$
$-0.44961436355736 \pm 1.22363585379352 j$	$-0.4496143 \pm 1.223636 j$
$-0.20330452525073 \pm 1.29600355151311 j$	$-0.2033045 \pm 1.296004 j$
$0.14750702212683 \pm 1.34841369533889 j$	$0.147507 \pm 1.348414 j$
$0.50634734941036 \pm 1.25979827310114 j$	$0.5063474 \pm 1.259798 j$
$0.83329614995319 \pm 1.04982507756824 j$	$0.8332961 \pm 1.049825 j$
$1.04560310563666 \pm 0.67293837999034 j$	$1.045603 \pm 0.6729383 j$
$1.20771535897979 \pm 0.24059979456907 j$	$1.207715 \pm 0.2405998 j$
$0.95139216401989 \pm 0.56315210274529 j$	$0.9513923 \pm 0.5631522 j$
$0.77837030003487 \pm 0.46073626392613 j$	$0.7783702 \pm 0.4607362 j$
$0.79640192035334 \pm 0.15865836018951 j$	$0.7964019 \pm 0.1586584 j$
$0.67627030631208 \pm 0.43523995090672 j$	$0.6762703 \pm 0.43524 j$
$0.46384030886663 \pm 0.58436750039291 j$	$0.4638403 \pm 0.5843675 j$
$0.27466939387352 \pm 0.68338074343342 j$	$0.2746694 \pm 0.6833807 j$
$0.08016782642482 \pm 0.73284236586261 j$	$0.08016782 \pm 0.7328424 j$
$-0.11813451163119 \pm 0.75307102211071 j$	$-0.1181345 \pm 0.753071 j$
$-0.26456653442537 \pm 0.72002392155667 j$	$-0.2645665 \pm 0.7200239 j$
$-0.41461446573694 \pm 0.60389882069075 j$	$-0.4146145 \pm 0.6038988 j$
$-0.70304843505842 \pm 0.14263861132902 j$	$-0.7030484 \pm 0.1426386 j$
$-0.63057376627246 \pm 0.30916579614562 j$	$-0.6305737 \pm 0.3091658 j$
$-0.54091513103870 \pm 0.46000078868751 j$	$-0.5409151 \pm 0.4600008 j$

Tab. 2. Comparison of zeros of the FIR “psychoacoustic” digital filter transfer function (the unequal digits of the CIA results are underlined).

4. Conclusion

Two high-order regular finite impulse response (FIR) digital filters have been tested by both MATLAB and CIA algorithms by means of the semisymbolic analysis. The results computed by MATLAB and CIA may be considered accurate at least to five initial valid digits. However, MATLAB is not able to analyze irregular filters.

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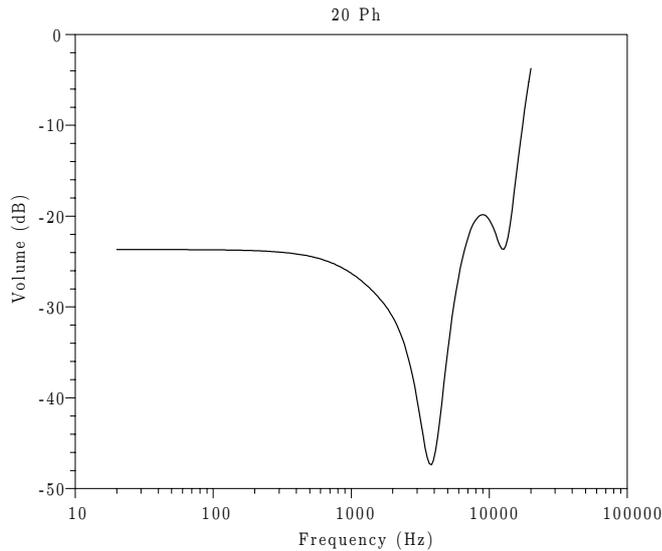


Fig. 3. Frequency response of the FIR “psychoacoustic” digital filter. (A part of the row operating at higher frequencies.)

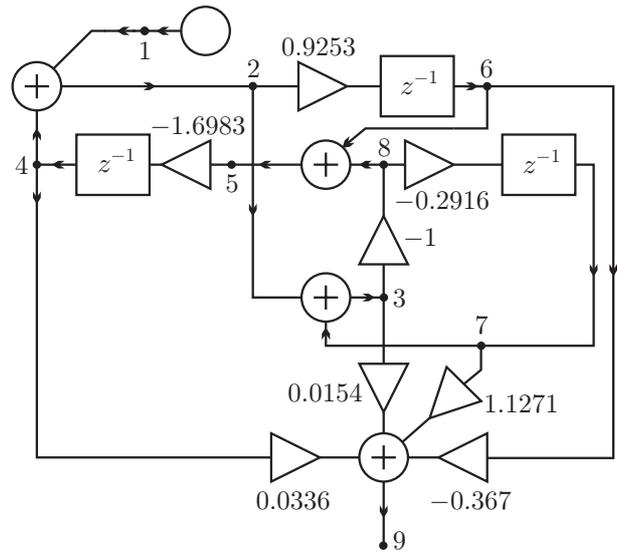


Fig. 4. Irregular infinite impulse response (IIR) digital filter designed by the method of “chained fractions”.

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Josef DOBEŠ received the Ph.D. degree in microelectronics at the Czech Technical University in Prague in 1986. From 1986 to 1992, he was a researcher of the TESLA Research Institute, where he performed analyses on algorithms for CMOS Technology Simulators. Currently, he works at the Department of Radio Electronics of the Czech Technical University in Prague. His research interests include the physical modeling of radio electronic circuit elements, especially RF and microwave transistors and transmission lines, creating or improving special algorithms for the circuit analysis and optimization, such as time- and frequency-domain sensitivity, poles-zeros or steady-state analyses, and creating a comprehensive CAD tool for the analysis and optimization of RF and microwave circuits.