Modeling the Substrate Skin Effects in Mutual RL Characteristics of General Asymmetric Coupled Interconnects

Hasan YMERI,[1] *B. NAUWELAERS*[1]*, Karen MAEX*[1,2]*, D. DE ROEST*[2]

[1]Dept. of Electrical Engineering (ESAT), Div. ESAT-TELEMIC, Katholieke Universiteit Leuven, Kasteelpark Arenberg 10, B-3001 Leuven-Heverlee, Belgium
[2]The Interuniversity Microelectronics Center (IMEC), Kapeldreef 75, B-3001 Leuven, Belgium

Hasan.Ymeri@esat.kuleuven.ac.be

Abstract. *The goal of this work was to model the influence of the substrate skin effects on the distributed mutual impedance per unit length parameters of multiple coupled on-chip interconnects. The proposed analytic model is based on the frequency-dependent distribution of the current in the silicon substrate and the closed form integration approach. It is shown that the calculated frequency-dependent distributed mutual inductance and the associated mutual resistance are in good agreement with the results obtained from CAD-oriented circuit modeling technique.*

Keywords

On-chip interconnects, lossy silicon, RL line characteristics, analytic model, skin effects.

1. Introduction

High-frequency effects such as delay, distortion and crosstalk, are found to be a limiting factor in the development of VLSI silicon integrated circuits. Due to the continuing decrease of the distances between components and the simultaneous increase of operating frequency, the crosstalk between components and/or blocks through the lossy substrate becomes stronger. As the operating speed of VLSI increases beyond few tens of GHz, the simple modeling approach for distributed RL parameters of on-chip interconnects including frequency-dependent behavior is crucial for the design and validation phase of high-speed VLSI circuits [1]. The ultimate performance of high-speed VLSI circuits is strongly influenced by the materials used for the substrate [3] .The advanced technology of multi-layer multichip modules (MCM), especially MCM-D on silicon substrate, has many advantages such as smaller size, lighter systems and improved performance, making it suitable for system-on-chip implementations. Silicon substrates have lower resistivity due to the requirements for the formation of active devices. In this case losses in the semiconductor substrate become significant and should be taken into account. In order to accomplish this, it is necessary to analyze and model the broadband characteristics [2, 4 - 9] of the on-chip interconnects on a silicon oxide-silicon substrate. With respect to the substrate conductivity and signal frequency, in general three wave propagation modes can be distinguished which lead to different formulas for the line parameters (quasi-TEM, slow-wave and skin-effect mode, respectively) [9]. In recent works [2, 4, 5] the authors succeeded to describe the frequency-dependent transmission line parameters of on-chip interconnects on lossy silicon substrate in a unified way. Therefore, simple and fast methods and formulas will be necessary to generate electrical models for the on-chip interconnects that accurately account for such effects as delay, crosstalk and resistive voltage drops.

In this letter, a new model that takes into account the influence of the substrate skin effects on distributed RL characteristics of coupled interconnects on a lossy silicon substrate is presented. To illustrate the accuracy of the model, the frequency-dependent mutual inductance and resistance of multiple coupled interconnects on a silicon substrate were computed using proposed analytic formulas, and compared with the solution by CAD-oriented circuit modeling approach.

2. New Frequency-Dependent Model

To develop an expression for series mutual impedance of general asymmetric coupled interconnect conductors on a lossy silicon substrate (see Fig. 1), the quasi-TEM wave is considered, and the magnetic vector potential **A** determinated as

$$\mathbf{H} = \frac{1}{\mu} rot\ \mathbf{A} \tag{1}$$

has only a *z*-component and satisfies the magnetic vector potential equation

$$\nabla^2 A_i - j\omega\mu_i\sigma_i A_i = -\mu J_z(x, y) \tag{2}$$

where $J_z(x,y)$ is unknown current density on the interconnect conductors. Results obtained from the full-wave analysis [2, 4] have shown that the influence of the finite substrate thickness t_{si} can be neglected for practical dimensions (i.e. $t_{si} \gg w_1, w_2$, $t_{si} \gg t_{ox}$, $t_{si} \gg T_1, T_2$, and $t_{si} \gg s$). The silicon substrate is therefore assumed to be infinitely thick in the following derivation. The w_1, w_2 are the widths and T_1, T_2 are the thickness of the metal strip conductors, s is the distance between the interconnect conductors and t_{ox} is the thickness of the silicon oxide.
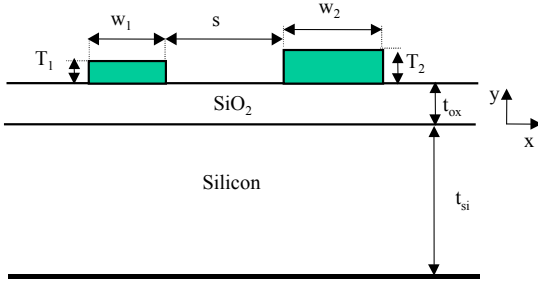

Fig. 1. Cross section of general asymmetric coupled interconnects on an oxide-semiconductor substrate.

In the framework of the quasi-TEM approach, the coupled on-chip interconnects on a lossy silicon substrate (Fig. 1), in the frequency domain, are characterized by the mutual per unit length series impedance $Z_{sm}(\omega)$

$$Z_{sm}(\omega) = j\omega L_m(\omega) + \Delta Z(\omega) \tag{3}$$

where ω is the angular frequency, L_m is the per unit length external mutual inductance of coupled on-chip interconnects on a lossy silicon substrate with zero resistivity (perfect conductor), and ΔZ

$$\Delta Z(\omega) = Z_{sim}(\omega) \tag{4}$$

is the perturbation part of the mutual series per unit length impedance that takes into account skin-effect phenomena in a lossy silicon substrate.

The external per unit length mutual inductance L_m is a frequency-independent real quantity and can be determined using quasi-magnetostatic analysis and image theory in the closed form as [10, 11, 14]

$$L_m = \frac{\mu}{2\pi} \ln \sqrt{\frac{(s_{eq} + r_{1eq} + r_{2eq})^2 + (H_{1eq} + r_{1eq} + H_{2eq} + r_{2eq})^2}{(s_{eq} + r_{1eq} + r_{2eq})^2 + (H_{1eq} + r_{1eq} - H_{2eq} - r_{2eq})^2}} \tag{5}$$

where μ is the magnetic permeability of the medium, $r_{ieq} = (w_i + T_i)/4$ ($i = 1, 2$), $H_{1eq} = t_{ox} + (T_1 - w_1)/4$, $H_{2eq} = t_{ox} + (T_2 - w_2)/4$, $s_{eq} = s + (w_1 - T_1)/4 + (w_2 - T_2)4$. The round-sectioned lines embedded in an electrically homogeneous medium are assumed, which is realistic for most microelectronic micrometer designs [14].

The silicon substrate per unit length impedance $Z_{sim}(\omega)$ is a frequency complex quantity and can be determined using the theory of special functions and integral equation method [14, 15].

The silicon substrate impedance given here makes the expression more tractable for use in micrometer and sub-micrometer designs.

Applying the method of separation of variables and expressing the magnetic field components of the line current as Fourier integrals, the solution of (2) that can lead to eqn. (6) for perturbation part of mutual impedance per unit length is given in the Appendix.

If a lossy silicon substrate has conductivity σ and relative permeability μ_r, then resulting integral expression (part of mutual impedance due to the "shadow currents" induced in the conductive silicon substrate beneath the interconnect lines) can be written as [4]

$$Z_{sim}(\omega) = j\omega \frac{\mu_r \mu_0}{\pi} \int_0^\infty \frac{e^{-2t_{ox}\lambda} \cos(s\lambda)}{\sqrt{\lambda^2 + jm_{si}^2} + \mu_r \lambda} d\lambda$$

$$m_{si} = \sqrt{\omega\mu\sigma} \tag{6}$$

where λ denotes the spectral domain variable (Fourier Transform domain), t_{ox} is the thickness of the silicon oxide, μ is the magnetic permeability of the substrate and s the distance between the coupled interconnect conductors.

It is convenient to reduce the number of the explicit parameters in the integral in (6) by setting $\xi = m_{si}s$, and $\eta = 2m_{si}t_{ox}$ to obtain

$$Z_{sim} = \frac{\omega\mu_r\mu_0}{\pi} I_\mu(\xi, \eta),$$

$$I_\mu(\xi, \eta) = j\int_0^\infty \frac{e^{-\eta\tau} \cos(\xi\tau)}{\sqrt{\tau^2 + j} + \mu_r\tau} d\tau. \tag{7}$$

where $\tau = \lambda/m_{si}$. In the case of a silicon substrate $\mu_r = 1$, the Laplace integral I_μ can be expressed in closed form. To do so, define r and θ such that $\xi + j\eta = re^{j\theta}$; then we get

$$I_1(\xi, \eta) = \int_0^\infty \left(\sqrt{\tau^2 + j} - \tau\right) e^{-\eta\tau} \cos(\xi\tau) d\tau$$

$$= \frac{\pi}{4r} e^{j\pi/4}\left[e^{-j\theta} K\left(re^{j\left(\frac{\pi}{4}+\theta\right)}\right) + e^{j\theta} K\left(re^{j\left(\frac{\pi}{4}-\theta\right)}\right)\right] - \frac{\cos(2\theta)}{r^2}. \tag{8}$$

Here $K(z)$ is shorthand for $H_1(z) - Y_1(z)$, where Y_1 is the Bessel function of second kind and order 1, and H_1 is the Struve function of order 1 (a particular integral of Bessel's equation). The function H_1 may be evaluated using the power series [15]

$$H_1(z) = \frac{2}{\pi}\left(\frac{z^2}{3} - \frac{z^4}{3^2 5} + \frac{z^6}{3^2 5^2 7} - \dots\right). \tag{9}$$

3. Numerical Results

In order to validate the derived new formulas, the frequency-dependent mutual per-unit-length inductance and

resistance [$Z_{sm}(\omega) = R_m(\omega) + j\omega L_m(\omega)$] for a coupled interconnect structure on a heavily doped CMOS substrate (resistivity ρ_{si} = 0.01 Ωcm) with a 3 μm oxide layer are considered. The cross sections of the conductors are 2 μm by 1 μm and 1 μm by 1 μm, and the thickness of the silicon substrate is 300 μm. The spacing between the two conductors is 2 μm. Fig. 2 shows the variation in the distributed mutual resistance, $R_m(\omega)$, as a function of frequency. Similarly, Fig. 3 shows the change in the distributed mutual inductance, $L_m(\omega)$, as a function of frequency. It is observed that the values of the mutual inductance and resistance per unit length, calculated from the new formulas, are found to be in good agreement with those of [8] (quasi-magnetostatic analysis and circuit modeling technique). The agreement is excellent over the entire frequency range of 0.4 - 10 GHz. The maximum deviations observed between the new analytic formulas and the EM simulation results of Figs. 2 and 3 correspond to relative error less than 2 %. The relative errors between the curves in Figs. 2 and 3 for R_m and L_m are shown in Fig. 4. As expected, a lossy silicon substrate has a significant impact on the frequency-dependent characteristics of multiple coupled on-chip interconnects and must be attributed to the skin effects in the substrate.
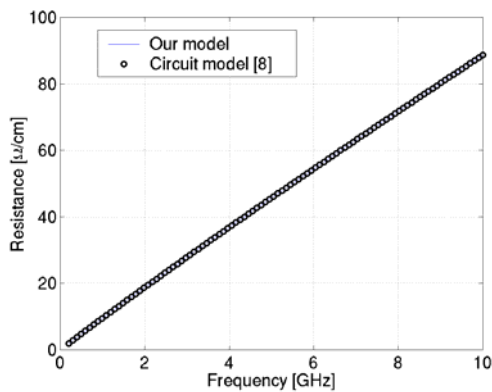

Fig. 2. Distributed mutual resistance per unit length of asymmetric coupled interconnects on a lossy silicon substrate as the function of a frequency.
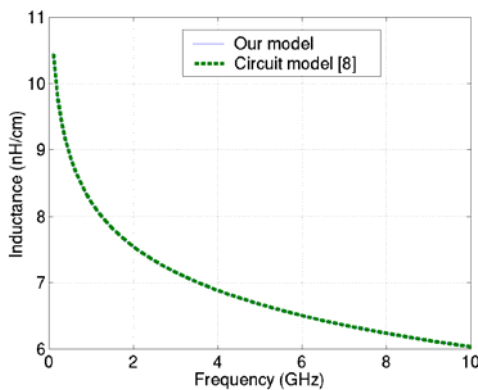

Fig. 3. Distributed mutual inductance per unit length of asymmetric coupled interconnects on a lossy silicon substrate as the function of a frequency.
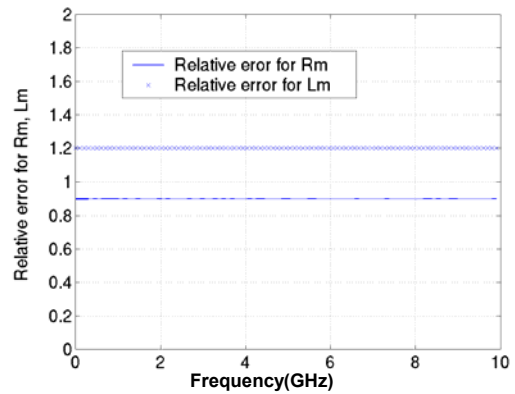

Fig. 4. Relative error between the curves (obtained by our analytic procedure and full wave solutions) in Figs. 2 and 3 for Rm and Lm.

4. Conclusion

A simple and accurate frequency-dependent analytic model is proposed to calculate the distributed mutual per unit length impedance components of general asymmetric coupled interconnects on a lossy silicon substrate over the entire frequency range. The model includes the frequency-dependent distribution of the induced currents in the conductive silicon substrate beneath the interconnect lines as the return current path. It is clear that the model developed in this letter can be used for the accurate estimation of the distributed mutual per unit length impedance parameters of the CMOS on-chip interconnects.

5. Appendix

Integral expression for the perturbation part of the mutual impedance per unit length (6) is derived as follows [11, 12].
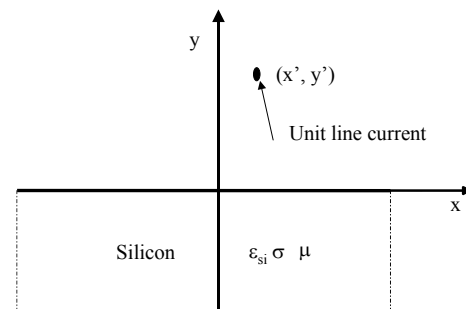

Fig. 5. Straight filament parallel to a semiconducting half-space.

For this reason a straight filament (unit current source) parallel to a semiconducting half-space as shown in Fig. 5 will be first regarded. The conductivity, permittivity and permeability of the silicon substrate are designated as σ, ε_{si}, μ, respectively.

A general solution of eqn. (2) may be looked for in the form

$$A_1(x,y) = \int_0^\infty [C_{11}(\lambda)e^{\lambda(y-y')} + C_{12}(\lambda)e^{-\lambda(y-y')}]\cos[\lambda(x-x')]d\lambda, \quad for \quad y \geq 0$$
(A1.a)

$$A_2(x,y) = \int_0^\infty [C_2(\lambda)e^{q(y-y')}\cos[\lambda(x-x')]d\lambda, \quad for \quad y \leq 0$$
(A1.b)

where $q = (\lambda^2 + j\omega\mu\sigma)^{1/2}$ and having considered that A_2 must vanish at large distance from the current filament, when y tends to $-\infty$. In order to determine the unknown coefficients C_{11}, C_{12}, and C_2, we have to take into account, that when the silicon semiconducting half-space is absent, the magnetic flux density $\boldsymbol{B}_0$ of a single current filament can be expressed in the form

$$B_{0x} = \pm\frac{\mu_0 I}{2\pi}\int_0^\infty e^{-|y'-y|\lambda}\cos[\lambda(x-x')]d\lambda$$
(A2.a)

$$B_{0y} = \frac{\mu_0 I}{2\pi}\int_0^\infty e^{-|y'-y|\lambda}\sin[\lambda(x-x')]d\lambda$$
(A2.b)

Using these expressions and imposing at the boundary surface $y = 0$, the continuity of the tangential components of the magnetic field and of the normal component of the magnetic flux density, the following expressions were obtained:

$$A_1(x,y) = \frac{\mu_0 I}{2\pi}\ln\frac{\sqrt{(x-x')^2+(y-y')^2}}{\sqrt{(x-x')^2+(y+y')^2}} + \frac{\mu I}{\pi}\int_0^\infty \frac{e^{-(y'+y)\lambda}}{\mu_r\lambda+q}\cos[\lambda(x-x')]d\lambda$$
(A.3)

$$A_2(x,y) = \frac{\mu I}{\pi}\int_0^\infty \frac{e^{-y'\lambda}e^{yq}}{\mu_r\lambda+q}\cos[\lambda(x-x')]d\lambda$$
(A.4)

As we can see above the magnetic vector potential is introduced in Maxwell's equations for the horizontal magnetic field intensity above the lossy silicon semiconducting substrate. This leads to the quasi-static voltage drop $\partial V/\partial z$ in the z-direction that also appears in the classical line equations,

$$\frac{\partial V}{\partial z} = -[Z_s]I$$
(A.5)

Hence, the total series impedance per unit length Z_s (self or mutual) of the interconnect lines can be derived. This method provides the best insight into the relation between the purely mathematical equations and the physical reality of the problem.

The magnetic vector potential is used in order to find the quasi-static potential drop $\partial V/\partial z$, at any point (x,y) in space along the line parallel to the z direction. This allows the total series self and mutual impedances per unit length, eq. (A. 5), to be evaluated.

The axial electric field intensity along the lossy silicon substrate is

$$E_{zs}(x, y = 0) = -j\omega A_1(x, y = 0) - \frac{\partial V(x, y = 0)}{\partial z}$$
(A.6)

and at any point (x,y) above the lossy silicon substrate

$$E_{za}(x,y) = -j\omega A_1(x,y) - \frac{\partial V(x,y)}{\partial z}.$$
(A.7)

Subtracting eqn. (A. 6) from eq. (A. 7), we get

$$E_{za}(x,y) = E_{zs}(x, y=0) - j\omega[A_1(x,y) - A_1(x,y=0)] - \frac{\partial}{\partial z}[V(x,y) - V(x,y=0)].$$
(A.8)

The last term in eqn. (A.8) represents the total scalar voltage drop, in the axial z-direction, of the distributed parameter circuit consisting of interconnects conductor and lossy silicon substrate, as in equation (A. 5).

From the definition of the magnetic vector potential

$$A_1(x,y) - A_1(x, y=0) = \mu_0\int_0^y H_{1x}(x,y)dy$$
(A.9)

where the x-component of the magnetic field intensity above the lossy silicon substrate is given by

$$H_{1x}(x,y) = \frac{1}{\mu_0}\frac{\partial A_1(x,y)}{\partial y}.$$
(A.10)

The final simplified equation for electric field component in z-direction above the lossy silicon substrate is

$$E_{za}(x,y) = -\frac{\partial V(x,y)}{\partial z} + \frac{\partial V(x,y=0)}{\partial z} - j\omega\frac{\mu_0 I}{2\pi}\ln\sqrt{\frac{(x-x')^2+(y'+y)^2}{(x-x')^2+(y'-y)^2}}$$
$$- j\omega\frac{\mu I}{\pi}\int_0^\infty \frac{e^{-(y'+y)\lambda}}{\mu_r\lambda+q}\cos[\lambda(x-x')]d\lambda$$
(A.11)

In order to develop expression for the mutual impedance per unit length of coupled interconnect conductors above the lossy silicon substrate a current I flowing in the interconnect conductor with width w_1 and thickness T_1, as depicted in Fig. 1, is now regarded. It can be assumed that the current density is constant over the conductor's cross section because the skin-effect in the strip or rounded conductor can be neglected due to its small dimensions (up to a few tens GHz). For the interconnect conductor with current I the axial electric field intensity is equal to the potential drop per unit length on the surface of the interconnect conductors.

The mutual impedance per unit length Z_{ik} between two circuits (conductors) is given by

$$Z_{ik} = -\frac{E_{zik}^{ind}}{I_k}$$
(A.12)

where E_{zik}^{ind} is the z component (axial) of the inductive part of the electric field produced by the k-th circuit (conductor k) carrying current I_k at the axis location of the i-th circuit (conductor i).

Substituting the coordinates for the interconnect conductors in equation (A. 11), rearranging the terms, we get

$$\frac{\partial V(x,y)}{\partial z} - \frac{\partial V(x,y=0)}{\partial z} = \frac{\partial V}{\partial z} = -Z_m I$$

$$= -I\left[j\omega\frac{\mu_0}{2\pi}\ln\sqrt{\frac{(s_{eq}+r_{1eq}+r_{2eq})^2+(H_{1eq}+r_{1eq}+H_{2eq}+r_{2eq})^2}{(s_{eq}+r_{1eq}+r_{2eq})^2+(H_{1eq}+r_{1eq}-H_{2eq}-r_{2eq})^2}} \right.+$$

$$\left. j\omega\frac{\mu I}{\pi}\int_0^\infty \frac{e^{-2t_{ox}\lambda}}{\mu_r\lambda+\sqrt{\lambda^2+j\omega\mu\sigma}}\cos(s\lambda)d\lambda \right]$$

$$(A.13)$$

Dividing both sides of eqn. (A. 13) by current I, it can be observed that the total series mutual impedance per unit length consists of two parts. The first term is the per unit length external mutual inductance of coupled on-chip interconnects on a lossy silicon substrate with zero resistivity (L_m given by eqn. (5)), whereas the remaining integral constitutes the perturbation part of the mutual series per unit length impedance that takes into account skin-effect phenomena in a lossy silicon substrate (Z_{sim} given by (6)).

References

[1] EDELSTEIN, D., SAI-HALASZ, G. A., MII, Y.-J. VLSI On-chip Interconnection Performance Simulations and Measurements. *IBM J. Res. Develop.,* 1995, vol. 39, pp. 383 – 401.

[2] GROTELUSCHEN, E., DUTTA, L. S., ZAAGE, S. Full-Wave Analysis and Analytic Formulas for the Line Parameters of Transmission Lines on Semiconductor Substrates. *Integration, the VLSI J.*, 1993, vol. 16, pp. 33 - 58.

[3] RUSSER, P., BIEBL, E., HEINRICH, W. Planar Millimeter Wave Circuits on Silicon. *Proceedings of the German IEEE Joint MTT/AP Chapter International Workshop on Silicon-Based High-Frequency Devices and Circuits*, Gunzburg, 1994, pp. 1 – 16.

[4] YMERI, H., NAUWELAERS, B., MAEX, K. Distributed Inductance and Resistance Per-Unit-Length Formulas for VLSI Interconnects on Silicon Substrate. *Microwave Opt. Technol. Lett.,* 2001, vol. 30, pp. 302 – 304.

[5] CHANG, L., CHENG, K., MATHEWS, T. Simulating Frequency-Dependent Current Distribution for Inductance Modeling of On-Chip Copper Interconnects. *Proc. Int. Symp. Physical Design,* 2000, pp. 117 – 120.

[6] WEE, J.-K., PARK, Y.-J., MIN, H.-S., CHO, D.-H., SEUNG, M.-H., PARK, H.-S. Modeling the Substrate Effect in Interconnect Line Characteristics of High-Speed VLSI Circuits. *IEEE Trans. Microwave Theory Tech.,* 1998, vol. 46, pp. 1436-1443.

[7] YMERI, H., NAUWELAERS, B., MAEX, K., DE ROEST, D., VANDENBERGHE, S. Accurate Analytic Expressions for Frequency-Dependent Inductance and Resistance of Single On-Chip Interconnects on Conductive Silicon Substrate. *Physics Letters A,* 2002, vol. 293, pp. 195-198.

[8] ZHENG, J., HAHM, Y.-C., TRIPATHI,V. K., WEISSHAAR, A. CAD-Oriented Equivalent-Circuit Modeling of On-Chip Interconnects on Lossy Silicon Substrate. *IEEE Trans. MTT.,* 2000, vol. 48, pp. 1443-1451.

[9] HASEGAWA, H., FURUKAWA, M., YANAI, H. Properties of Microstrip Line on Si-SiO$_2$ System. *IEEE Trans. Microwave Theory Tech.,* 1971, vol. 19, pp. 869 - 881.

[10] KORN, G. A., KORN, T. M. *Mathematical Handbook for Scientists and Engineeres.* New York: McGraw-Hill, 1951.

[11] TEGOPOULOS, J. A., KRIEZIS, E. E. *Eddy Currents in Linear Conducting Media*. Amsterdam: Elsevier, 1985.

[12] YMERI, H., NAUWELAERS, B., MAEX, K., DE ROEST, D., VANDENBERGHE, S. Frequency-Dependent Expressions of Inductance and Resistance of Microstrip Line on Silicon Substrate. *Microwave Opt. Technol. Letters*, 2002, vol. 33, pp. 349 – 352.

[13] EO, Y., EISENSTADT, W. High Speed VLSI Interconnection Lines and Packages in High Speed Integrated Circuits. *IEEE Trans. Comp., Packag., Manufact. Technol. B*, 1993, vol. 16, pp. 552 – 562.

[14] YMERI, H., NAUWELAERS, B., MAEX, K., DE ROEST, D. New Modeling Approach of On-Chip Interconnects for RF Integrated Circuits in CMOS Technology. *Microelectronics International,* 2003 vol. 20**,** pp. 41 – 44.

[15] ABRAMOWITZ, M., STEGUN, I. A. *Handbook of Mathematical Functions*, New York: Dover, 1965.