Estimator Comparison of the Nakagami-*m* Parameter and Its Application in Echocardiography

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Abstract: *This article deals with the comparison of various estimators of the m parameter from the Nakagami distribution. This kind of distribution has been used in many engineering applications and we present another possible application in biomedical engineering, particularly the ultrasound tissue characterization in the echocardiographic application. Matlab 6.5 was used as a proper tool for fast and efficient scientific research.*

Keywords

Nakagami distribution, Matlab, maximum likelihood, ultrasound, tissue characterization.

1. Introduction

The Nakagami-*m* distribution has founded many applications in technical sciences. It has been shown by extensive empirical measurement that this distribution is an appropriate model for radio links [2]. The next fast growing area of Nakagami-*m* distribution application is the ultrasound tissue characterization. The envelope of the ultrasound radiofrequency (RF) signal could be described by this distribution and the parameters can be used to distinguish between various kinds of tissues, e.g. detection and identification of abnormalities in breast, liver or kidney. However, in echocardiography little work has been done. In this article we compare various estimation methods for the Nakagami *m* parameter (Section 2 and 3). Section 4 deals with the use of the estimation in ultrasound tissue modeling. Section 5 presents some results for parametric tissue imaging in echocardiography.

2. Nakagami-*m* Distribution and Estimators

The probability density function (PDF) of the Nakagami-*m* distribution for an envelope V_i of ultrasonic RF signal is given by [2], [5], [6], [8]

$$p(V)=\frac{2m^m}{\Gamma(m)\Omega^m}V^{2m-1}\,\mathrm{e}^{-\left(\frac{mV^2}{\Omega}\right)}\quad V\geq 0\,, \tag{1}$$

where Ω and m are the two parameters of this distribution and $\Gamma(m)$ is the Gamma function. In radio channels modeling, the parameter m is constrained so that $m \geq 1/2$ [2]. But this condition may be violated in ultrasound applications [6], [7]. The parameter Ω represents the average power of the envelope signal and can be estimated as

$$\Omega=\frac{1}{N}\sum_{i=1}^{N}V_i^2\,. \tag{2}$$

This Ω estimator is well accepted, but there are many estimators for the second parameter m, which determines the shape of the PDF function. An estimate ($\tilde{m}$) of this parameter can be evaluated through the Maximum Likelihood (ML) method. It has been shown that this estimate leads to [11]

$$\ln\tilde{m}-\psi(\tilde{m})=y \tag{3}$$

where

$$\psi(x)=\frac{\partial\ln(\Gamma(x))}{\partial x} \tag{4}$$

is called the digamma function. This function can be easily evaluated in Matlab using the function *psi*(k, x), where k is the order of the derivative. The right side of (3) is

$$y=\ln\left(\frac{\mu_2}{B}\right) \tag{5}$$

where

$$\mu_k=\frac{1}{N}\sum_{i=1}^{N}V_i^k\,, \tag{6}$$

and

$$B=\left(\prod_{i=1}^{N}V_i^2\right)^{1/N}. \tag{7}$$

Solving the equation (3) is not easy due to the fact that there is no simple expansion of digamma function. One possibility is to use the Tolparev-Polyakov estimation rising from the second-order approximation of $\psi(\tilde{m})$. This estimator is [1], [9]

$$\tilde{m}_{TP} = \frac{1+\sqrt{1+(4/3)\ln\left(\frac{\mu_2}{B}\right)}}{4\ln\left(\frac{\mu_2}{B}\right)} . \qquad (8)$$

The next possibility is to use the Lorenz estimator [1]:

$$\tilde{m}_{LOR} = \frac{4.4}{\sqrt{\mu_2^{dB} - (\mu_1^{dB})^2}} + \frac{17.4}{\left(\mu_2^{dB} - (\mu_1^{dB})^2\right)^{1.29}} , \qquad (9)$$

where

$$\mu_k^{dB} = \frac{1}{N}\sum\left(20\log_{10} V_i\right)^k . \qquad (10)$$

The next approximation was proposed by Greenwood and Durand in 1960 and it is given by [11]:

$$\tilde{m}_{GD} = \frac{(0.50008 + 0.16488y - 0.05442y^2)}{y}$$

for $0 < y \le 0.5772$ and

$$\tilde{m}_{GD} = \frac{8.898919 + 9.05995y + 0.9775373y^2}{y\left(17.79728 + 11.96847y + y^2\right)} ,$$

for $0.5772 < y < 17$. The maximum relative errors of this expression are 0.0088 % and 0.0054 %, resp. [11].

An alternative technique to solving the ML equation (3) is by recursive equation, as suggested by Bowman [11]

$$\tilde{m}_{BOW} = \tilde{m}_i = \frac{\tilde{m}_{i-1}\left\{\ln\left(\tilde{m}_{i-1}\right) - \psi\left(\tilde{m}_{i-1}\right)\right\}}{y} , \qquad (11)$$

This algorithm converges rapidly and is quite powerful.

From the moment method the so call inverse normalized variance estimator can be derived [1], [6], [9]. This is the most widely used estimator, which is given by:

$$\tilde{m}_{INV} = \frac{\mu_2^2}{\mu_4 - \mu_2^2} . \qquad (12)$$

In [3], Cheng and Beaulieu proposed generalized moment estimator, which is based on non-integer moments. This is similar to (12) and is given as

$$\tilde{m}_p = \frac{\mu_{1/p}\mu_2}{2p\left(\mu_{2+1/p} - \mu_{1/p}\mu_2\right)} , \qquad (13)$$

where $p > 0$.

These are the mostly used estimators, particularly the $\tilde{m}_{INV}$, although its performance is not as good as for other estimators, as we will see in next section.

3. Estimators Comparison

In this section the six above estimators are compared in order to decide, which is more convenient and accurate. For this purpose the Nakagami random number generator, based on Monte-Carlo simulation, was created. For the fixed parameter $\Omega = 1$ and any fixed m from the set {0.25, 0.5, 0.75, 1, ..., 15}, the signal of length 1 million samples was generated using Matlab. The PDF of one generated sequence for $m = 3$ and the corresponding theoretical function is shown in Fig. 1. These sets were stored on the hard disc for the following consequential simulation.

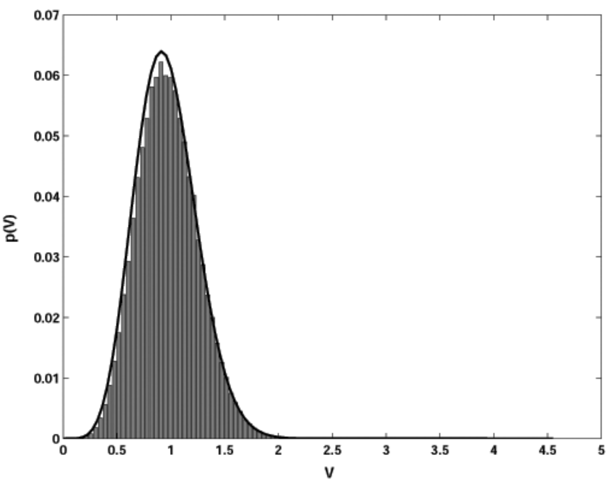

Fig. 1. The PDF of one generated sequence for $m = 3$ (bar graph) and the corresponding theoretical function (solid line).

From these sets a 200 sequences of length $N = 500$ and $N = 2000$ were randomly chosen and the estimation was performed using the six above listed estimators. The comparison was done by plotting the estimated value $\tilde{m}$ as a function of true value m.

Fig. 2a) shows the dependencies for N=500 and we can conclude that the estimators $\tilde{m}_{INV}$ and $\tilde{m}_{LOR}$ are less accurate. However, all estimators give similar results for $m \le 4$. For higher values, the estimators $\tilde{m}_{BOW}$, $\tilde{m}_{GD}$, $\tilde{m}_{TP}$, seem to be the most appropriate. Fig. 2a) shows only the $\tilde{m}_{BOW}$ estimation, because it gives almost the same results as $\tilde{m}_{GD}$.

Fig. 2b) shows the results for $N = 2000$ and the same conclusion can be drawn. The comparison of Fig. 2a) and b) shows that longer sequences (higher N) result in more accurate estimations.

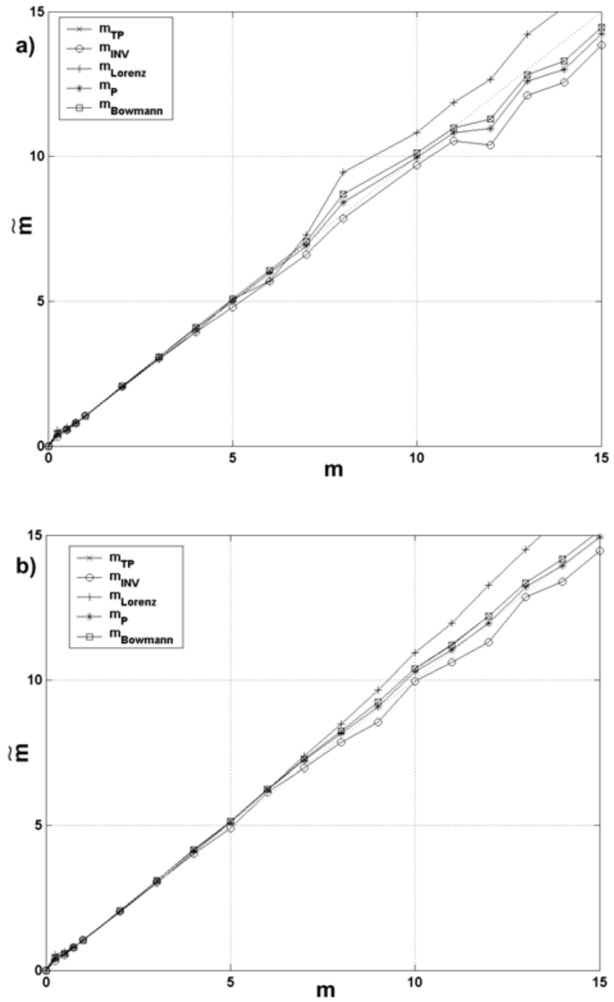

Fig. 2. Comparison of the estimated and the true values of the Nakagami m parameter for a) N = 500, b) N = 2000.

If we should make a decision, which estimation method is the best, we would choose the $m^{\sim}_{BOW}$ and $m^{\sim}_{GD}$ estimators, that give almost the same results and are more accurate, especially for m < 1.

4. Experiments on Ultrasound Simulation

Computer simulation was carried out to verify the usefulness of the Bowman and Greenwood-Durand estimators for ultrasound tissue characterization. A one-dimensional discrete scattering model was used in our simulation [6]. A set of 20 A-scans of length 3 cm was generated as a model of real tissue with different scattering conditions. Along each A-scan, uniformly distributed scatters were positioned and the received echo signal (RF signal) was computed as a superposition of echoes from all the scatters:

$$v(t) = \sum_{i=1}^{N} \alpha_i \, p\left(t - 2\frac{x_i}{c}\right) \tag{14}$$

where α_i is the amplitude of the i^{th} scatter assumed to have Gamma distribution [6] and c is the ultrasound velocity 1540 m/s. The emitted ultrasound wave is represented by $p(\,t)$. It is modeled in a frequency domain as a Gaussian-shaped spectrum:

$$P(f) = \frac{1}{\beta^2} e^{-\frac{(f-f_o)^2}{2\beta^2}} + \frac{1}{\beta^2} e^{-\frac{(f+f_o)^2}{2\beta^2}} \tag{15}$$

where β is the bandwidth and f_0 is the center frequency. The inverse Discrete Fourier Transform is performed to obtain the time function.

The transmit frequency f_0 = i3 MHz was used. Tissue types with various numbers of scatters were simulated and the Nakagami m parameter was estimated for each type. Before that the envelope detection of simulated RF data had been performed, using the theory of analytic signal and Hilbert transformer [4]. No additional processing was done (envelope dynamic compression or decimation).

χ^2 tests were conducted to test the hypothesis that the envelope of the RF signal was Nakagami distributed. This test was performed for each fixed scatter density n (in a resolution cell) from the set n = {1,2...15}. The two estimates: $m^{\sim}_{BOW}$ and $m^{\sim}_{GD}$ were computed and a χ^2 test performed for each of these estimates and corresponding simulated images. In order to reduce the estimate variation, the procedure was repeated 50 times. The average values of this statistic are given in Tab. 1. The Nakagami hypothesis seems to be acceptable within the limits of error.

The Chi-square is calculated as the sum of the squares of the differences of the observed values from the mean value divided by the mean value itself. More details could be found in [11].

Here, we present only the Bowman estimator $m^{\sim}_{BOW}$. The $m^{\sim}_{GD}$ estimator was tested too, but both of them provide practically the same values. The other estimation techniques (including Tolparev-Polyakov estimator) lead to less appropriate values, because they have lower χ^2 values that in some cases lie under the critical value.

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
χ^2	851	593	1146	57	28	6228	401	56	24	103	2375	662	1686	539	107

Tab. 1. χ^2 values for the Bowman estimator $\widetilde{m}_{BOW}$.

5. Experiments on Real RF Data

A simple test was applied to real echocardiography data to investigate whether the Nakagami distribution is appropriate for the characterization of cardiac tissues.

5.1 Data Description

The data were measured with the *GE VingMed System 5* scanner in cooperation with the Faculty Hospital, Masaryk University Brno. The envelope images were saved on the computer hard disc before any scan conversion or dynamic compression and could be directly used for Nakagami parameters estimation.

The images were stored in a matrix, where the number of columns is the number of A-scans and the number of rows is determined by the acquisition depth and sampling frequency. In our case, the size of the matrix was 486 × 35, corresponding to the depth span 15 cm and the view angle 45° (see Fig. 3).

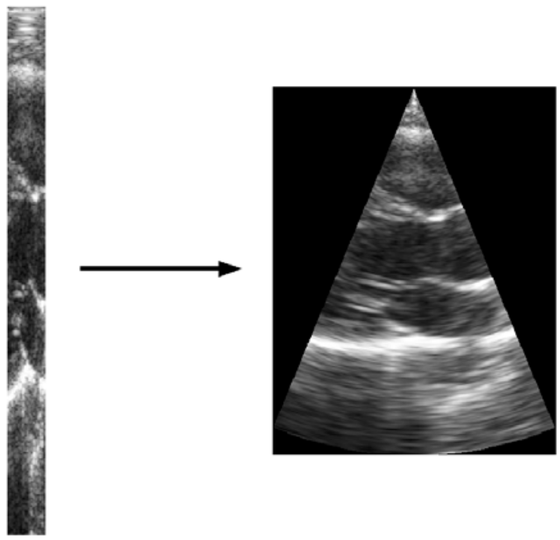

Fig. 3. Scan conversion was done using the bilinear interpolation.

5.2 *m* and *Ω* Estimation

The m and Ω parameters were computed within a sliding window, moving along the image, before scan conversion. The size of this window is important, because it determines the spatial resolution of the resulting parametric image and the accuracy of the estimate. The larger the window, the higher is the accuracy, but lower spatial resolution. The number of columns was set to 3; therefore three A-lines are included in the computation. This is, in fact, the minimum width. The more A-lines are added, the more blurred results are obtained. The number of rows is not so critical because the axial resolution is higher. However, the value about 11 was selected in this application. It corresponds to the height about 3.3 mm.

Fig. 4a) shows the original B-mode image of the heart (long-axis view). Fig. 4b) and c) show the corresponding parametric images of the m and Ω. One can see, that the Ω-image has brighter values for the specular reflections and higher echogenity regions; and dark values for regions with no specular reflections, because it represents the average power within the window. The low values at the edges of the different tissues mean, that only few scatters are present, which is due to the presence of specular reflections. The parts of a tissue with higher scatter densities have high values of m and therefore are brighter.

The logarithmic compression was utilized before the visualization. Moreover, the matrix of m parameter is median filtered (3×3 mask) to reduce the variation of estimated parameters. After that, Ω-image and m-image were created by bilinear interpolation.

6. Conclusion

Various estimators has been studied and compared in this article. The Greenwood-Durand and Bowman estimators seem to be the most appropriate m estimators. The model based on Nakagami-m distribution has been explored as a promising model for ultrasonic tissue characterization in echocardiography. The model is easy to use, because the computation is straightforward, compared to other models (e.g. K-distribution). The advantage of this model is also given by its ability to incorporate several scattering conditions and makes it therefore attractive for ultrasound characterization.

Acknowledgment

This work has been supported by the grant No.B2813303 of the Czech Academy of Sciences and the grant No.102/02/0890 of the Grant Agency of the Czech Republic and the Research Programme CEZ J22/98:262200011. The authors are very grateful to Doc. MUDr. Václav Chaloupka for measuring the ultrasound data and to E.M.S.company for providing the ultrasound machine to enable the digital data acquisition.

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Fig. 4. a) B-mode image of the heart (long axis view). b) Image of the parameter Ω. c) Image of the parameter *m*.

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