Extrapolation of Incomplete Image Data with Discrete Orthogonal Transforms

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Abstract. *In image processing and transmission, interpolation and extrapolation are of great importance whenever missing pixels have to be filled in, and many methods have been proposed to solve this problem. In this paper we present a method for extrapolating the missing data with an existing set of basis functions of a selected orthogonal transform. The best extrapolation is found according to linear approximation theory as a weighted sum of basis functions, where coefficients of the sum are solutions of the derived matrix equation.*

Keywords

Object-based image processing, discrete orthogonal transform, extrapolation.

1. Introduction

In the world nowadays, a great deal of transmission of image data takes place. This transmission is not always error-free, and some portion of data is not transmitted. Besides, missing data can be present in the image from many other reasons - resampling, elimination of spurious objects etc. Missing pixels in an image can be ordered either on a regular or irregular mesh or in blocks. We can call image data with missing pixels or blocks incomplete (gappy) data. Our goal is to find a description of an image based on existing data so that from this description, also values of missing data can be derived.

Discrete orthogonal transformation is a method that can be used to describe or characterize a data set in a way that decreases redundancy of image data [1]. In this paper we will show how the transformation can be used to exploit the redundancy inherent in image data sets to repair or analyze marred or gappy data.

We freely switch between the terms interpolation and extrapolation, because the scheme is suited for both. Which one of those we get depends on using appropriate segment window.

The procedure of orthogonal transformation is typically used to provide a more effective coordinate system for representing data than more general and usually much larger coordinate systems that can result from multipurpose data gathering techniques. For example, a high-resolution digital image of L gray levels and size MxN has L^{MN} possible image configurations. Even small values of L, M and N produce an enormous coordinate system that is suited to representing every such image. A set of images has statistically many common features that are present in each of the images, therefore much smaller space can be used to represent them.

2. Extrapolation of Missing Data in 2D Case

Let's denote the original image data as $x(n_1, n_2)$. Using theory of linear approximation we can write the approximation using a discrete orthonormal basis with functions $u(n_1, n_2, k_1, k_2)$ in a segment w [1] as

$$x(n_1,n_2)_w \approx x_D(n_1,n_2)_w = \\ = \sum_{k_1=0}^{D_1-1}\sum_{k_2=0}^{D_2-1} a(k_1,k_2)\cdot u(n_1,n_2,k_1,k_2)\cdot w(n_1,n_2) \quad (1)$$

where $w(n_1, n_2)$ is a rectangular window with the value of 1 for pixels from the segment, otherwise 0, n_1=0, 1, 2, …N_1-1; n_2=0, 1, 2, …N_2-1, $D_1 \leq N_1$, $D_2 \leq N_2$, $a(k_1, k_2)$ are approximation coefficients.

Likewise, the complete image data $g(n_1, n_2)$, including the missing data, can be approximated as

$$g(n_1,n_2) \approx g_D(n_1,n_2) = \sum_{k_1=0}^{D_1-1}\sum_{k_2=0}^{D_2-1} b(k_1,k_2)\cdot u(n_1,n_2,k_1,k_2). \quad (2)$$

Our objective is to find a set of sequential coefficients $\{b(k_1, k_2)\}$ in order to minimize the mean square error $E=\|\mathbf{g}-\mathbf{g}_D\|^2$ in the segment w. Because the coefficients $b(k_1, k_2)$ are determined using only known data in the segment w, let's define the inner product on the segment as

$$(\mathbf{q},\mathbf{s})_\mathbf{w} = \{(\mathbf{w}\cdot\mathbf{q}),(\mathbf{w}\cdot\mathbf{s})\} \quad (3)$$

and its corresponding norm on the segment is

$$\|\mathbf{s}\|^2 = (\mathbf{s}\cdot\mathbf{s})_\mathbf{w}. \quad (4)$$

It is clear that

$$(\mathbf{q},\mathbf{s})_{\mathbf{w}_1} \neq (\mathbf{q},\mathbf{s})_{\mathbf{w}_2} \tag{5}$$

for the same data **q** and **s**, but different window functions $\mathbf{w}_1 \neq \mathbf{w}_2$.

Let's minimize the error E. By substituting $\mathbf{g}_D$ from (2) we obtain

$$E = \|\mathbf{g}\|^2_{\mathrm{w}} - 2\sum_{k_1=0}^{D_1-1}\sum_{k_2=0}^{D_2-1} b(k_1,k_2)\cdot\lceil \mathbf{g},\mathbf{u}(k_1,k_2)\rceil_{\mathrm{w}} + $$

$$+\sum_{i_1=0}^{D_1-1}\sum_{i_2=0}^{D_2-1}\sum_{j_1=0}^{D_1-1}\sum_{j_2=0}^{D_2-1} b(i_1,i_2)b(j_1,j_2)\cdot\lceil \mathbf{u}(i_1,i_2),\mathbf{u}(j_1,j_2)\rceil_{\mathrm{w}} \tag{6}$$

where $\mathbf{u}(i_1, i_2)$ is the base function of DOT as a matrix containing elements $u(n_1, n_2, i_1, i_2)$.

We minimize the error with respect to each $b(l_1, l_2)$ from the amount of $D_1 \cdot D_2$ basis functions. Then, for the error, we obtain from (6)

$$\frac{\partial E}{\partial b(l_1,l_2)} = -\lceil \mathbf{g},\mathbf{u}(l_1,l_2)\rceil_{\mathrm{w}} +$$

$$+\sum_{k_1=0}^{D_1-1}\sum_{k_2=0}^{D_2-1} b(k_1,k_2)\cdot\lceil \mathbf{u}(k_1,k_2),\mathbf{u}(l_1,l_2)\rceil_{\mathrm{w}} = 0 \tag{7}$$

for $l_1 = 0,1,2,...D_1 - 1$, $l_2 = 0,1,2,...D_2 - 1$,

and from (7) we have the system of linear equations

$$\sum_{k_1=0}^{D_1-1}\sum_{k_2=0}^{D_2-1} b(k_1,k_2)\cdot\lceil \mathbf{u}(k_1,k_2),\mathbf{u}(0,0)\rceil_{\mathbf{w}} = \lceil \mathbf{g},\mathbf{u}(0,0)\rceil_{\mathbf{w}}$$

$$\sum_{k_1=0}^{D_1-1}\sum_{k_2=0}^{D_2-1} b(k_1,k_2)\cdot\lceil \mathbf{u}(k_1,k_2),\mathbf{u}(0,1)\rceil_{\mathbf{w}} = \lceil \mathbf{g},\mathbf{u}(0,1)\rceil_{\mathbf{w}}$$

$$\sum_{k_1=0}^{D_1-1}\sum_{k_2=0}^{D_2-1} b(k_1,k_2)\cdot\lceil \mathbf{u}(k_1,k_2),\mathbf{u}(1,0)\rceil_{\mathbf{w}} = \lceil \mathbf{g},\mathbf{u}(1,0)\rceil_{\mathbf{w}}$$

$$\sum_{k_1=0}^{D_1-1}\sum_{k_2=0}^{D_2-1} b(k_1,k_2)\cdot\lceil \mathbf{u}(k_1,k_2),\mathbf{u}(D_1-1,D_2-1)\rceil_{\mathrm{w}} =$$

$$= \lceil \mathbf{g},\mathbf{u}(D_1-1,D_2-1)\rceil_{\mathrm{w}} \tag{8}$$

Let's define

$$M(i_1,i_2,j_1,j_2) = \lceil \mathbf{u}(i_1,i_2),\mathbf{u}(j_1,j_2)\rceil_{\mathbf{w}} \tag{9}$$

$$f(i_1,i_2) = 2\lceil \mathbf{g},\mathbf{u}(i_1,i_2)\rceil_{\mathbf{w}} \tag{10}$$

for each n_1, n_2 in the segment.

If we order the elements of $M(i_1, i_2, j_1, j_2)$ into 2D matrix in such a way, that we create the ordering of basis functions with respect to (i_1, i_2), then we can write the system of linear equations in (8) as

$$\mathbf{Mb} = \mathbf{f}\,. \tag{11}$$

The system of equations in (10) is linear dependent, therefore we should select one of the values rationally. It is efficient to use such a base (of DOT), which has one constant function $u(n_1, n_2, i_1, i_2)$. For example, DCT II and many more transforms give the DC element applying the basis function $u(n_1, n_2, 0, 0)$. This DC element corresponds to mean luminance of the given image region (segment).

During the extrapolation procedure, it is necessary to identify missing data in order to distinguish between missing data and present image data with zero intensity. In our approach, this is done through masking. Mask values for missing data are zero and criteria are evaluated for masked image, therefore we approximate to all known pixels regardless of their intensity value. Therefore this algorithm, although derived from approximation theory, by using a sufficient number of basis functions yields interpolation/extrapolation.

3. Example

To demonstrate the results of extrapolation, we used gray-level original image of human face and rather artificial regular pattern of missing pixels that creates very disturbing corruption of image data for a human observer. In the example, the ratio of missing pixels is 15% and DCT transform is used for approximation. Output images demonstrate results for different amounts of utilized base functions.

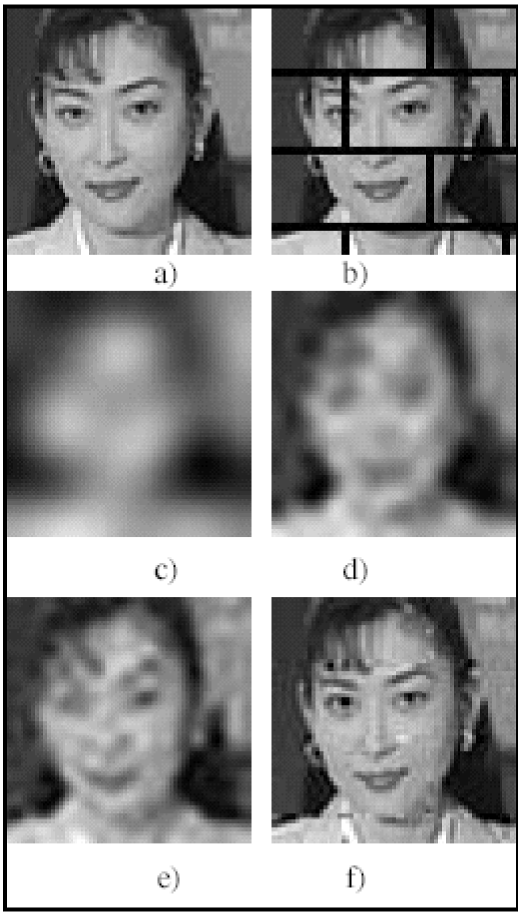

Fig. 1. Examples of extrapolation on image: a) original, b) gappy image (missing pixels are represented by black color); image extrapolated with DCT using: c) 1%, d) 5%, e) 10%, f) 50% of basis functions.

4. Discussion

We have presented a scheme for image extrapolation when only partial data is available. The orientation of the algorithm towards object oriented image description and PCA-like description it is well suited to be included in modern image coding and error concealment algorithms. Utilization of other transforms, for example wavelet [6], could bring further improvements. The algorithm is non-iterative, which ensures less computational complexity than in iterative schemes used for shape independent image processing.

Since the performance of the method depends on an internal structure of the segment, a use with sophisticated segment description methods may yield perspective results. Triangulation [4] can provide a description that is simple yet is more capable to cover segments than using square blocking.

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