An Adaptive Block-Based Eigenvector Equalization for Time-Varying Multipath Fading Channels

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Abstract. *In this paper we present an adaptive Block-Based EigenVector Algorithm (BBEVA) for blind equalization of time-varying multipath fading channels. In addition we assess the performance of the new algorithm for different configurations and compare the results with the least mean squares (LMS) algorithm. The new algorithm is evaluated in terms of intersymbol interference (ISI) suppression, mean squared error (MSE) and by examining the signal constellation at the output of the equalizer. Simulation results show that the BBEVA performs better than the non-blind LMS algorithm.*

Keywords

Channel Equalization, Blind equalization, Multipath Fading Channels, Mobile Radio Communications.

1. Introduction

Adaptive equalization of time-varying channels is an important step in the design of reliable and efficient data communication systems [1-7]. When the communications environment is highly nonstationary, however, it may become impractical to use the classical training sequence equalizers. For this reason, blind adaptive channel equalization algorithms that do not rely on training sequences have been developed [3, 4, 6, 7]. In this paper we explore blind equalization using higher order statistics (cumulant) approach.

We consider a complex, discrete baseband communications system. The channel impulse response at time n is modeled as an FIR filter of length M, and is denoted as $h(n,m)$. The received signal $x(n)$ can be expressed as:

$$x(n) = \sum_{m=0}^{M-1} h(n,m)s(n-m) + v(n) \tag{1}$$

where $s(n)$ is the Quadrature Phase Shift Keyed (QPSK) transmitted data symbols and $v(n)$ is additive white Gaussian noise (AWGN) with power spectral density $N_0/2$.

In [8] a closed-form eigenvector solution to the problem of blind equalization was proposed followed by an iterative technique called EigenVector Algorithm (EVA) for blind equalization in [9]. The iterative update of the equalizer weights $\mathbf{w} \in X^{L\times 1}$ is given by the so called EVA equation:

$$\lambda \mathbf{w} = \mathbf{R}^{-1}\mathbf{C}_4\mathbf{w} \,. \tag{2}$$

The equalizer weights are obtained by choosing $\mathbf{w}$ as the eigenvector of $\mathbf{R}^{-1}\mathbf{C}_4$ with the maximum corresponding eigenvalue λ. In (2), $\mathbf{R}^{-1} \in X^{L\times L}$ is the inverse of the autocorrelation matrix:

$$\mathbf{R} = \begin{pmatrix} r_{xx}(0) & \cdots & r_{xx}(1-L) \\ \vdots & & \vdots \\ r_{xx}(L-1) & \cdots & r_{xx}(0) \end{pmatrix}, \tag{3}$$

and $\mathbf{C}_4 \in X^{L\times L}$ is the 4th order cross-cumulant matrix:

$$\mathbf{C}_4 = \begin{pmatrix} c_4^{yx}(0,0) & \cdots & c_4^{yx}(1-L,0)^* \\ \vdots & & \vdots \\ c_4^{yx}(L-1,0) & \cdots & c_4^{yx}(1-L,1-L) \end{pmatrix}. \tag{4}$$

Here $c_4^{yx}(i_1,i_2)$ is defined as:

$$\begin{aligned} c_4^{yx}(i_1,i_2) &= E\{|y(n)|^2 x^*(n+i_1)x(n+i_2)\} \\ &- r_{yy}(0)r_{xx}(i_2-i_1) - r_{yx}(i_2)r_{yx}^*(i_1) \\ &- \bar{r}_{yx}^*(i_2)\bar{r}_{yx}^*(i_2) \end{aligned} \,. \tag{5}$$

The variables i_1 and i_2 are integers with arbitrary values and $y(n)$ is the equalizer output. The parameters r_{yy}, r_{yx} and $\bar{r}^*_{yx}$ denote autocorrelation, cross-correlation and a modified cross-correlation sequence, respectively:

$$\begin{aligned} r_{yy}(i) &= E\{y^*(n)y(n+i)\} \\ r_{yx}(i) &= E\{y^*(n)x(n+i)\} \\ \bar{r}_{yx}^*(i) &= E\{y(n)x(n+i)\} \end{aligned} \tag{6}$$

Here i is an arbitrary integer. Ideally, when the algorithm has converged, the resulting weights will maximize the kurtosis $|c_4^{yy}(0,0)|$ of the equalizer output $y(n)$, producing an impulse response of the total system ($h^* w$) of a delayed and scaled Dirac pulse. The estimation of $\mathbf{R}$ and $\mathbf{C}_4$ is described in detail in [9].

In this paper we present a modified version of the EigenVector Algorithm for Blind Equalization (EVA) [10], and extend its application to the equalization of *time-varying multipath fading* channels. The new modified iterative algorithm, called Block-Based EVA (BBEVA), is shown in Fig. 1. We will also compare the performance of the algorithm with the non-blind LMS algorithm. In addition, we carry out simulations to investigate the effects of the different building blocks on the performance of the proposed algorithms.

The paper is structured as follows. In Section 2, we present the BBEVA equalizer and the associated building blocks of the full system. Section 3 presents the simulation results to evaluate the performance of the algorithm with different configurations and comparisons with the LMS algorithm. Finally, Section 4 summarizes the paper and presents future research possibilities.

2. BBEVA Equalizer Development

The new modified iterative algorithm, suggested for the equalization of time-varying multipath fading channels, is called Block-Based EVA (BBEVA). The complete BBEVA setup is shown in Fig. 1.

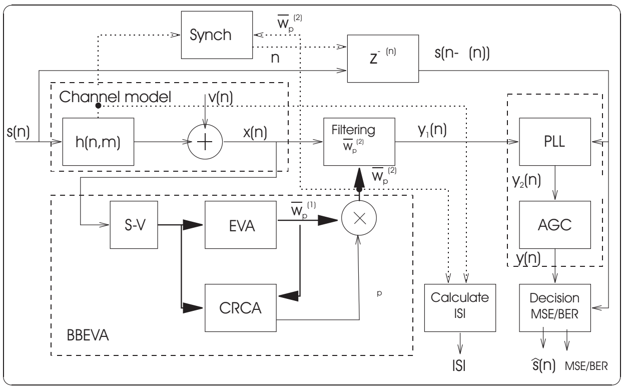

Fig. 1. The complete BBEVA system setup.

The new BBEVA algorithm operates on data blocks of size *B*, in which the signals are assumed to be stationary. The BBEVA algorithm calculates the optimal weights (one set for each block) as:

$$\mathbf{w}_p = \left[w_p^{(1)}(0),\ldots,w_p^{(1)}(L-1)\right]^T \tag{7}$$

using the data $\mathbf{x}_p=[x(pB),\ldots,x(pB-B+1)]$. Here, all vectors and matrices are functions of time index n and/or the block index p. The calculation of the optimal weights is performed by use of the EVA in an iterative approach as shown in Eq. (2).

In order to construct an efficient BBEVA system, some building blocks (see Fig. 1.) were employed and evaluated to ensure the proper operation of the system. These are the Constellation Rotation CAnceller (CRCA), the Phased Locked Loop (PLL), the Amplitude Gain Controller (AGC) and the synchronization block. These different blocks are explained briefly below and their effect on the performance of the algorithm will be investigated and evaluated by computer simulations in Section 3.

2.1 Constellation Rotation Canceller (CRCA)

The residual ISI is defined as:

$$Q(n) = \frac{\sum_{k\neq k_f(n)} |f(n,k)|^2}{|f(n,k_f(n))|^2}, \tag{8}$$

where $f(n,k)$ is the convolution between the channel impulse response and the equalizer impulse response at time n in block p:

$$f(n,k) = \sum_m h(n,m) w_p^{(2)}(k-m),\; p = \left\lfloor \frac{n}{B} \right\rfloor. \tag{9}$$

The function $k_f(n)$ is the index of the "tap" of $f(n,k)$ with the greatest magnitude:

$$k_f(n) = \arg\max_k |f(n,k)|. \tag{10}$$

Since EVA only has knowledge about the current data block, the resulting constellation will be independent of past blocks. If ISI is suppressed and the channel is slowly time-varying, it can be assumed that the following statement will hold:

$$\sum_{l=0}^{L-1} w_{p-1}^{(1)}(l)x(pB+i-l) \approx \exp(-j\theta_p)\sum_{l=0}^{L-1} w_p^{(1)}(l)x(pB+i-l) \tag{11}$$

for all p and $i\in[0,B\text{-}1]$. In order words, there can be a phase shift of the constellation between blocks. This problem is addressed by the introduction of a CRCA, which estimates θ_p for every block by calculating a Probability Density Function (PDF) for θ_p and choosing the value of θ_p corresponding to the peak in the PDF. The result is used to adjust the weights to the correct phase, giving $\mathbf{w}_p^{(2)}$. This would ensure that the resulting equalized signal to have a stable constellation.

2.2 Phase Locked Loop (PLL)

The signal after the EVA and the CRCA will have a phase ambiguity and suffer from slow phase variations because of imperfect equalization. The former means that it is impossible to know which of the four constellations should be assigned to which symbols, without any a *priori* information, such as the use of pilot signals. The latter means that the phase, from symbol to symbol, drift slightly due to the imperfections and variations of the channel. These two problems are the motivation behind the use of the PLL. The PLL is implemented as a Proportional-Integration (PI) regulator which adjusts the phase by multiplying $y_1(n)$ with a factor $\exp(j\Theta(n))$:

$$\Theta(n) = \Theta(n-1) + Ke(n), \tag{12}$$

where $e(n)$ is the phase error $<d(n)/y_1(n)$, and $d(n)$ equals $s(n)$ or $\hat{s}(n)$ depending on whether pilots or decision feedback is used. In this paper, pilots assumed to be available for the use by the PLL.

2.3 Amplitude Gain Controller (AGC)

Due to amplitude variations in the constellation, an AGC has to be used. The AGC is implemented as a PI-regulator with preset amplitude as its target signal.

2.4 Synchronization

To make it possible to estimate MSE, the total delay of the system must be known. The delay fluctuates in a very slow manner, i.e. $y(n) \approx s(n-\Delta(n))$. In the system, the delay is assumed known to the MSE estimator; this can be seen in Fig. 1. where the "Synch" block has knowledge about the channel.

3. Simulation Assumptions and Results

Monte Carlo computer simulations of the BBEVA system presented in Fig. 1 were carried out in order to assess the performance of the equalizer. The channel used in the simulation is shown in Fig. 2. The signal-to-noise ratio (SNR) is set to 20 dB, and the number of QPSK transmitted symbols over the channel in each realization is 15000.

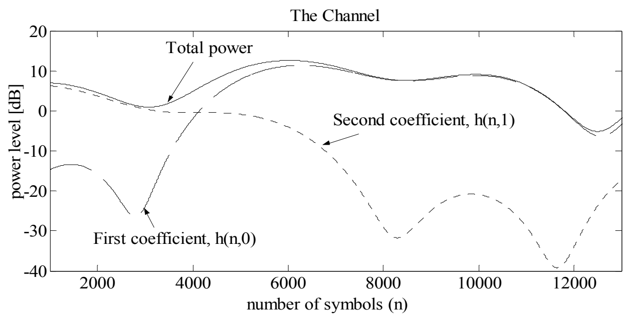

Fig. 2. The channel used in the simulations.

The performance of the BBEVA is compared with a LMS equalizer in terms of intersymbol interference suppression and mean squared error (MSE), and by examining the constellation at the equalizer output. The MSE and ISI plots (Fig. 3) are the average of 25 realizations. The adaptation constant for the LMS was set to 0.01. It is clear from Fig. 3 that the BBEVA equalizer performs better than the LMS at each time instant by achieving better suppression of ISI and noise, respectively. These results are confirmed by the tighter signal constellation of the equalized signal achieved by the BBEVA as compared to the LMS at each time instant (Fig. 4), demonstrating the potential of the BBEVA algorithm.

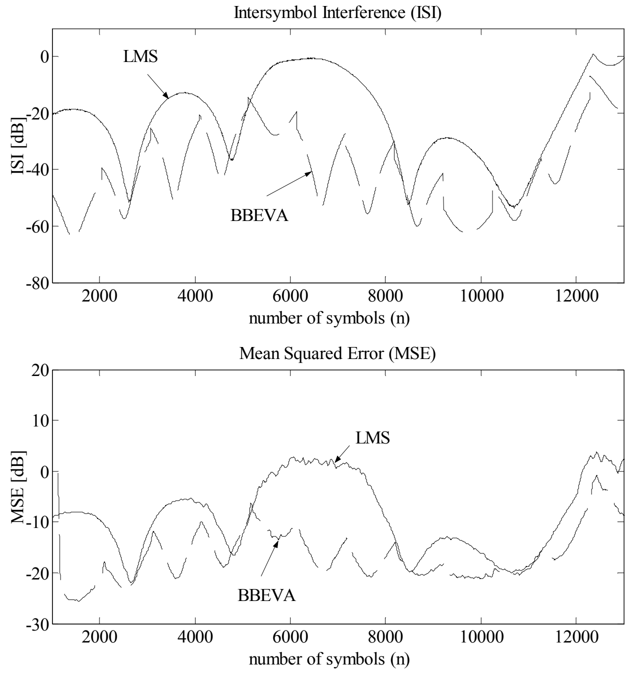

Fig. 3. The residual ISI (top) and MSE (bottom) for BBEVA and LMS algorithms.

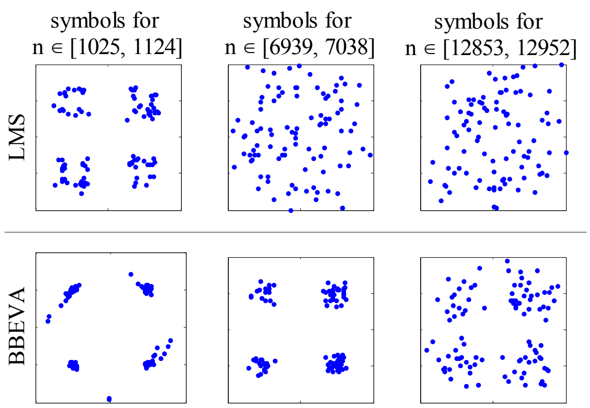

Fig. 4. Constellation of the equalized signal for LMS and BBEVA algorithms at different time (sample) instances.

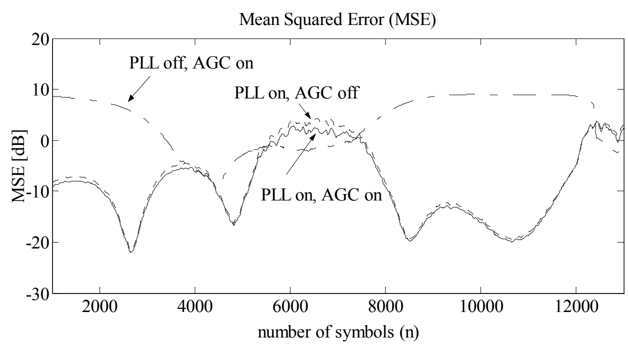

Fig. 5. The MSE for the LMS algorithm for different configurations.

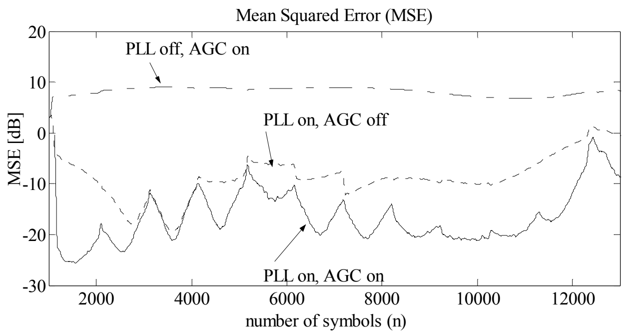

Fig. 6. The MSE for the BBEVA algorithm for different configurations.

The simulations presented above do not show how much of the performance can be accredited to the equalizing (BBEVA or LMS) algorithms and to the PLL and AGC, respectively. Therefore the simulator is configured accordingly with the aim to investigate this issue. The LMS and BBEVA equalizers are simulated for three cases (Figs. 5 and 6): (1) original configuration corresponding to the full setting in Fig. 3 (PLL and AGC *on*), (2) PLL turned *on* and the AGC *off* and (3) PLL *off* and AGC *on*.

For the LMS algorithm (Fig. 5), it is noted that the best performance of the system is obtained when both the PLL and the AGC are in operation. Turning off the AGC has a very minor impact on the performance. On the other hand, turning off the PLL degrades the performance dramatically.

The same configurations are simulated for the BBEVA algorithm (Fig. 6). Again, the best performance is obtained when both the PLL and AGC are active. Turning off the AGC has more negative impact on the performance of BBEVA than to the LMS. Finally, as with the LMS, it was noticed that turning off the PLL causes considerable deterioration on the performance of BBEVA. The reason for this is that the phase ambiguity is not corrected for when the PLL is *off*.

In conclusion, for proper operation of the algorithms in time-varying multipath fading channels, the original configuration and settings (PLL and AGC are active) should be used.

4. Conclusions

In this paper we have presented a Block-Based EigenVector Algorithm (BBEVA) for blind equalization of time-varying multipath fading channels. Simulation results show that BBEVA performs better than the LMS algorithm and that the incorporation of a PLL is of a paramount importance for the proper operation of the algorithms. Comparisons with other blind algorithms such as the Constant Modulus Algorithm (CMA) and the introduction of antenna arrays comprise future research.

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