General Framework and Advanced Information Theoretical Results on Eigenmode MIMO Channel Inversion

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Abstract. *This paper provides general and deep investigation of adaptation strategies based on the channel inversion policy regarding wide variety of channel models. Our novel approach to the eigenmode space MIMO channel inversion policy relies on the eigenmode space reduction providing zero transmission outage probability regardless of the instantaneous channel fading realization. Very detailed survey of the features of channel capacity is provided in analytical closed form expressions supported by many particular numerical results (Alamouti scheme is included). The correlated MIMO channel is involved into our treatment as well. We also address the trade-off between the capacity and transmission outage probability. The novel results are developed in the general framework with exhaustive summary of well known SISO and SIMO results.*

Keywords

Channel Inversion, MIMO, Adaptive Modulation, Ergodic Channel Capacity, Transmission Outage Probability.

1. Introduction

There is extremely strong demand for wireless systems enabling higher transmission rate, robustness against detrimental fading effects, flexibility and reliability with reasonable costs. The adaptive algorithms employed in different parts of the communication chain seem to be the answer to such challenge.

1.1 Recent Results—Literature Navigation

There are plenty of papers concerning on the adaptive algorithms of various kinds where information-theoretical insights in conjunction with practical applications are addressed. Let us mention some key papers to help the reader interested in adaptive modulations to guide through the huge amount of results.

Since any adaptation rule strongly depends on the channel model which is admitted into the consideration, be referred to the landmark paper [1] to very comprehensive information-theoretical investigation of the fading channels. Also some adaptation rules and corresponding capacities are developed there, including the inversion policy of both, truncated and total versions. In [2], there is discussed the capacity of the channel when CSI[1] is perfectly known at the transmitter and in [3], similar results are obtained in a little different way. Both these papers are still strongly recommended to read, however the special emphasis is given on the optimal *waterfilling* strategy there. Very deep and exhaustive investigation of optimal power control in fading channel with delay-limited observation is given in [4]. The relations derived mainly in the aforementioned literature are then applied to the Rayleigh and Nakagami distribution of the channel fading e.g. in [5, 6, 7]. We will recall some selected results obtained therein also in our tutorial part of this paper. Very comprehensive investigation of adaptive systems together with the diversity techniques (MRC—Maximum Ratio Combining, SC—Selection Combining) at the receiver side with different distributions of channel fading can be found in [8]. Very good and general approach to the adaptive modulation design is shown in [9], where also some points related to the channel inversion are implicitly mentioned. Moreover, the cases of average BER versus instantaneous BER constraint and discrete versus continuous rate adaptation are compared there so the more reasonable approach than the pure information-theoretical one is revealed. The special case of adaptive modulation using variable MQAM constellation based on the properly chosen BER approximation is developed in [10]. This paper is well written and easy for understanding so it could serve to the reader as the first touch to the real adaptation modulation design. Moreover, in [11], there are discussed some points to the channel fading estimation and prediction methods. All these approaches are developed assuming perfect CSI. In [12], there are provided some interesting numerical results taking into account the error of channel estimation, clipping and co-channel interference with some proposals to reduce such effects. Impact

[1] Channel State Information

of imperfect CSI prediction on adaptive modulation design is widely investigated in [13] and other useful literature concerning the realistic propagation scenario is e.g. [14, 15]. Finally, in [16, 17], there are discussed the efficiency and applicability of the channel inversion technique in multiuser scenario.

These were some key papers from our perspective. We have tried to mention the papers related to the channel inversion policy but also those with a bit wider scope involving adaptive transmission algorithms in general. In the *tutorial part* of the paper, we will refer to selected results obtained recently in these papers and also discussed in very detailed context in [18].

Next, the *novel contributions* are revealed in the second part of our paper. The significant original extensions regarding MIMO eigenmode space channel inversion, outage versus capacity trade-off [19], closed form for MIMO truncated channel inversion, reduced eigenmode total channel inversion, (see also [20, 21]), and the influence of channel correlation in MIMO to the features of eigenmode channel inversion are also covered.

1.2 Service Applications Regarding the Channel Models

Anyway, the theoretical investigations should be driven by real motivation. In information theory, we always dream of the optimal scheme. But the optimal solutions with regard to some single point of view have usually some weaknesses as well, which might cause serious problems from the overall system perspective. In real service or application requirements, there are very often more competing objective demands. In such a case, very valuable scheme is the one, fulfilling more distinctive goals with reasonable performance in all points. We know the optimal (capacity achieving) strategy, so called *waterfilling allocation* (regardless the number of dimensions over which the allocation is performed). But such strategy suffers from many bad features such as the non-zero transmission outage probability, non-constant received power based on the channel realization etc.

In this paper we are going to develop the theory of channel inversion adaptation. That perfectly corresponds to such higher level service which calls for constant achievable rate for arbitrary instantaneous channel fading. Such feature is assured by the inversion symbol energy allocation procedure performed at the transmitter side. One can argue whether such goal is feasible and easily implementable and whether the capacity loss might be still acceptable. We will show that for some particular channels there is no way how to ensure the non-zero transmission outage probability under the total channel inversion policy. To reduce the problem we can follow the truncation approach. But still, for acceptable transmission outage probability the capacity is very low.

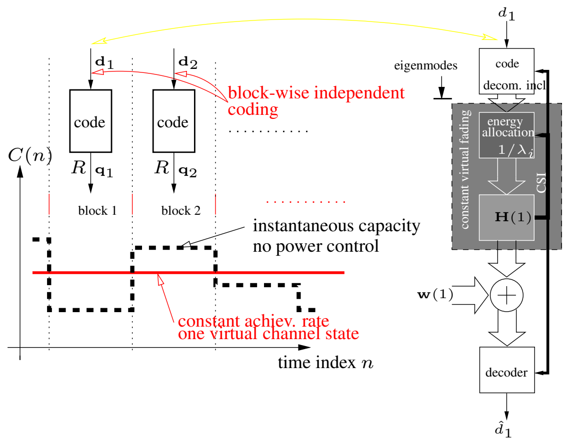

Fig. 1. System model and coding approach.

We will show other ways how to obtain totally invertible channels (sub-channels) providing very promising capacity value and warranted zero transmission outage probability.

The whole development of our approach supposes the long term average symbol energy limit. This is the crucial prerequisite which means the energy can be allocated in each block independently and differently as long as the average of transmitted energy meets the total limit. So the maximum achievable rates we investigate are so called *long term (average) or ergodic capacities*. The transmission outage probability is given by the portion of the time we do not transmit any energy/information. The transmission is interrupted whenever our optimization procedure resolves the signal would be corrupted by so deep fades that there is more beneficial to save energy for better channel occurrence (truncation strategy).

Please do not confuse with *delay-limited* channel model. The investigation of delay-limited condition (finite transmitted frame) yields very different capacity versus outage meaning with distinctive information-theoretical origin. We are not interested in the CDF[2] of the channel capacity, i.e. so called *outage capacity* (capacity versus outage, ϵ-capacity) for finite frame observation window. Such quantity specifies what the probability that given frame (finite number of channel realizations) supports required constant rate is.

In Fig. 1, there is depicted our system model illustrating the coding idea and symbol energy allocation goal. The channel model is flat and block-fading. We assume instantaneous capacity development in time regarding channel realizations. In our approach, there is strictly used the single code (the length of codewords is sufficient to be *comparable*[3] to the additive noise ergodic interval only) and the information is not jointly encoded via very long codewords spanning more block-fading realizations. The symbol energy evaluation relies on perfect CSI.

[2] Cumulative Distribution Function
[3] Theoretically infinitely long.

1.3 Particular Contents of the Paper

The starting point of the paper is the comprehensive summary of the physical and stochastic description of investigated channel models. We review both the channel statistics for SISO[4], SIMO[5] with SC (Selection Combining) and MRC (Maximum Ratio Combining), and also the Alamouti scheme [22] with $N_T = N_R = 2$ for its special importance. We are going to discuss the achievable rates for both truncated and total channel inversion. The closed formulas of some capacities are also recalled. In the case of MIMO we provide novel closed form expression for capacity of truncated channel inversion parametrized by the cut-off value λ_T. The following items clarifies what the tutorial and novel areas of this paper are:

Tutorial Part

- SISO channel models—Nakagami, Rayleigh,
- SIMO channel models—MRC, SC,
- MIMO channel model, IID & Correlated Rayleigh,
- basic idea of truncated & total inversion,

Novel Part

- total & truncated channel eigenmode space inversion,
- full- & reduced space joint PDF,
- marginal PDFs of ordered/unordered reduced eigenmode space,
- total inversion of eigenmode subspace,
- subspace marginal PDFs for correlated MIMO channel,
- correlated channel & inversion options,
- closed form expression for truncated full eigenmode space channel inversion,
- trade-off between transmission outage probability and ergodic capacity,
- Alamouti scheme under channel inversion—closed form capacity term.

2. SISO Channel Models

We start with SISO channel models. Naturally many adaptive (not only channel inversion) rules were developed exclusively for scalar channels. As we will show later, sometimes the extension to the MIMO channel is straightforward and sometimes with some very hard objections.

2.1 Nakagami Channel Model

First, the Nakagami distribution is introduced, mostly for its very general form including some a bit more specific and very often used distributions. The absolute value of the scalar channel gain value α in Nakagami channel model parametrized by the value of $m \geq 1/2$ corresponds to the probability density function

$$p(\alpha) = 2\left(\frac{m}{\Omega}\right)\frac{\alpha^{2m-1}}{\Gamma(m)}e^{(-m\alpha^2/\Omega)}, \quad \alpha \geq 0. \tag{1}$$

In this term, it holds $\Omega = \mathrm{E}[\alpha^2]$. Symbol $\Gamma(.)$ denotes the Gamma function. To regard the power conditions in the channel we define the instantaneous received signal to noise ratio (SNR) when no symbol energy allocation is performed just as the scaled square of the channel gain $\gamma = \bar{\mathcal{E}}\alpha^2/\sigma^2$. The symbol $\bar{\mathcal{E}}$ is the maximum long term average symbol energy limit. The variance of the AWGN[6] is σ^2. Then γ becomes very useful quantity because it includes both, the influence for the *power channel gain* α^2 and the maximum energy available at the transmitter over the noise variance. The system investigation for different SNR conditions is then very straightforward. For γ, we can immediately write so called Gamma distribution

$$p(\gamma) = \left(\frac{m}{\bar{\gamma}}\right)^m \frac{\gamma^{m-1}}{\Gamma(m)}e^{(-m\gamma/\bar{\gamma})}, \quad \gamma \geq 0 \tag{2}$$

where $\bar{\gamma} = \mathrm{E}[\gamma]$ is the average received signal to noise ratio where no power/energy adaption is applied. For the special case $m = 1/2$, (1) proceeds to the one-side Gaussian distribution ("worst case channel distribution").

2.2 Rayleigh Channel Model

Substituting $m = 1$ into (1) we arrive at so called *Rayleigh* distribution which is physically originated by rich scattering in the channel without the *Line of Sight* [23]. It is given by

$$p(\alpha) = 2\frac{\alpha}{\Omega}e^{(-\alpha^2/\Omega)}, \quad \alpha \geq 0. \tag{3}$$

Thus, the instantaneous received SNR γ is then distributed exponentially

$$p(\gamma) = \frac{1}{\bar{\gamma}}e^{(-\gamma/\bar{\gamma})}, \quad \gamma \geq 0. \tag{4}$$

[4] Single Input Single Output
[5] Single Input Multiple Outputs
[6] Additive White Gaussian Noise

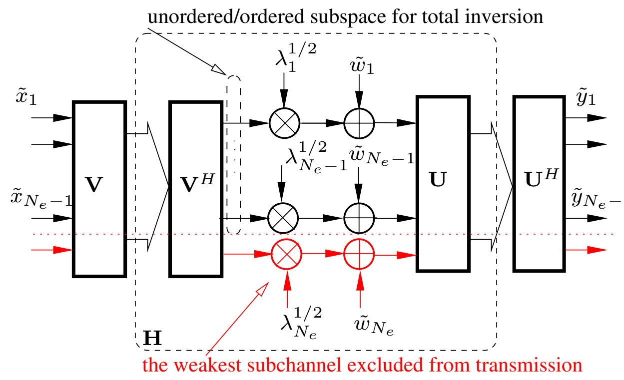

Fig. 2. SVD, reduced space of eigenmodes.

3. SIMO Channel Models–Combining at the Receiver

When the communication system chain is equipped by single transmitting and multiple (N_R) receiving antennas, we cope with the SIMO channel. We introduce exclusively two main processing techniques employed in the receiver which has special importance for the simplicity of implementation and reasonable performance.

3.1 Maximum Ratio Combining

Maximum Ratio Combining (MRC) processing relies on coherent processing in each branch of the receiver (signal from each receiving antenna). This method can be seen as the optimal combining at the receiver side since it provides the sufficient statistics to perform ML detection but with lower dimension. When MRC is employed and the average received SNR in each branch is the same (i.e. $\overline{\gamma}_k = \overline{\gamma},\ k = 1, ..., N_R$), than it holds for the total received SNR distribution

$$p_{mrc}(\gamma) = \frac{\gamma^{N_R-1} e^{\frac{-\gamma}{\overline{\gamma}}}}{(M-1)!\overline{\gamma}^{N_R}}, \quad \gamma \geq 0. \tag{5}$$

3.2 Selection Combining

Instead of performing MRC, we can consider much simpler but suboptimal strategy called *selection combining*. From N_T independently received and potentially independently faded replicas of transmitted signal it is chosen the one, which outperforms the others in some parameter of interest. Naturally, the received SNR is the most reasonable one. In SC, some information is not used in the detection of the transmitted signal so the worse performance is expected. Under the same assumptions we have made in the above mentioned case, PDF of γ is

$$p_{sc}(\gamma) = \frac{N_R}{\overline{\gamma}} \left(1 - e^{-\frac{\gamma}{\overline{\gamma}}}\right)^{N_R-1} e^{-\frac{\gamma}{\overline{\gamma}}}, \quad \gamma \geq 0. \tag{6}$$

Using the Binomial expansion, we could rewrite (6) into the equivalent form

$$p_{sc}(\gamma) = \frac{M}{\overline{\gamma}} \sum_{k=0}^{M-1} (-1)^k \binom{M-1}{k} e^{\left(-\frac{(1+k)\gamma}{\overline{\gamma}}\right)}. \tag{7}$$

Mutually different power gains in paths are discussed in [24].

4. MIMO Channel Model

Assuming N_T antennas at the transmitter and N_R at the receiver the flat fading channel corresponds to the model

$$\mathbf{y} = \mathbf{Hx} + \mathbf{w}. \tag{8}$$

The channel matrix $\mathbf{H}$ is of the dimension $N_R \times N_T$. The noise vector has covariance matrix $\mathbf{R_w} = \sigma^2\mathbf{I}$. The SVD (Singular Value Decomposition) of the channel matrix yields $\mathbf{y} = \mathbf{UDV}^H\mathbf{x} + \mathbf{w}$, [25]. The eigenmode space is of the dimension $N_e = \min(N_T, N_R)$. Similarly we use the notation $N_n = \max(N_R, N_T)$. Matrices $\mathbf{U} \in \mathbb{C}^{N_R \times N_R}$, $\mathbf{V} \in \mathbb{C}^{N_T \times N_T}$ are both unitary and the matrix $\mathbf{D} \in \mathbb{R}^{N_R \times N_T}$ is non-negative and diagonal with entries given as non-negative square roots of eigenvalues of Wishart matrix

$$\mathbf{W} = \begin{cases} \mathbf{HH}^H, & N_R < N_T \\ \mathbf{H}^H\mathbf{H}, & N_R \geq N_T \end{cases}. \tag{9}$$

The ordered eigenvalues are denoted explicitly with index as λ_i and it holds $\lambda_1 > \lambda_2 > ... > \lambda_{N_e}$. When unordered full space of eigenmodes is assumed, the symbol λ is used to denote the general eigenmode gain with the common marginal distribution. Accordingly, $\tilde{\lambda}$ will denote the general subchannel gain of unordered subspace of eigenmodes. New equivalent variables are $\tilde{\mathbf{y}} = \mathbf{U}^H\mathbf{y}$, $\tilde{\mathbf{x}} = \mathbf{V}^H\mathbf{x}$, $\tilde{\mathbf{w}} = \mathbf{U}^H\mathbf{w}$ and equivalent input-output equation is $\tilde{\mathbf{y}} = \mathbf{D}\tilde{\mathbf{x}} + \tilde{\mathbf{w}}$. The symbol energy adaptation (or power adaptation) is performed at the level of vector $\tilde{\mathbf{x}}$. In Fig. 2, there is clarified what the full space of eigenmodes (sub-channels) is and how the unitary transformations are applied. The reduced eigenmode space is also mentioned and we address details later in the paper.

4.1 IID Rayleigh MIMO Channel

The distributions (both, marginal and joint) for unordered case are denoted as $p_u(.)$ and for ordered case as $p_o(.)$. In order to derive the ergodic capacity of the MIMO channel in the Rayleigh fading with known CSI at the transmitter, we have to review the distribution of squares of singular values of the channel matrix $\mathbf{H}$, i.e. of the eigenvalues of the matrix $\mathbf{W}$. It can be proved that these ordered eigenvalues are described by joint *Wishart distribution* with PDF given as

$$p(\lambda_1, \cdots, \lambda_{N_e}) = K_{N_e,N_n} \prod_{i=1}^{N_e} e^{-\lambda_i} \lambda_i^{N_n-N_e} \prod_{i<j}^{N_e} (\lambda_i - \lambda_j)^2. \tag{10}$$

The constant K_{N_e,N_n} depends on N_e, N_n, i.e. on the numbers of antennas at the both sides, and is given by

$$K_{N_e,N_n} = \frac{\pi^{N_e(N_e-1)}}{\Gamma_{N_n}(N_n)\Gamma_{N_e}(N_e)} \tag{11}$$

where $\Gamma_a(b)$ is the complex multivariate gamma function. The general integration that yields the PDF of the k-th eigenvalue is given by the following

$$p_k(\lambda_k) = \int_{\lambda_k}^{\infty}\int_{\lambda_{k-1}}^{\infty}\cdots\int_{\lambda_3}^{\infty}\int_{\lambda_2}^{\infty}\cdots\int_0^{\lambda_k}\cdots$$

$$\cdots\int_0^{\lambda_{N_e-1}} p(\lambda_1,\cdots,\lambda_{N_e})d\lambda_{N_e}\cdots d\lambda_{k+1}d\lambda_1 d\lambda_2\cdots d\lambda_{k-1}. \tag{12}$$

For the unordered eigenmodes the common share marginal distribution of the eigenvalue of $\mathbf{W}$ is [26]

$$p_u(\lambda) = \frac{1}{N_e}\sum_{k=0}^{N_e-1}\frac{k!\lambda^{N_n-N_e}e^{-\lambda}}{(k+N_n-N_e)!}\left(L_k^{N_n-N_e}(\lambda)\right)^2. \tag{13}$$

Based on [27], eq. 8.970.1, the equivalent form of Laguerre polynomial can be used as

$$L_k^n(x) = \frac{1}{k!}e^x x^{-n}\frac{d^k}{dx^k}\left(e^{-x}x^{k+n}\right) \tag{14}$$

$$= \sum_{i=0}^{k}(-1)^i\binom{k+n}{k-i}\frac{x^i}{i!}. \tag{15}$$

We follow the approach of [29], we want to rewrite squared Laguerre polynomial into the direct form. We can use the eq. 8.976.3 from [27] which claims[7]

$$(L_k^n(\lambda))^2 = \frac{\Gamma(k+n+1)}{2^{2k}k!}\sum_{l=0}^{k}\frac{(2l)!\binom{2k-2l}{k-l}}{l!\Gamma(n+l+1)}L_{2l}^{2n}(2\lambda). \tag{16}$$

Now, putting (14) into (16) we can arrive at (skipping details given in [29])

$$p_u(\lambda) = \frac{1}{N_e}\sum_{k=0}^{N_e-1}\sum_{l=0}^{k}\sum_{i=0}^{2l}\left\{\Theta(i,l,k)\lambda^{N_n-N_e-i}e^{-\lambda}\right\} \tag{17}$$

where

$$\Theta(i,l,k) = \frac{(-1)^i(2l)!\binom{2k-2l}{k-l}\binom{2N_n-2N_e+2l}{2l-i}}{2^{2k-i}l!i!(N_n-N_e+l)!}. \tag{18}$$

4.2 Correlated Rayleigh MIMO Channel

Following e.g. [30], we assume the correlation at the receiver given by correlation matrix $\mathbf{\Sigma}$. It can be shown the joint PDF is given by

$$p(\lambda_1,...,\lambda_{N_{\min}}) = K\frac{|\mathbf{E}(\boldsymbol{\lambda},\boldsymbol{\mu})|\,|\mathbf{V}_1(\boldsymbol{\lambda})|}{|\mathbf{V}_2(\boldsymbol{\mu})|}\prod_{j=1}^{N_e}(x_j^{N_n-N_e}) \tag{19}$$

where the constant K is

$$K = K_{N_e,N_n}\prod_{j=1}^{N_e}(j-1)!\,|\mathbf{\Sigma}|^{-N_e}. \tag{20}$$

Next, we need to define also the matrix $\mathbf{V}_1$

$$\mathbf{V}_1(\boldsymbol{\lambda}) = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \lambda_1 & \lambda_2 & \cdots & \lambda_{N_e} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_1^{N_e-1} & \lambda_2^{N_e-1} & \cdots & \lambda_{N_e}^{N_e-1} \end{bmatrix}. \tag{21}$$

Based on this expression the matrix $\mathbf{V}_2$ corresponds to

$$\mathbf{V}_2(\boldsymbol{\mu}) = \mathbf{V}_1(-[\mu_1^{-1},...,\mu_{N_e}^{-1}]). \tag{22}$$

To finish our correlated model it remains to define

$$\mathbf{E}(\boldsymbol{\lambda},\boldsymbol{\mu}) = \begin{bmatrix} e^{-\lambda_1/\mu_1} & e^{-\lambda_2/\mu_1} & \cdots & e^{-\lambda_{N_e}/\mu_1} \\ e^{-\lambda_1/\mu_2} & e^{-\lambda_2/\mu_2} & \cdots & e^{-\lambda_{N)_e}/\mu_2} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-\lambda_1/\mu_{N_e}} & e^{-\lambda_2/\mu_{N_e}} & \cdots & e^{-\lambda_{N_e}/\mu_{N_e}} \end{bmatrix} \tag{23}$$

where we have used the vector notation $\boldsymbol{\mu} = [\mu_1,...,\mu_{N_e}]^T$, $\mu_1 \geq \mu_2 \geq ... \geq \mu_{N_e}$ are ordered eigenvalues of the correlation matrix $\mathbf{\Sigma}$.

4.3 Alamouti Scheme $N_T = N_R = 2$

When orthogonal Alamouti scheme for MIMO system is employed with $N_T = N_R = 2$, we find that the total received SNR for each symbol is (see e.g. [28])

$$\gamma = \sum_{i=1}^{4}\gamma_i. \tag{24}$$

We assume the Rayleigh fading between each pair of the transmitting and the receiving antennas, i.e. $\gamma_i, i = 1,2,3,4$ are exponentially distributed and γ is χ^2 distributed.
Thus, its PDF is

$$p_A(\gamma) = \frac{\gamma^3 e^{-\gamma/\bar{\gamma}}}{3!\bar{\gamma}^4}. \tag{25}$$

5. Total Channel Inversion

If CSI is used by the transmitter to ensure the constant received power, then the time-varying channel appears at the receiver side as an ordinary time-invarying channel

[7] There is very small typo in [29], the index i should be replaced by 1.

with AWGN. The long term maximum average symbol energy is denoted as $\bar{\mathcal{E}}$ then the adaptive formula for a transmitted power is given by the relation [3, 1]

$$\mathcal{E}(\gamma) = \frac{K}{\gamma}. \tag{26}$$

The ergodic symbol energy constraint has to be accomplished as

$$\int_0^\infty \frac{K}{\gamma} p(\gamma) d\gamma = \bar{\mathcal{E}}. \tag{27}$$

It implies the necessary condition

$$K = \frac{\bar{\mathcal{E}}}{\mathrm{E}[1/\gamma]}. \tag{28}$$

Substituting (28) into the capacity term we can obtain the generic way to get the channel capacity as

$$C_{\mathrm{tot}} = \log_2(1+K) = \log_2\left(1 + \frac{\bar{\mathcal{E}}}{\mathrm{E}\left[1/\gamma\right]}\right). \tag{29}$$

The problem is especially in the expectation in (28) which *diverges* for some kinds of distribution, e.g. for SISO Rayleigh fading. In the section about reduced eigenmode space channel inversion for MIMO channel, we will show that the same holds for the weakest sub-channel under both, IID and correlated MIMO Rayleigh channel model. Figures with some of these distributions appear also in the numerical results section.

6. Truncated-Inversion

A disadvantage of the channel inversion is especially the necessity to utilize large power for the sake of the large fade compensation. Thus, we show a modification of previous method called in the literature the *truncated inversion*. We obtain the truncated inversion by an introduction of the marginal value γ_0, which is a boundary between the executing of the inversion and the interruption of the transmission. Afterwards, the power is allocated by the formula[8]

$$\mathcal{E}(\gamma) = \frac{K(\gamma_T)}{\gamma} \mathcal{U}(\gamma - \gamma_T) \tag{30}$$

where the parametrized value of K has to fulfill the condition which is parametrized by the cut-off value γ_T and the total average energy $\bar{\mathcal{E}}$

$$K(\gamma_T) = \frac{\bar{\mathcal{E}}}{\int_{\gamma_T}^\infty 1/\gamma p(\gamma) d\gamma}. \tag{31}$$

To get the capacity we have to find such γ_T which maximizes the ergodic capacity (see e.g. [3])

$$C_{\mathrm{trc}} = \max_{\gamma_T} \log_2[1 + K(\gamma_T)] \mathrm{Pr}(\gamma \geq \gamma_T). \tag{32}$$

The transmitter adapts its transmitted symbol energy to maintain the received SNR to be constant in time, from that follows that we do not adapt the rate in time. For the receiver, the channel seems to be time-unvarying channel with AWGN. The advantage of this implementation is its simplicity. The disadvantage lies in the sub-optimality with regards to the achievable capacity. The other very important and sometimes omitted fact is that the truncation yields usually very high probability of outage.

7. Channel Capacity under Inversion for SISO & SIMO

To make this paper self-contained, in this section, we provide the exhaustive review of the results obtained in classical papers about adaptive modulation [5, 6, 10, 3].

7.1 Rayleigh SISO

In Rayleigh case, there is the zero capacity of total channel inversion since in order to invert the channel, the long term average symbol energy constraint should have to go to infinity. The solution is to truncate the inversion for very deep fades. Substituting (4) into (32) we easily derive the term for the capacity[9] of channel inversion with the truncation below the value γ_T

$$C_{\mathrm{trc}}^{\mathrm{siso}} = \log_2\left(1 + \frac{\overline{\gamma}}{E_1(\gamma_T/\overline{\gamma})}\right) e^{-\frac{\gamma_T}{\overline{\gamma}}}. \tag{33}$$

7.2 Rayleigh SIMO–MRC

We can derive the achievable capacity with maximum ratio combining and *the total channel inversion* (substitution of (5) into (29))

$$C_{\mathrm{tot,mrc}}^{\mathrm{simo}} = \log_2\left(1 + (N_R - 1)\overline{\gamma}\right). \tag{34}$$

Substitution of (5) into (32) gives us the capacity under the *truncated inversion* and for $N_R \geq 2$ it holds[10]

$$\begin{aligned} C_{\mathrm{trc,mrc}}^{\mathrm{simo}} &= \log_2\left(1 + \frac{(N_R-1)!\overline{\gamma}}{\Gamma(N_R-1, \gamma_T/\overline{\gamma})}\right) \frac{\Gamma(N_R, \gamma_T/\overline{\gamma})}{(N_R-1)!} \\ &= \log_2\left(1 + \frac{(N_R-1)\overline{\gamma}}{\mathcal{P}_{N_R-1}(\gamma_T/\overline{\gamma})} \mathcal{P}_{N_R}(\gamma_T/\overline{\gamma})\right). \end{aligned} \tag{35}$$

[8] $\mathcal{U}(\cdot)$ is the unit step function

[9] $E_n(x) = \int_1^{+\infty} t^{-n} e^{-xt} dt, x \geq 0$ is the exponential integral of order n.

[10] $\Gamma(.,.)$ is the complementary incomplete Gamma function, and $\mathcal{P}_k(x) = e^{-x} \sum_{j=0}^{k-1} \frac{x^j}{j!}$ denotes the Poisson distribution.

7.3 Rayleigh SIMO–SC

Analogously, using (6) and general formula (29), we find that the total inversion capacity with selection combining is

$$C^{\rm simo}_{\rm tot,sc} = \log_2\left(1+\frac{\overline{\gamma}}{N_R L_1}\right) \tag{36}$$

where

$$L_1 = \lim_{u\to 0^+}\sum_{k=0}^{N_R-1}(-1)^k\binom{N_R-1}{k}E_1((1+k)u/\overline{\gamma}). \tag{37}$$

And the truncated inversion capacity is (substituting (6) into (32))

$$C = \log_2\left(1+\frac{\overline{\gamma}}{N_R L_2}\right)\Pr(\gamma\geq\gamma_T) \tag{38}$$

$$L_2 = \sum_{k=0}^{N_R-1}(-1)^k\binom{N_R-1}{k}E_1((1+k)\gamma_T/\overline{\gamma}). \tag{39}$$

Let be devised to [5] and also to references inside for analysis done in details. Therein, one can find the graphical comparison of capacities listed above.

7.4 Nakagami SISO

Under general Nakagami fading (1), the total inversion capacity follows from the distribution (2) and general formula (29)

$$C = \log_2\left(1+\frac{m-1}{m}\overline{\gamma}\right) \tag{40}$$

analogously one may obtain the capacity for truncated version and $\forall m \geq 1$

$$C^{\rm siso}_{\rm trc} = \log_2\left(1+\frac{\overline{\gamma}\Gamma(m)}{m\Gamma(m-1,m\gamma_T/\overline{\gamma})}\right)\frac{\Gamma(m,m\gamma_T/\overline{\gamma})}{\Gamma(m)}. \tag{41}$$

7.5 Discrete Rate Adaptation with Total and Truncated Channel Inversion

For completeness, we state the total inversion with only finite set of constellations [6]. The expected achievable spectral efficiency is

$$R_{\max} = \log_2\left(\left\lfloor 1+\frac{G}{\int_{\gamma_T}^{\infty}(1/\gamma)p(\gamma)d\gamma}\right\rfloor_{\mathcal{M}}\right) \tag{42}$$

where $\lfloor x\rfloor_{\mathcal{M}}$ denotes the highest number in set $\mathcal{M}$ less or equal to x where $\mathcal{M}$ denotes the set of numbers of points in the available constellations. The bit error rate limit which has to be met in our adaptation is chosen as BER_0 and $G = -1.6/\ln(5\mathrm{BER}_0)$. Obviously, the spectral efficiency under this policy has one of values $\log_2 M$, $M\in\mathcal{M}$, depending on discrete steps in $\overline{\gamma}$.

The truncated inversion gives the spectral efficiency

$$\begin{aligned} R_{\max} &= \max\nolimits_{\gamma_T}\log_2\left(\left\lfloor 1+\frac{G}{\int_{\gamma_{\rm T}}^{\infty}(1/\gamma){\rm p}(\gamma){\rm d}\gamma}\right\rfloor_{\mathcal{M}}\right)\\ &\times\ \Pr(\gamma\geq\gamma_T). \end{aligned} \tag{43}$$

8. Channel Capacity under Channel Inversion for MIMO

For the full eigenmode space, there is no way how to totally invert the sub-channels since for the unordered case it holds

$$\int_0^{\infty}\frac{1}{\lambda}p_u(\lambda)d\lambda\to\infty. \tag{44}$$

The mean symbol energy necessary to invert the channel diverges. For ordered case, the same harmful feature appears only with the weakest eigenmode. Our solution is to exclude the last (weakest) eigenmode from the transmission and the reduced subset of eigenmodes is then totally invertible for both ordered as well as unordered case. In the following subsections, we are going to derive the capacities of these mentioned algorithms more in details. For completeness, we also mention the waterfilling capacity and the special case of Alamouti scheme.

8.1 Optimal 2D Waterfilling in Space and Time

The optimal power allocation in ergodic space-time communication scenario is obvious. Optimization yields *waterfilling in space and time* [31, 32]. Under ergodic consideration, there is the *constant water level* μ for any particular channel realization within the 2D support (time and space). Such power level has to fulfill the criterion[11]

$$\bar{\mathcal{E}} = \int_0^{\infty}\sum_{i=1}^{N_e}\left(\mu-\frac{\sigma^2}{\lambda_i}\right)^+ p_u(\lambda_1,...,\lambda_{N_e})d\lambda_1...d\lambda_{N_e}. \tag{45}$$

The total long-term mean energy limit is $\bar{\mathcal{E}}$. It does not matter whether ordered or unordered eigenvalues are considered. The instantaneous amount of power allocated in sum over all eigenmodes is varied. Such policy exhibits the highest achievable long time average capacity

$$C_{\rm wf} = \mathrm{E}_{\boldsymbol{\lambda}}\left[\sum_{i=1}^{N_e}\log_2\left(\frac{\mu\lambda_i}{\sigma^2}\right)^+\right] \tag{46}$$

but it fails whenever the support of given constant number of eigenmodes with constant capacity achievable in each block is required strictly for any channel realization $\mathbf{H}$.

[11] $(x)^+ = \max(0,x)$

8.2 Alamouti Scheme

Employing (25) and (29) we have derived the simple closed form capacity for total channel inversion with $N_T = N_R = 2$ Alamouti scheme [22] as

$$C^{\mathrm{A}}_{\mathrm{tot}} = \log_2 (1 + 3\bar{\gamma}). \tag{47}$$

Using (32), the closed form for truncated version parametrized by the cutoff value λ_T can be found as

$$\begin{aligned} C^{\mathrm{A}}_{\mathrm{trc}} &= \log_2 \left(1 + \frac{6\bar{\gamma}^3}{e^{-\lambda_T/\bar{\gamma}}(\lambda_T^2 + 2\lambda_T\bar{\gamma} + 2\bar{\gamma}^2)}\right) \\ &\times \frac{e^{-\lambda_T/\bar{\gamma}}(\lambda_T^3 + 3\lambda_T^2\bar{\gamma} + 6\lambda_T\bar{\gamma}^2 + 6\bar{\gamma}^3)}{6\bar{\gamma}^3}. \end{aligned} \tag{48}$$

8.3 Full Eigenmode Space with Truncated Inversion

8.3.1 Unordered Eigenmode Space

The capacity derivation assumes the unordered set of eigenmodes obtained via channel decomposition. Then based on (31) and (30), the threshold λ_T needs to be optimized as

$$\lambda_T = \arg\max_{\check{\lambda}_T} C^{\mathrm{un}}_{\mathrm{trc}}(\check{\lambda}_T) \tag{49}$$

where the truncated channel inversion capacity utilizing all channel eigenmodes is

$$C^{\mathrm{un}}_{\mathrm{trc}}(\lambda_T) = N_e \mathrm{E}_\lambda \left[\log_2\left(1 + \frac{K(\lambda_T)}{\sigma^2}\mathcal{U}(\lambda - \lambda_T)\right)\right] \tag{50}$$

for given signal to noise ratio $\bar{\mathcal{E}}/\sigma^2$. Once the threshold is found in advance the transmission off-line utilizing known probabilistic channel description, the on-line power allocation is indeed very simple.

We can derive what the capacity of such truncated version of unordered eigenmode space channel inversion is. The goal is to perform the expectation in (50). First the term for parametrized constant $K(\lambda_T)$ is necessary to be found. For inverse value it has to hold

$$K^{-1}(\lambda_T) = \frac{N_e}{\bar{\mathcal{E}}} \int_{\lambda_T}^{\infty} \frac{1}{\lambda} p_u(\lambda) d\lambda. \tag{51}$$

Based on the fact

$$\int_{\lambda_T}^{\infty} \lambda^{N_n - N_e - i - 1} e^{-\lambda} d\lambda = \Gamma[N_n + i - N_e, \lambda_T] \tag{52}$$

using $X = N_n - N_e$, we can rewrite (51) into

$$K^{-1}(\lambda_T) = \sum_{k=0}^{N_e-1}\sum_{l=0}^{k}\sum_{i=0}^{2l}\left\{\frac{\Theta(i,l,k)}{\bar{\mathcal{E}}}\Gamma[X + i, \lambda_T]\right\}. \tag{53}$$

The final closed term for capacity of channel inversion in MIMO IID Rayleigh channel parametrized by the cut-off value is given by

$$\begin{aligned} C^{\mathrm{un}}_{\mathrm{trc}}(\lambda_T) &= \sum_{k=0}^{N_e-1}\sum_{l=0}^{k}\sum_{i=0}^{2l}\left\{\frac{\Theta(i,l,k)}{\bar{\mathcal{E}}}\Gamma[X + 1 + i, \lambda_T]\right\} \\ &\times \log_2\left(1 + \frac{\bar{\mathcal{E}}}{\sigma^2}K(\lambda_T)\right). \end{aligned} \tag{54}$$

8.3.2 Ordered Eigenmode Space

When the truncation is done on the ordered full space, the situation is complicated since the thresholds differ among eigenmodes. The optimization searches the vector of truncation $\boldsymbol{\lambda}_T = [\lambda_T^1, ..., \lambda_T^{N_e}]$ such that

$$\boldsymbol{\lambda}_T = \arg\max_{\check{\boldsymbol{\lambda}}_T} C^{\mathrm{ord}}_{\mathrm{trc}}(\check{\boldsymbol{\lambda}}_T). \tag{55}$$

The capacity term in the ordered case is

$$C^{\mathrm{ord}}_{\mathrm{trc}}(\boldsymbol{\lambda}_T) = \mathrm{E}_{\boldsymbol{\lambda}}\left[\sum_{i=1}^{N_e}\log_2\left(1 + \frac{K_i(\lambda_T^i)}{\sigma^2}\mathcal{U}(\lambda_i - \lambda_T^i)\right)\right] \tag{56}$$

where $K_i(\lambda_T^i)$ has to be chosen to satisfy the total power constraint

$$\bar{\mathcal{E}} = \mathrm{E}_{\boldsymbol{\lambda}}\left[\sum_{i=1}^{N_e}\frac{K_i(\lambda_T^i)}{\lambda_i}\mathcal{U}(\lambda_i - \lambda_T^i)\right]. \tag{57}$$

Obviously such optimization suffers from infeasibility and moreover the average allocated power is different among eigenmodes.

8.4 Eigenmode Subspace with Total Inversion

8.4.1 Unordered Eigenmode Subspace

The subspace channel is the set of the $N_e - 1$ strongest eigenmodes of the original MIMO channel. After subtracting the weakest one, the remaining eigenmodes can be considered again ordered or unordered. The unordered case capacity is

$$C^{\mathrm{un}}_{\mathrm{sub}} = (N_e - 1)\log_2\left(1 + \frac{\bar{\mathcal{E}}}{(N_e - 1)\sigma^2\mathrm{E}_{\tilde{\lambda}}[1/\tilde{\lambda}]}\right). \tag{58}$$

8.4.2 Ordered Eigenmode Subspace

When eigenmodes are ordered within the subspace we can evaluate the capacity as

$$C^{\mathrm{ord}}_{\mathrm{sub}} = \sum_{i=1}^{N_e-1}\log_2\left(1 + \frac{K_i}{\sigma^2}\right) \tag{59}$$

where the values K_i have to meet the total ergodic power limit

$$\bar{\mathcal{E}} = \mathrm{E}_{\lambda_1,...,\lambda_{N_e-1}}\left[\sum_{i=1}^{N_e-1}\frac{K_i}{\lambda_i}\right]. \tag{60}$$

9. Numerical Results

In Fig. 3, there are depicted 2D charts with joint Wishart distributions of full and reduced eigenmode space. For the full space with two eigenmodes $N_T = N_R = 2$, one can observe the nonzero values along the line $\lambda_1 = 0$ or $\lambda_2 = 0$. For the reduced space with $N_T = N_R = 3$, there are zero values along these lines. That corresponds to the influence of the weakest eigenmode feature which causes the infeasibility of the total inversion of the full set of eigenmode disregarding the number of antennas (modes).

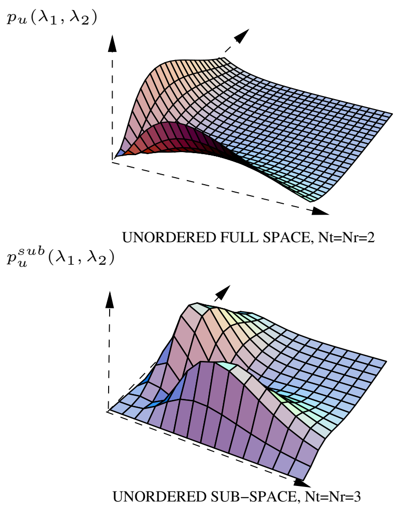

Fig. 3. Full- and reduced-space, joint probability density functions.

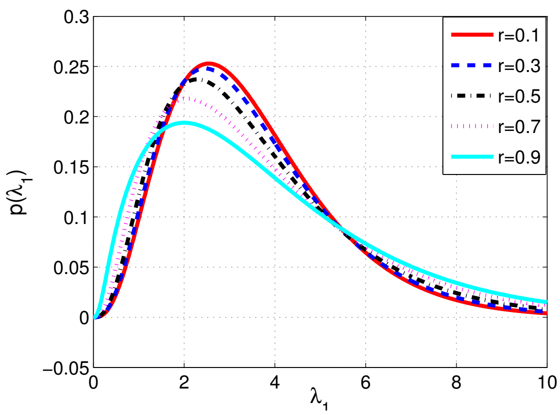

Fig. 4. The strongest eigenmode marginal density function, $p(\lambda_1)$ under correlated fading, $N_T = N_R = 2$.

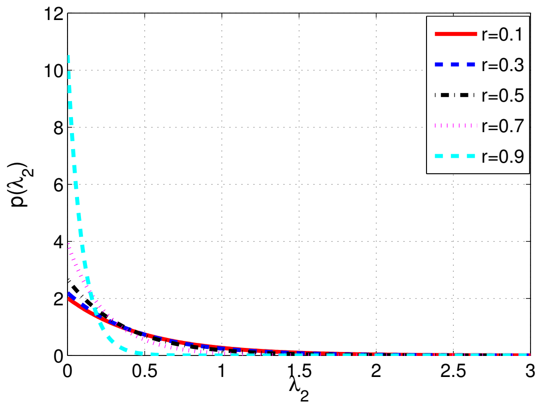

Fig. 5. The weakest eigenmode marginal density function, $p(\lambda_2)$ under correlated fading, $N_T = N_R = 2$.

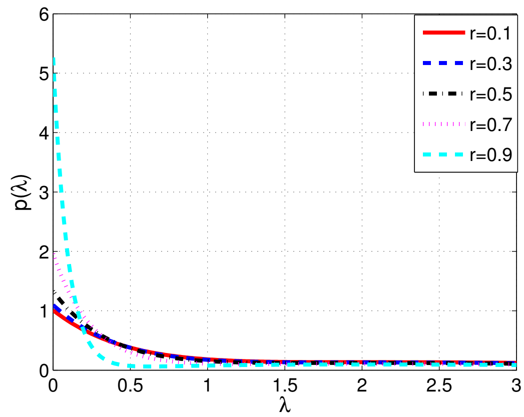

Fig. 6. The shared eigenmode marginal density function (unordered case), $p(\lambda)$ under correlated fading, $N_T = N_R = 2$.

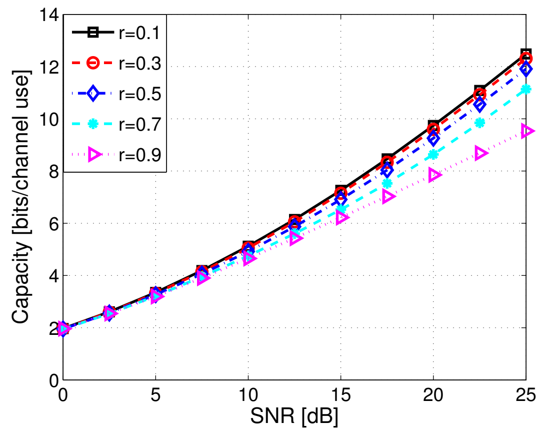

Fig. 7. Capacity—Eigenmode full-space truncated channel inversion under correlated fading, $N_R = N_T = 2$.

In Fig. 4, there is shown the strongest eigenmode marginal density for $N_T = N_R = 2$ under increasing correlation. It is clear that this density stands totally invertible even for the highest correlation. Similarly, in Fig. 5 the influence of the correlation on the weakest eigenmode density and the shared marginal density for unordered eigenmode space is brought out in Fig. 6. These densities stay non-invertible for arbitrary correlation coefficient.

In Fig. 7, there is sketched the capacity curve for full eigenmode space and different correlation assuming $N_T = N_R = 2$. We can see the capacity loss is significant mostly for very strong correlation and high SNR. Very interesting observation follows from Fig. 8. There is a negligible gap among capacity curves so that although the influence on the marginal distribution of the strongest eigenmode is pretty high, the capacity stays resistant. In Fig. 9 and Fig. 10, very similar situation but for the case of $N_T = N_R = 3$ is shown. One can see a stronger influence of correlation on the capacity. Moreover, based on all these capacity figures it is clear that although the subspace capacity is always below the truncated full space capacity, the loss is very small and almost unrecognizable in most of SNR values. The emphasis should be given to the fact that in the subspace eigenmode channel inversion, there is zero transmission outage probability so that the total sustainability of the transmission might be guaranteed. See e.g. [19] for the charts of such very high probability in the case of truncated inversion. The performance of the inversion employed with the orthogonal Alamouti code [22] is provided in Fig. 11. There is truly negligible gap between the capacity of truncated and total inversion and also the loss in comparison with optimal waterfilling is very small. The trade-off between the outage probability and the capacity of the truncated inversion is illustrated in Fig. 12. One can find out, the designer is free to pick out two options with given capacity, the more burst like with higher occurrence of transmission interruption or the more sustainable transmission.

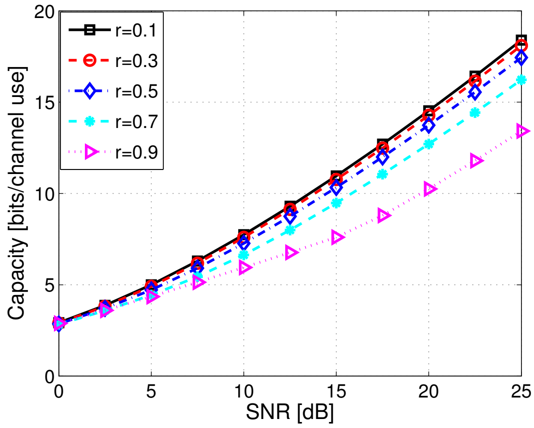

Fig. 9. Capacity—Eigenmode full-space truncated channel inversion under correlated fading, $N_R = N_T = 3$.

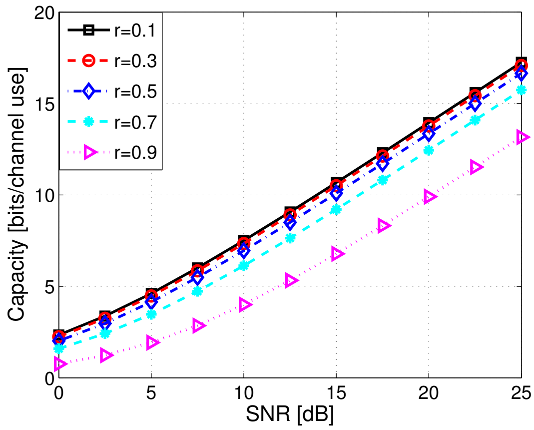

Fig. 10. Capacity—Eigenmode sub-space total channel inversion under correlated fading, $N_R = N_T = 3$.

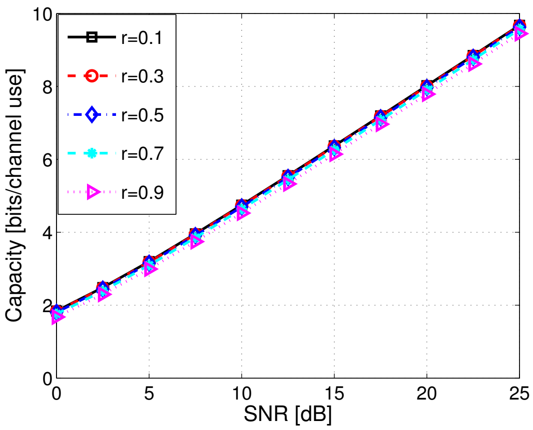

Fig. 8. Capacity—Eigenmode sub-space total channel inversion under correlated fading, $N_R = N_T = 2$.

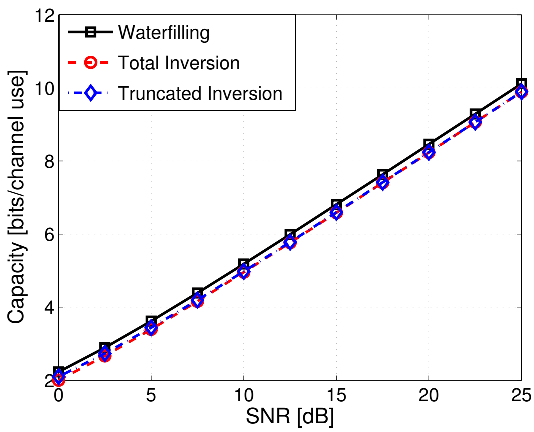

Fig. 11. Capacity—Alamouti scheme with optimal waterfilling allocation versus sub-optimal inversion, $N_R = N_T = 2$.

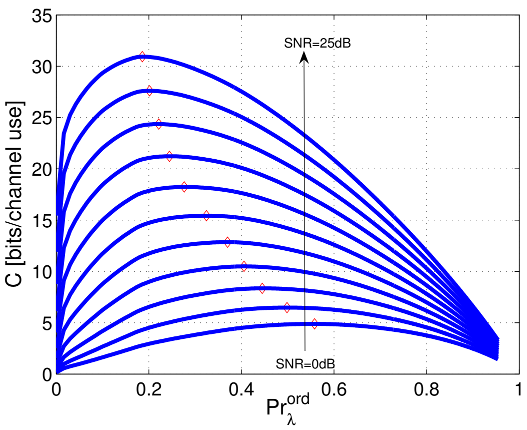

Fig. 12. Transmission outage probability versus capacity trade-off, $N_T = N_R = 5$, $\mathrm{SNR} = \{0; 2.5, ..., 25\}$ dB.

10. Conclusions

In this paper, the concept of MIMO eigenmode channel inversion policy is developed in very general framework including even the SISO and SIMO results. To provide zero outage transmission probability and constant achievable rate there is our conclusion that it is absolutely sufficient to exclude only the weakest eigenmode from the transmission. The remaining eigenmode sub-set becomes totally invertible regardless particular fading realization. This paper proves that there is negligible capacity gap of subspace eigenmode total inversion with zero outage compared to the optimized full space total inversion with unacceptable probability of outage. Very novel and useful extension to the correlated MIMO Rayleigh channel is provided. We have found the joint and marginal distributions even for correlated case. Somewhat more surprising fact is that we have also proved that the correlation does not make more eigenmodes non-invertible and so that the only one which has to be removed is again just the weakest eigenmode. The capacity of correlated MIMO Rayleigh channel is also carefully studied. Other novel points of the paper are the closed form expressions for truncated full space eigenmode inversion and for Alamouti $N_T = N_R = 2$ scheme. Such expression enables analytical investigation of mutual capacity versus transmission outage probability trade-off.

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