Analog Group Delay Equalizers Design Based on Evolutionary Algorithm

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Abstract. *This paper deals with a design method of the analog all-pass filter designated for equalization of the group delay frequency response of the analog filter. This method is based on usage of evolutionary algorithm, the Differential Evolution algorithm in particular. We are able to design such equalizers to be obtained equal-ripple group delay frequency response in the pass-band of the low-pass filter. The procedure works automatically without an input estimation. The method is presented on solving practical examples.*

Keywords

Analog all-pass filters, group delay frequency response, Differential Evolution algorithm.

1. Introduction

The genetic and evolutionary algorithms were found out as a powerful tool in design and optimization of electrical circuits and systems. These algorithms are global minimization algorithms which simulate an evolutional process in the nature [5]. They store a number of representations of solutions to a problem in a so-called 'population' matrix. The genetic algorithms can minimize not only a standard function, but also highly nonlinear and partly non-differentiable functions with many local minima.

These algorithms are time-consuming, but, on the other hand, computationally robust and lead to optimum results in the solution of complicated design problems. Excellent results have been obtained e.g. in a transfer function approximation problem solution in many cases of analog or digital filters design. For example, Martinek and Vondraš have dealt with the approximation of the analog filter transfer function with concurrent requirements for magnitude and group delay frequency response in the papers [2], [3] and mainly in the paper [4]. Storn proposed a method for the design of a digital IIR filter with concurrent requirements for magnitude and group delay frequency response using the Differential Evolution (DE) algorithms.

Here we propose a different solution of the similar problem in analog domain. The magnitude response and the group delay are optimized individually in cascade-coupled filter blocks. At this point, the optimization can be effectively done by adjusting parameters of each section.

The design procedure of an analog filter with requirements for magnitude and group delay frequency response can be divided into two parts. The design of the analog filter by using a standard method is the first well-established part and the design of the additional analog all-pass filter is the second part. It is applied in connection with the "main" filter for equalization of the group delay frequency response of the filter. In general, an approximation of the equalizer transfer function represents an exigent mathematical problem. The present design methods are based on usage of classical numerical methods, where a good estimation of the initial approximation to ensure convergence of the numerical method is needed. These methods are mostly mathematically complicated for programming and implementation. The disadvantage of these classical methods is also that they can converge to local optimum. Several such design procedures have been published in papers [7], [8], [9].

Therefore, a new very simple method of the equalizing all-pass filter design based on evolutionary algorithm is presented in this contribution. The procedure is split to two parts. At first, we calculate the analog all-pass filter transfer function to obtain the total group delay frequency response (of the low-pass filter with applied all-pass filter) which has unequal-ripple extremes in the pass-band. Subsequently, the estimation of extremes is utilized for the numerical solver to search correct shifted values of the all-pass filter transfer function complex poles to be obtained equal-ripple form of the group delay in the pass-band. Thus, the procedure allows designing filters with equal-ripple group delay frequency response in the pass-band. The Differential Evolution algorithm is used like numerical solver in both parts. The method will be demonstrated on practical examples. The whole design procedure has been programmed in the MATLAB environment.

2. The Differential Evolution Algorithm

The Differential Evolution algorithm belongs to the evolutionary algorithms group. The algorithm was developed by K. Price and for the first time was presented in

1995. In the conference First International Contest of Evolutionary Computation (1stICEO) held in Nagoya in May 1996, this algorithm turned out to be the best evolution type of algorithm for the real-valued functions solving.

The Differential Evolution (DE) algorithm is a parallel direct search method which uses floating-point number representation to find continuous parameters. This technique utilizes *NP* *D*-dimensional parameter vectors

$$\underline{x}_{i,G}, \quad i = 1,2,\ldots,NP \tag{1}$$

as a population for each generation *G*. It means, for each iteration step of the minimization process. *NP* does not change during the evaluation. The initial population is chosen randomly if nothing is known about the system. This population is generated in the MATLAB as $NP \times D$ matrix, where *NP* is the number of members of the population and *D* is the number of unknown variables in the solved task. Each matrix element is generated as a random number with uniform probability distribution in the form:

$$x_{i,j} = \min + r \cdot (\max - \min) \tag{2}$$

where $i = 1 \ldots NP$, $j = 1 \ldots D$, and the range of the variables is (min..max).

DE generates new parameter vectors by adding the weighted difference vector between two population members to a third member. If the resulting vector yields a lower objective function value than a predetermined population member, the newly generated vector replaces the vector which it was compared with. The compared vector can, but need not be part of the generation process mentioned above.

Several variants of DE exist. But, we have found out that the variant *DE*/*best*/1/*bin* is the most efficient version for the solved task. Therefore, the scheme of the variant will be described in greater detail. It works as follows:

1. For each target vector $\underline{x}_{i,G}$, $i = 1, 2, \ldots, NP$, a trial vector $\underline{v}$ is generated according to

$$\underline{v} = \underline{x}_{best,G} + F \cdot \left(\underline{x}_{r_2,G} - \underline{x}_{r_3,G}\right) \tag{3}$$

with r_2, $r_3 \in [1, NP]$, integer and mutually different, and $F>0$. $\underline{x}_{\text{best,G}}$ is the best vector, which was obtained as the best solution in the previous algorithm running. The integers r_2 and r_3 are chosen randomly from the interval [1, *NP*] and are different from the running index *i*. *F* is a real invariable which controls the amplification of differential variation ($\underline{x}_{r2,G}$ - $\underline{x}_{r3,G}$).

2. Each element of the chosen target vector $\underline{x}_{i,G}$, $i = 1, 2, \ldots, NP$, and each element of the trial vector $\underline{v}$ is selected (the first element of both, the second element of both…) and for each couple a random number with uniform probability distribution in the range ($0 \div 1$) is generated. This random number is compared with the crossover constant *CR*. If the random number is greater than *CR*, then the corresponding element of the vector $\underline{x}_{i,G}$ is inserted into the new vector $\underline{u}_{i,G}$. Otherwise the corresponding element of the trial vector $\underline{v}$ is inserted to the new vector $\underline{u}_{i,G}$. Thus, the new vector $\underline{u}_{i,G}$ is obtained. It introduces crossover operator in evolution theory. Fitness value of the vector $\underline{u}_{i,G}$ is compared with fitness value of the target vector $\underline{x}_{i,G}$. The vector with lower fitness value is chosen to the new population $G+1$.

This DE process is repeated until an acceptable solution is found or a preselect number of generations is performed.

3. First Part: Estimation of Group Delay Equalizer

The input requirement is to design a low-pass filter with pre-assigned cutoff frequency, which satisfies requirements for the magnitude frequency response. Such filter can be proposed using some development system, for instance using Syntfil Maple library [6]. We will denote a transfer function of the designed filter in the paper like $H(s)$ by variable *s* defined as:

$$s = \Sigma + j \cdot \Omega . \tag{4}$$

Thus, *s* is complex variable meaning normalized frequency.

3.1 An Analog All-Pass Filter Definition

Generally, the group delay frequency response of the analog filter can be equalized by an analog all-pass filter in cascade connection with the previous main filter. General transfer function of the even order analog all-pass filter can be defined in the form:

$$A(s) = \frac{\prod_{i=1}^{\frac{n}{2}} \left(s - \alpha_i + j \cdot \beta_i\right) \cdot \left(s - \alpha_i - j \cdot \beta_i\right)}{\prod_{i=1}^{\frac{n}{2}} \left(s + \alpha_i + j \cdot \beta_i\right) \cdot \left(s + \alpha_i - j \cdot \beta_i\right)} \tag{5}$$

where *n* is the order of the all-pass filter

General transfer function of the odd order analog all-pass filter can be defined in the form:

$$A(s) = \frac{\left(-s + \alpha_0\right) \cdot \prod_{i=1}^{\frac{n}{2}} \left(s - \alpha_i + j \cdot \beta_i\right) \cdot \left(s - \alpha_i - j \cdot \beta_i\right)}{\left(s + \alpha_0\right) \cdot \prod_{i=1}^{\frac{n}{2}} \left(s + \alpha_i + j \cdot \beta_i\right) \cdot \left(s + \alpha_i - j \cdot \beta_i\right)} \tag{6}$$

where the order of the all-pass filter is $n+1$.

The total group delay of the cascade connection of the low-pass filter and the all-pass filter is defined as a sum of the group delays of each of them, so that:

$$\tau(\Omega) = \tau_{allpass}(\Omega) + \tau_f(\Omega) = -\operatorname{Re}\left[\frac{A'(s)}{A(s)} + \frac{H'(s)}{H(s)}\right]_{s = j \cdot \Omega} \tag{7}$$

where $A(s)$ denotes the transfer function of the all-pass filter and $A'(s)$ denotes the derivative $A(s)$ by the variable s, $H(s)$ denotes the transfer function of the low-pass filter and $H'(si$ denotes the derivative $H(s)$ by the variable s.

3.2 The Analog All-Pass Filter Estimation Principle

Fundamental of the presented procedure is the computation of the complex poles of the transfer function of the all-pass section with the group delay frequency response which equalizes the group delay frequency response of the low-pass filter. We find the minimum of the $\Delta\tau$ using the Differential Evolution algorithm. The mathematical formulation of the objective function is

$$F(\underline{x}) = \max[\tau(\Omega)] - \min[\tau(\Omega)] + P_1 \quad (8)$$

where vector $\underline{\boldsymbol{x}}$ is composed of real and imaginary parts of the complex poles of the all-pass filter transfer function. P_1 is the penalty function, which can be computed by

$$P_1 = \sum_{i=1}^{N} \begin{cases} 20000 - 100 \cdot x_i & \text{if} \quad x_i < 0 \\ 0 & \text{otherwise} \end{cases}. \quad (9)$$

N labels the number of the unknown searched variables and x_i are elements of the vector $\underline{\boldsymbol{x}}$.

The penalty function P_1 is included into the objective function to guarantee stability of the analog all-pass filter. As it is known, the real parts of the complex poles must be located in the left part of the complex plane in the variable s. Here we have used penalty function published in [4].

The vector $\underline{\boldsymbol{x}}_{opt}$, for which the objective function $F(\underline{x})$ has minimum, is the wanted solution of the analog all-pass filter estimation problem.

3.3 Practical Example

The method usability to the all-pass filter design will be presented on the solution of the following example. It is required to equalize the group delay frequency response of the analog low-pass filter, which meet requirements to the magnitude frequency response. The filter is proposed by applying the Cauer approximation. The transfer function of the 6th order analog low-pass filter is defined:

$$H(s) = \sum_{i=0}^{6} b_i \cdot s^i \Bigg/ \sum_{i=0}^{6} a_i \cdot s^i \quad (10)$$

where coefficients are arranged in Tab. 1.

a_6=1	b_6=0.0045594
a_5=1.1440986	b_5=0
a_4=2.3731640	b_4=0.0760678
a_3=1.8230555	b_3=0
a_2=1.5491043	b_2=0.2355832
a_1=0.6668399	b_1=0
a_0=0.2091171	b_0=0.1974193

Tab. 1. Coefficients of the low-pass filter.

The group delay of the filter is shown in Fig. 1. The group delay error (the difference between the maximum and minimum of the group delay in the pass-band) is $\Delta\tau_f$=25.06 s in the pass-band and we want to equalize the group delay frequency response to receive a better group delay error.

The optimization task was solved using the DE algorithm initial settings: NP=150, CR=0.9, F=0.9, the range of the unknown variables (real and imaginary parts of complex poles) was (0 ÷ 1). The group delay frequency responses were sampled at 1024 equidistant points in the pass-band. We have decided for illustration of the described method to use all-pass filters of the order 4, 5 and 6 defined by appropriate terms (5), (6).

The algorithm found the final values shown in Tab. 2. The transfer functions obtained using these values were simulated in MATLAB. The resultant group delay frequency responses of the cascaded connections of the filter and the all-pass filters are shown in Figs. 2, 3 and 4.

4th order all-pass filter	5th order all-pass filter	6th order all-pass filter
α_1=0.216576734	α_0=0.188387743	α_1=0.091233209
β_1=0.204041274	α_1=0.182087386	β_1=0.776032459
α_2=0.169770328	β_1=0.362592308	α_2=0.150423689
β_2=0.639848225	α_2=0.123357832	β_2=0.473666694
	β_2=0.720882752	α_3=0.169704235
		β_3=0.152475683

Tab. 2. The found parameters of the all-pass filters.

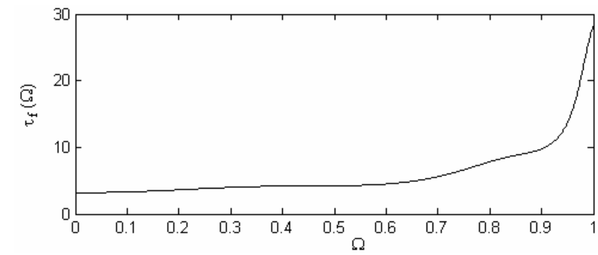

Fig. 1. The group delay of the low-pass filter.

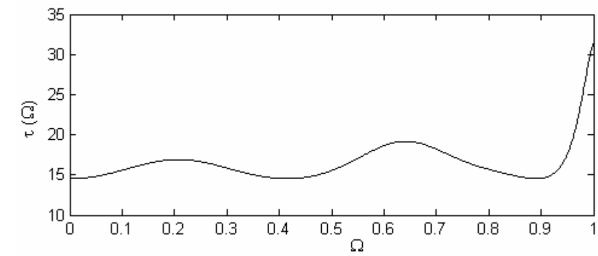

Fig. 2. The group delay of cascaded connection of the low-pass filter with the applied all-pass filter of the order 4.

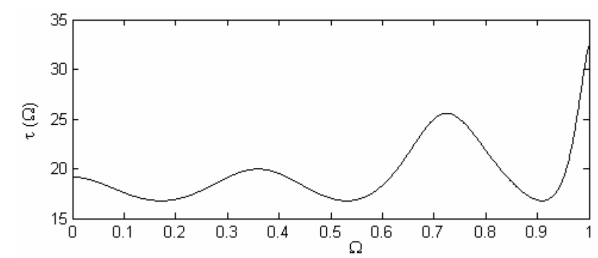

Fig. 3. The group delay of cascaded connection of the low-pass filter with the applied all-pass filter of the order 5.

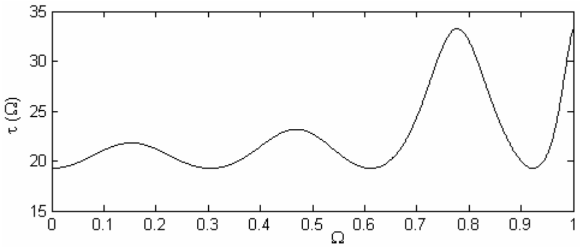

Fig. 4. The group delay of cascaded connection of the low-pass filter with the applied all-pass filter of the order 6.

4. Second Part: Final Design of Group Delay Equalizer

In the previous part, after low-pass filter group delay equalization, we have obtained lower value of group delay error in all cases, but group delay frequency responses have unequal-ripple form in the pass-band. Now, we will utilize the positions of the found extremes in Figs. 2, 3 and 4 to search correctly shifted complex poles of the all-pass filter transfer functions to achieve equal-ripple form of the group delay frequency responses. Let us have a pass-band interval of the group delay frequency responses (low-pass filter + appropriate all-pass filters) sampled at N equidistant points. It is plotted for each applied all-pass filter in the Fig. 5, 6 and 7. In the figures, the group delay frequency responses sampling at the N=1024 points was used.

Now, a so called error function for the sampled responses will be created:

$$e_k = \begin{cases} \tau(0) + (-1)^k \cdot \varepsilon - \max[\tau(\Omega_k)] & \text{for even k} \\ \tau(0) + (-1)^k \cdot \varepsilon - \min[\tau(\Omega_k)] & \text{for odd k} \end{cases} \tag{11}$$

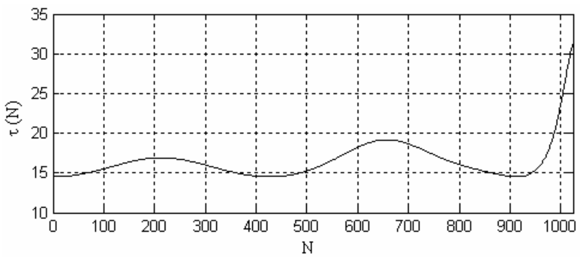

Fig. 5. The group delay of cascaded connection of the low-pass filter with the applied all-pass filter of the order 4.

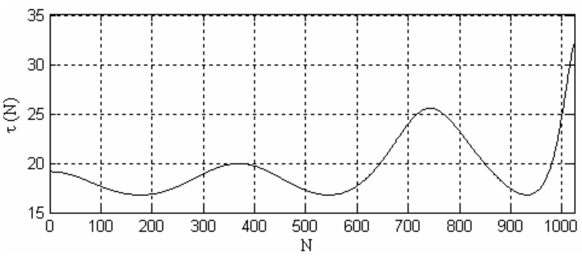

Fig. 6. The group delay of cascaded connection of the low-pass filter with the applied all-pass filter of the order 5.

$$\Omega_k \in (p_k - N1_k, p_k + N2_k) \tag{12}$$

where k=1,..,n+2, p_k denotes the position (sample) of the appropriate k^{th} extreme of the sampled response, $\tau(0)$ is the constant value of the group delay, which is approximated in Chebyshev sense, ε is the ripple of the group delay. Ω_k denotes the interval in which an appropriate extreme will be searched. $N1_k$ is the lower bound of the interval of Ω_k and $N2_k$ is the upper bound of the interval of Ω_k. Mostly, the optimum values are $N1_k$=100 points and $N2_k$=80 points. Only $N1_1$=0, $N2_1$=0, $N1_{n+2}$=0 and $N2_{n+2}$=0. We take into account sampling at N=1024 points. The values of $N1_k$ and $N2_k$ have to be re-counted for sampling by using different number of sampled points.

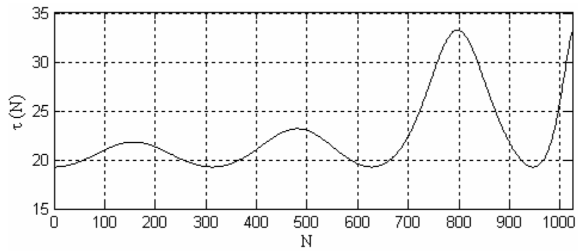

Fig. 7. The group delay of cascaded connection of the low-pass filter with the applied all-pass filter of the order 6.

Important: in case of odd order all-pass filter design, the parameter k=0,..,n+1. Thus, $N1_0$=0, $N2_0$=0, $N1_{n+1}$=0 and $N2_{n+1}$=0.

As mentioned above, DE algorithm will be used as a numerical solver. Therefore, an objective function will be defined. DE algorithm searches complex poles of the all-pass filter transfer function and constants $\tau(0)$ and ε to obtain the minimum of the objective function. Thus, the minimization process of the objective function leads to the design of the all-pass filter.

The mathematical formulation of the objective function for an even order all-pass filter is defined:

$$F(\underline{x}) = \left(\sum_{k=1}^{n+2} [e_k]^2 \right) + P_2 \,. \tag{13}$$

As mentioned above, for odd all-pass filter design, the objective function is modified so that k=0,..,n+1. The vector $\underline{\boldsymbol{x}}$ is composed of real and imaginary parts of complex poles of the all-pass filter transfer function and the constants $\tau(0)$ and ε. P_2 is the penalty function, which can be computed by:

$$P_2 = \sum_{i=1}^{M} \begin{cases} 20000 - 100 \cdot x_i & \text{if} \quad x_i < 0 \\ 0 & \text{otherwise} \end{cases} . \tag{14}$$

M labels the number of the unknown searched variables and x_i are elements of the vector $\underline{\boldsymbol{x}}$.

The penalty function P_2 is included into the objective function to guarantee stability of the analog all-pass filter. As it is known, the real parts of the complex poles must be located in the left part of the complex plane in the variable s. The penalty function P_2 also ensures positive values of initial group delay $\tau(0)$ and ripple ε.

The vector $\underline{x}_{opt}$, for which the objective function $F(\underline{x})$ has minimum, is the wanted solution of the analog all-pass filter design problem.

4.1 Final Results

The optimization task was solved using the initial settings: *NP*=150, *CR*=0.9, *F*=0.9, range of the unknown variables (real and imaginary parts of complex poles, constants $\tau(0)$ and ε) was (0÷1). The group delay frequency responses were sampled at 1024 equidistant points in the pass-band.

The algorithm found the final values shown in Tab. 3. The transfer functions obtained using these values were simulated in MATLAB. The resultant group delay frequency responses of the cascaded connections of the filter and the all-pass filters are shown in Figs. 8, 9 and 10.

4th order all-pass filter	5th order all-pass filter	6th order all-pass filter
α_1=0.084167941	α_0=0.083085265	α_1=0.088472270
β_1=0.191913543	α_1=0.085397763	β_1=0.757905454
α_2=0.083895124	β_1=0.344152333	α_2=0.084955920
β_2=0.598699067	α_2=0.085975762	β_2=0.453002355
	β_2=0.695630880	α_3=0.083676763
		β_3=0.146892823
$\Delta\tau$= 17.912s	$\Delta\tau$= 16.518s	$\Delta\tau$= 14.850s

Tab. 3. The found parameters of the final all-pass filters.

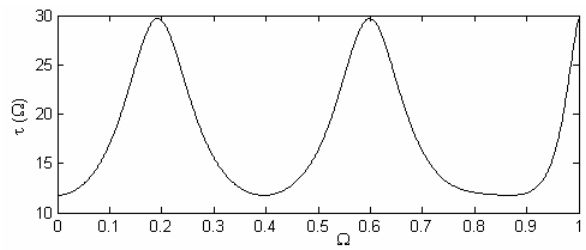

Fig. 8. The group delay of cascaded connection of the low-pass filter with the applied all-pass filter of the order 4.

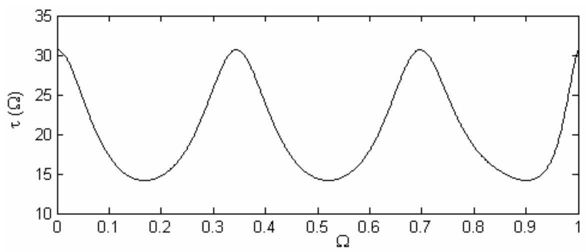

Fig. 9. The group delay of cascaded connection of the low-pass filter with the applied all-pass filter of the order 5.

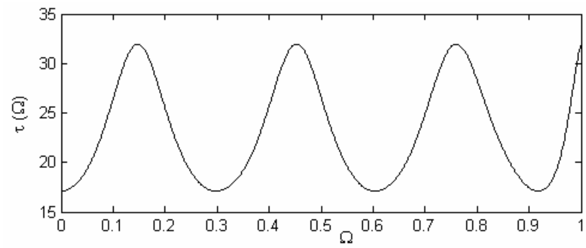

Fig. 10. The group delay of cascaded connection of the low-pass filter with the applied all-pass filter of the order 6.

5. Conclusions

A new unconventional method for the design of the analog group delay equalizers was presented in this paper.

The method allows designing such all-pass filter to achieve an equal-ripple form of the total group delay frequency response in the pass-band. It is a very good property of the method developed for equalization of analog filters or another delay lines.

The procedure consists of two parts. At first, the analog all-pass filter transfer function is calculated to be obtained the total group delay frequency response (of the low-pass filter with the applied all-pass filter) which has unequal-ripple extremes in the pass-band. Thus, the response is not equiripple. Hence, the equalization is performed in the sense of achievable difference between maxima and minima of the resulting response. Therefore, we have to apply the second step of the procedure as to achieve equiripple form of the resulting group delay.

The procedure works automatically without an input estimation. Moreover, in this paper we applied one of the most efficient types of evolutionary algorithms, the so called Differential Evolution algorithm. Due to the fact that the method is based on usage of evolutionary algorithm, the method is much more robust against convergence to the local extremes than classical numerical methods.

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References

[1] DAVÍDEK, V., LAIPERT, M., VLČEK, M. *Analog and Digital Filters (Analogové a číslicové filtry in Czech).* ČVUT Praha, 2000.

[2] MARTINEK, P., VONDRAŠ, J. New approach to filters and group delay equaliser transfer function design. In *The 8th IEEE Int. Conf. on Electronics, Circuits and Systems ICECS 2001*, vol. 1, p. 70.

[3] MARTINEK, P., VONDRAŠ, J. Multi-criterion filter design via Differential Evolution method for function minimization. In *1st IEEE Int. Conf. on Circuits and Systems for Communications ICCSC'02 Proceedings.* St. Petersburg (Russia): Saint-Petersburg State Technical University, 2002, vol. 1, p. 106-109.

[4] STORN, R. Differential Evolution Design of an IIR-Filter with Requirements for Magnitude and Group Delay. *Technical Report* TR-95-026, ICSI, May 1995.

[5] MICHALEWICZ, Z. *Genetic Algorithms + Data Structures = Evolution Programs.* Springer-Verlag, Berlin Heidelberg, .1996.

[6] HOSPODKA, J., BIČÁK, J. Syntfil - Synthesis of electric filters in Maple. In *MSW 2004* [CD-ROM]. Waterloo, 2004, vol. 1. http://syntfil.feld.cvut.cz/syntfil.html

[7] GREGORIAN, R., TEMES, G. Design techniques for digital and analog all-pass circuits. *IEEE Trans. on Circuits and Systems*, 1978, vol. 25, no. 12, p. 981-988.

[8] HENK, T. The generation of arbitrary-phase polynomials by recurrence formulae. *J. Circuit Theory Appl.*, 1981, vol. 9, p. 461-478.

[9] ZAPLATÍLEK, K., HÁJEK, K., DENK, M. Optimal all-pass network design using numerical optimization loop. In *Proc. of the 11th Electr. Devices Systems Conf. EDS 2004*, Brno, 2004, p. 98-103.