On Canonical Structures of ARC Biquadratic Filters with Single Transconductor

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Abstract. *The paper deals with simple canonical structures of the second order ARC filters employing only single transconductor (OTA) and passive components R and C. A systematic design procedure of this circuits based on the given autonomous networks is described. Several appropriate general autonomous circuits are presented and studied.*

Keywords

Analogue circuits, active RC filters, biquads, transconductors.

1. Introduction

Classical well known ARC filters based on standard voltage operational amplifier (not counting special high speed one) have been used only in low frequency applications, lower some hundred kHz. There an electronic tuning is complicated and they are not so suitable for full integration. To overcome these drawbacks the conventional opamp should be replaced by other modern functional block. At this time the dominant one used in analogue integrated filters is the transconductor or the operational transconductance amplifier (OTA). There the OTA has form of a chip in bipolar, CMOS or in GaAs technologies. On the other hand the OTA is also commercially available as a building block (e.g. OPA 660), which can be supplemented by passive elements *R* and *C*.

Over the last two decades the circuits based on the OTA's have been widely investigated. A large number of the OTA-RC or the OTA-C filters have been developed, with different design approach in mind [1], [2], [3] etc. Despite the wealth of publications the synthesis of these circuits is still an active topic. In this paper the simple structures of the second order filter (biquad) with only one single transconductor, several resistors *R* and maximally two capacitors *C*, what is a qualification of the canonical structure, are discussed. The given systematic design procedure is based on generalized autonomous network which is corresponding with suitable characteristic equation as was shown in [6] for the biquads based on current conveyors and also in [7] for the biquads with opamps.

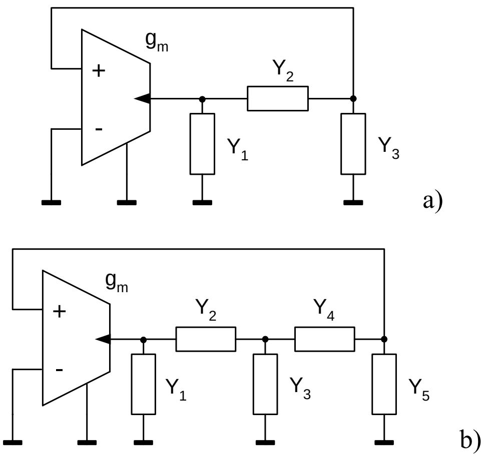

Fig. 1. General autonomous circuits of the single transconductor ARC biquads OTA1.
a) the model with three admittances OTA1Y3,
b) the model with five admittances OTA1Y5.

2. General Autonomous Circuits

To design biquadratic ARC filters and or oscillators we firstly choose an appropriate general autonomous circuit with the characteristic equation (CE) in the second-order general form (1) or in the following standard one (2)

$$a_2 s^2 + a_1 s + a_0 = 0, \quad (1)$$

$$s^2 + \frac{\omega_0}{Q} s + \omega_0^2 = 0 \cdot \quad (2)$$

There ω_0 is the undamped natural frequency and Q is the quality factor. The autonomous circuit contains only one transconductor **(OTA1)**, with general transconductance ($\pm g_m$), and certain number of general admittances Y_i (Fig. 1) in feedback loop. The simplest model **OTA1Y3** (Fig. 1a) contains only three passive admittances, namely two grounded (Y_1, Y_3) and one floating (Y_2). By routine symbolical nodal analysis (det **Y** = 0) the following CE is obtained

$$Y_1 Y_2 + Y_1 Y_3 + Y_2 Y_3 + g_m Y_2 = 0 \cdot \quad (3)$$

There we firstly suppose that the admittances are realized with single R or C only. Nevertheless, if more R and C components are used (usually in parallel combination) then more filter architectures can be obtained. It is the reason for a modification of the autonomous circuit OTA1Y3 (Fig. 1a) to the model with four admittances OTA1Y4-1, where admittance $Y_1=Y_{1A}+Y_{1B}$. Then the CE has the form

$$Y_{1A}Y_2 + Y_{1B}Y_2 + Y_1Y_3 + Y_2Y_3 + g_mY_2 = 0 \tag{4}$$

Other modification **OTA1Y4-3** has similarly $Y_3=Y_{3A}+Y_{3B}$. Note that decomposition of the $Y_2=Y_{2A}+Y_{2B}$ (**OTA1Y4-2**) is also possible but not so practical.

A more complex model with five admittances **OTA1Y5** is shown in Fig. 1b. This autonomous circuit gives more design flexibility and more filter structures can be achieved due to this resulting CE

$$\begin{aligned} &Y_1Y_2Y_4 + Y_1Y_2Y_5 + Y_1Y_3Y_4 + Y_1Y_3Y_5 + Y_1Y_4Y_5 + \\ &+ Y_2Y_3Y_4 + Y_2Y_3Y_5 + Y_2Y_4Y_5 + g_mY_2Y_4 = 0 \end{aligned} . \tag{5}$$

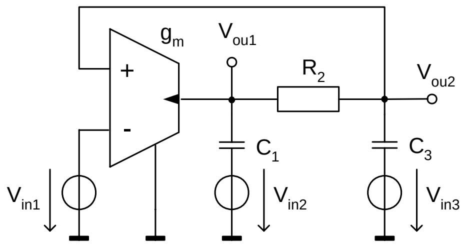

Fig. 2. ARC biquad with the structure OTA1Y3.

3. Structures with Three Admittances

From the characteristic equation (3) we can readily derive different filter structures choosing there the particular admittances Y_i, to obtain desired form (1) or (2) and postulating only two capacitors for the 2nd order canonical circuit. The combination $\{C_1, C_2, R_3\}$ is not acceptable, due to resulting CE in the form

$$s^2C_1C_2 + s(C_1G_3 + C_2G_3 + C_2g_m) = 0 \tag{6}$$

where the desired component a_0 (without s) is not there. Similarly the other combination $\{R_1, C_2, C_3\}$ can not be also used. On the other hand the variant $\{C_1, R_2, C_3\}$ (Fig. 2) is very suitable. In this case the CE (3) becomes of this desired shape

$$s^2C_1C_3 + sG_2(C_1 + C_3) + G_2g_m = 0 \; . \tag{7}$$

Comparing the formulas (6) and (2) the following design equations are done

$$\omega_0 = \sqrt{\frac{g_mG_2}{C_1C_3}} \; (8), \text{ and } \quad Q = \sqrt{\frac{g_m}{G_2}}\frac{\sqrt{C_1C_3}}{C_1 + C_3} . \tag{9}$$

For convenience of the design, low cost and minimizing the sensitivities we can set $C_1 = C_3 = C$. Then

$$\omega_0 = \frac{1}{C}\sqrt{g_mG_2} \; (10), \text{ and } \quad Q = \frac{1}{2}\sqrt{\frac{g_m}{G_2}} . \tag{11}$$

Choosing the value of the capacitance C the conductances are calculated by

$$G_2 = \frac{\omega_0 C}{2Q} \; (12), \text{ and } \quad g_m = 2Q\omega_0C \cdot \tag{13}$$

Parameter sensitivity with respect to variations of the passive component values are found to be very low, namely

$$\begin{aligned} &S^{\omega_0}_{g_m} = S^{\omega_0}_{G_2} = -S^{\omega_0}_{C_1} = -S^{\omega_0}_{C_2} = -S^{Q}_{G_2} = S^{Q}_{g_m} = \frac{1}{2} , \\ &S^{Q}_{C_1} = -S^{Q}_{C_3} = \frac{1}{2}\frac{C_3 - C_1}{C_1 + C_3}, \quad S^{Q}_{C}\big|_{C_1=C_3} = 0 \end{aligned} . \tag{14}$$

From the given autonomous circuits (Fig 1a) we can build a frequency filter so that we disconnect some grounded branch (inverting the input of the OTA and or Y_1, Y_3) and we put an independent input voltage source there (Fig. 2). It is based on the known idea that the CE remains the same if an ideal voltage source is connected into an arbitrary branch. Then we try to find a suitable output port examining all node voltages of this circuit. The Fig. 2 indicates all possibilities of driving (inputs) and loading (outputs) of this filter structure. The first suitable variant **OTA1Y3V12** has the input V_{in1} ($V_{in2} = V_{in3} = 0$, these sources are shorted). The output V_{ou2} and the following transfer function with low pass (LP) character was derived

$$\frac{V_{ou2}}{V_{in1}} = \frac{G_2g_m}{s^2C_1C_3 + sG_2(C_1 + C_3) + G_2g_m} . \tag{15}$$

The modification **OTA1Y3V11** with the output at the port V_{ou1} has not the transfer function in the desired form. The similar variant **OTA1Y3V22** with the input V_{in2} and the output V_{ou2} can be used as BP, because the numerator $N(s)=sC_1G_2$ and denominator $D(s)$ are the same as in (15). The other analyses indicate that output V_{out1} and the input V_{in3} are not suitable for standard types of our biquad.

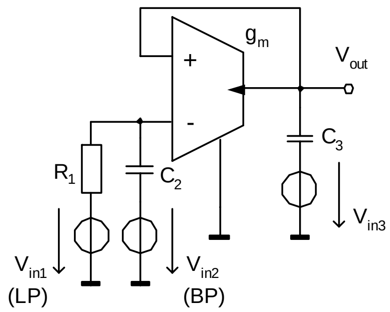

Fig. 3. Modified structure OTA1Y3M.

Another modification of the structure with three admittances **OTA1Y3M** is shown in Fig. 3. There two passive admittances are replaced on the input of the OTA. For the input V_{in1} the voltage transfer function of low-pass type is

$$\frac{V_{out}}{V_{in1}} = \frac{G_1g_m}{s^2C_1C_3 + s(C_1g_m + C_3G_1) + G_1g_m} . \tag{16}$$

For the input V_{in2} it is band-pass type with the same denominator (16) and the numerator in this form

$$N(s) = sC_1g_m \; . \tag{17}$$

Note that this circuit OTA1Y3M is not practical, due to the low value of the quality factor Q_{max} = 0,5.

4. Structures with Four Admittances

A modification of the biquad OTA1Y3 (Fig. 2) into the version with four admittances **OTA1Y4-1** is quite straightforward – the other resistor R_1 is parallel connected to the capacitor C_1 as shown in Fig. 4a. There all possible inputs and one output are depicted as a result of the routine symbolical nodal analysis using computer tool SNAP. The basic transfer functions are LP and BP types as

$$\frac{V_{out}}{V_{in1}} = \frac{G_2 g_m}{D(s)} \quad (18), \quad \frac{V_{out}}{V_{in2}} = \frac{sC_1G_2}{D(s)} \quad (19), \quad \frac{V_{out}}{V_{in3}} = \frac{G_1G_2}{D(s)}, \quad (20)$$

with the denominator

$$D(s) = s^2C_1C_3 + s[G_2(C_1 + C_3) + C_3G_1] + G_2(g_m + G_1). \quad (21)$$

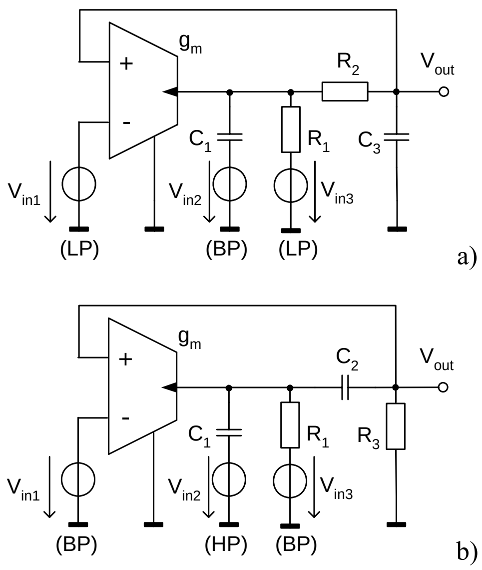

Fig. 4. ARC biquads with the structure OTA1Y4-1.

Here, a convenient design is also setting $C_1 = C_3 = C$ and $R_1 = R_2 = R$. Then for given parameters ω_0 Q and the chosen value of the C we can determine

$$R = \frac{3Q}{\omega_0 C} \quad (22), \text{ and } \quad g_m = 3Q\omega_0 C - \frac{\omega_0 C}{3Q}. \quad (23)$$

In opposite case the parameters of the biquad are given by

$$\omega_0 = \frac{1}{C}\sqrt{G(G + g_m)} \quad (24), \text{ and } \quad Q = \frac{\sqrt{G(G + g_m)}}{3G}. \quad (25)$$

The parameter sensitivities of this structure are given by the following expressions

$$S_{G_2}^{\omega_0} = -S_{C_1}^{\omega_0} = -S_{C_2}^{\omega_0} = \frac{1}{2}, \quad S_{G_2}^{Q} = -S_{C_1}^{Q} = S_{C_3}^{Q} = -\frac{1}{6},$$

$$S_{g_m}^{\omega_0} = S_{g_m}^{Q} = \frac{1}{2}\left(1 - \frac{1}{9Q^2}\right), \quad S_{G_1}^{\omega_0} = \frac{1}{18Q^2},$$

$$S_{G_1}^{Q} = -\frac{1}{3} + \frac{1}{18Q^2} \quad (26)$$

Other version of the structure OTA1Y4-1 is shown in Fig. 4b. The transfer function was derived as (27) with BP character

$$\frac{V_{out}}{V_{in1}} = \frac{sC_2 g_m}{s^2C_1C_2 + s[C_2(g_m + G_1 + G_3) + C_1G_3] + G_1G_3}. \quad (27)$$

Similarly the other transfer functions have the same denominator $D(s)$ (27), but different numerators with HP and BP character

$$\frac{V_{out}}{V_{in2}} = \frac{s^2C_1C_2}{D(s)} \quad (28), \text{ and } \quad \frac{V_{out}}{V_{in3}} = \frac{sC_2G_1}{D(s)}. \quad (29)$$

A convenient design is $C_1 = C_2 = C$ and $R_1 = R_3 = R$. For this case the design equations are

$$\omega_0 = \frac{G}{C} \quad (30), \text{ and } \quad Q = \frac{G}{3G + g_m}. \quad (31)$$

The expression (31) indicates that for practical values of the factor Q > 1, the desired transconductance must be negative with the value $g_m < -2G$. It means that the OTA input terminals need to be interchanged.

Another biquad structure, with four admittances, is **OTA1Y4-3** associated with

$$Y_1 = G_1, \quad Y_2 = sC_1, \quad Y_3 = G_3 + sC_3, \quad (32)$$

whose transfer function is similar to the expression (27). There is an interchange of the index number 3 and 1 only. Therefore this circuit has the same properties as OTA1Y4-3 given above.

It was shown above that these structures with four admittances give more possibilities in design, but in other hand they have the higher parameter sensitivities as the structures with three admittances. The same is right for the structures with five admittances described down.

5. Structures with Five Admittances

The structure **OTA1Y5** (Fig. 1b) has the CE (5), which is little complicated and therefore a selection of the particular admittance is not so easy. On the other hand this gives more flexibility and more 2nd order filters can be generated, but a question is their practical realization. As an illustrative example the biquad in Fig. 5 was derived. For this circuit the transfer function has BP character

$$\frac{V_{out}}{V_{inp}} = \frac{sg_mG_4C_2}{D(s)}. \quad (33)$$

The resulting denominator

$$D(s) = s^2C_2C_3(G_4 + G_5) + \\ + sC_2[(G_4 + G_5)G_1 + G_4(g_m + G_5)] + G_1G_4G_5 \quad (34)$$

offers more possibilities in design and tuning. For the equal capacitances and passive conductances, the design equations are found to be

$$G = 1{,}4142\,\omega_0 C \quad (35), \text{ and} \quad g_m = G\left(-5 + \frac{1{,}4142}{Q}\right), \quad (36)$$

for given parameters ω_0 Q and chosen value of the C. The expression (36) indicates that the desired transconductance must be negative.

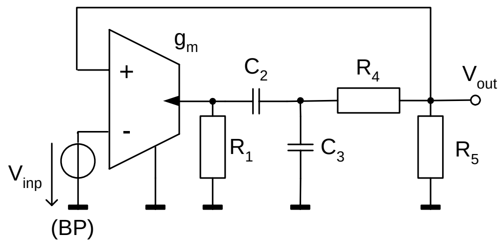

Fig. 5. Band-pass ARC biquad with the structure OTA1Y5.

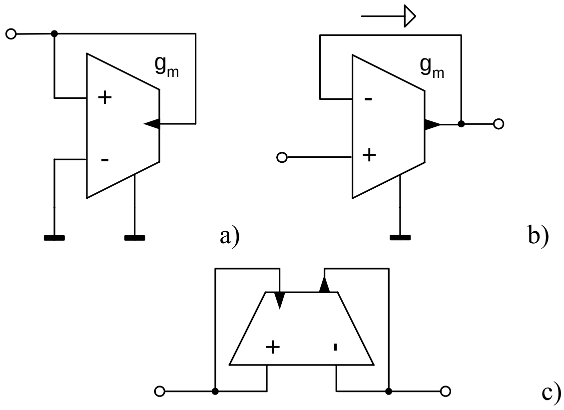

Fig. 6. R-OTA equivalent for simulation of the resistors a) grounded, b) unilateral, c) floating.

6. OTA-C Filters from these structures

The structures given above may not be fully integrated and easy electronically tuned due to the fact that they contain discrete resistors and a single OTA only. But these structures can be ingeniously retransformed for monolithic implementation replacing discrete R with R-OTA equivalent. The resistors can be simulated using the OTA's as shown in Fig. 6. The circuit in Fig. 6a is equivalent to a grounded resistor with the value $R=g_m^{-1}$. The other equivalent R-OTA on Fig. 6b is floating but unilateral resistor only. Finally full floating resistor simulation requires two OTA's as shown in [2] or one with balance output (Fig. 6c).

As an illustrating example the OTA-C low-pass filter is shown in Fig. 7. This circuit was derived from Fig. 2 replacing the discrete resistor R_2 by the OTA equivalent (Fig. 6c). The transfer function is the same as the expression (15), just the constant G_2 is replaced with the transconductance g_2, which can be electronically changed, too. The resistor substitution also retains the sensitivity property of the original prototype single OTA filter.

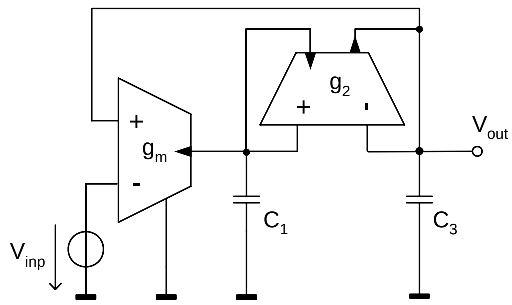

Fig. 7. OTA-C low-pass filter derived from Fig. 2.

In the high order OTA-C filter is better to be used as a building block, instead the here given biquad, the OTA-C integrator and the state-variable multi-loop feedback structures, what is in detail described in [1].

7. Structure for Commercial Building Block OTA & VB

A commercially available modern building IC block, as OPA 660, LM 13700 etc., includes two sub-blocks, namely the OTA and a voltage buffer (VB). The second mentioned VB can be also suitably used in our structure **OTA1VB1Y3**, as shown in Fig. 8, which is a good modification the structure OTA1Y3 (Fig. 1a). This general autonomous circuits (Fig. 8) has the following CE

$$Y_1Y_2 + Y_1Y_3 + g_mY_2 = 0\,, \tag{37}$$

which is simpler comparing with (3), there is no part Y_2Y_3.

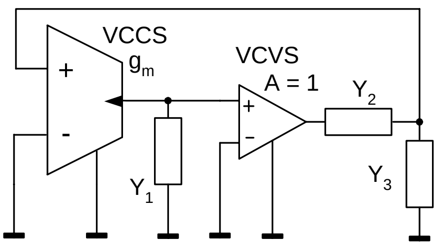

Fig. 8. General autonomous circuits with transconductor, voltage buffer and three admittances OTA1VB1Y3.

From the CE (37), using the procedure given above, a lot of different filter structures can be anew readily derived.

One of them is shown in Fig. 9, choosing there the particular admittances as

$$Y_1 = sC_1 + G_1\,, \quad Y_2 = sC_2\,, \quad Y_3 = G_3\,. \tag{38}$$

The desired second order polynomial is obtained

$$D_2(s) = s^2C_1C_2 + s[C_1G_3 + C_2(G_1 + g_m)] + G_1G_3\,. \tag{39}$$

From (39) the pole frequency (40) and quality factor (41) can be written in terms of the network parameters as

$$\omega_o = \frac{1}{\sqrt{C_1C_2R_1R_3}} \quad (40), \qquad Q = \frac{\sqrt{C_1G_1C_2G_3}}{C_1G_3 + C_2(G_1 + g_m)}\,. \quad (41)$$

The expression (41) indicates that the Q factor can be easily adjusted by the transconductance g_m.

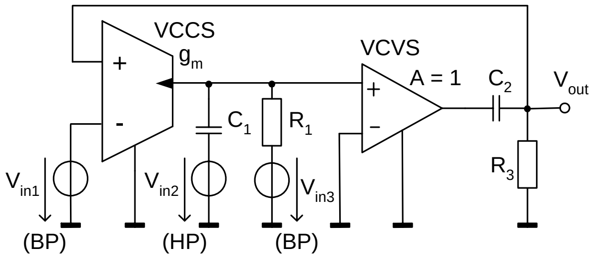

Fig. 9. ARC biquad with the structure OTA1VB1Y4-1.

8. Realization and Simulation

To evaluate the performance of above structures, the real circuit was designed with the commercially available LM 13700, realizing the structure **OTA1VB1Y4-1** (Fig. 9). The proposed circuit in Fig. 10 was simulated with PSpice using the professional macro model of the OTA (LM 13700). Resulting magnitude responses (Fig. 11) have confirmed the symbolical analysis and theoretical assumptions. There the g_m of the OTA was adjusted, with external DC current I_B. Two curves are given in the Fig. 11, namely for I_B = 200 and 350 µA. The change of this current gives the possibility of tuning the Q parameter of this filter.

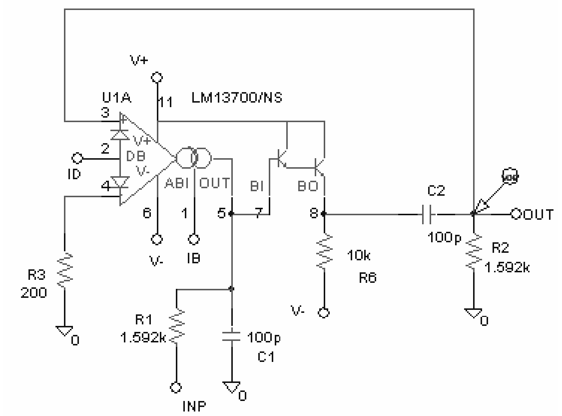

Fig. 10. Realization of the ARC biquad structure OTA1VB1Y4-1 with LM 13700.

9. Conclusion

The systematic uniform synthesis based on the general autonomous network with suitable characteristic equation is really good way how to obtain the 2nd order ARC filters or oscillators with modern functional blocks, as it has been shown here for the transconductors. The given OTA-based biquadratic filters have been found to be little sensitive, with small influence of the non-ideality. They can operate at higher frequency, can be electronically tunable (g_m is controlled by I_{DC}) and can be implemented in IC technologies. One single OTA means smaller chip area, lower power consumption and lower noise. In IC design grounded capacitors are preferred. There the discrete resistors are replaced with given equivalent OTA simulation.

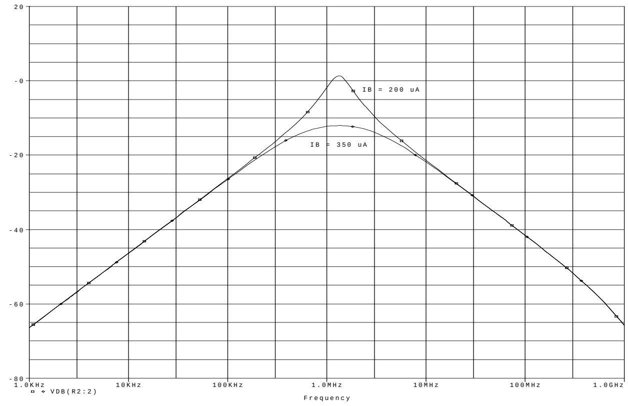

Fig. 11. Magnitude responses of the ARC biquad with LM13700, for two values of the current I_B = 200 and 350 µA.

Acknowledgements

The research was financially supported by the Grant Agency of the Czech Republic under grant No. 102/04/0442 and by the Ministry of Education of the Czech Republic under the research program MSM 00216305.

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