

L-System Tool for Generating Fractal Antenna Structures with Ability to Export into EM Simulators

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Abstract. An L-System (Lindenmayer system) is a scheme primarily developed in the area of the computer science for simulating the development of biological structures. It has also been found very useful for generating the geometry of various fractal antennas. A Matlab environment has been used for both implementing an in-plane L-systems algorithm and for creating appropriate files for widely used EM simulators like the IE3D and the CST Microwave Studio. Finally, the performance of the developed script is demonstrated on two fractal microstrip patch antennas.

Keywords

Fractal antennas, fractals, L-Systems, EM simulation, microstrip patch antennas.

1. Introduction

In computer science terms, an L-System [1] is a context-free, recursive, text substitution scheme, followed by a geometric interpretation. A simple L-System starts with a seed S, say the letter F, and has one or more rules R₁ to R_n to replace the initiatory seed. A simple replacement rule might be, for example R: F → F-F-FF+F-F-F (meaning of used characters is described later in detail).

The rule is then recursively applied many times (depending on the desired iteration level) to produce a series of strings of the increasing complexity. In order to produce fractals, the strings generated by L-Systems have to contain the necessary information about the figure geometry. A graphic interpretation of strings is based on assigning the language to the motion of an imaginary turtle. This interpretation is used to produce fractal images [2] and has been implemented in our Matlab script. Examples are given below together with an explanation of the L-System language alphabet.

2. L-System Language

Let us describe the specific language (alphabet) used in the presented simple L-System Matlab script.

- F move forward a step of the length d ;
- + turn to the left by a specified angle θ ;
- turn to the right by a specified angle θ .

Moreover, advanced letters are also defined with the following meanings:

- G move forward a step of length e ;
- > turn to the left by a specified angle φ ;
- < turn to the right by a specified angle φ ;
- [push the current state of the turtle onto a stack;
-] pop a state from the stack and make it the current state of the turtle.

The last two letters allow us to make a so-called *bracketing string* [2] resulting in branched structures (tree like).

In order to make the usage of the script more convenient, and in accordance with a common notation [2], some advanced letters are implemented:

- L = +F-F-F+
- R = -F+F+F-
- D = --F++F
- E = F--F++
- X, Y = do nothing

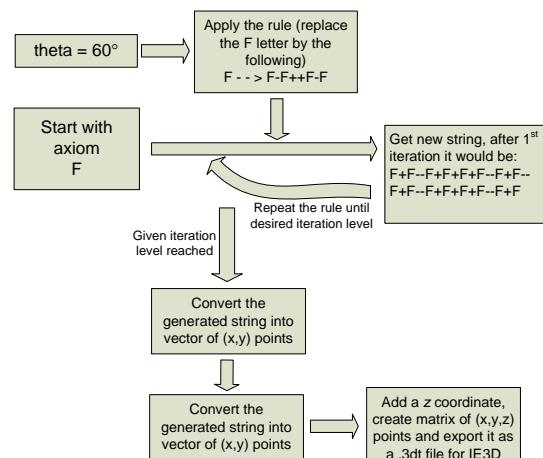


Fig. 1. A flowchart showing the script generating the Koch curve.

Fig. 1 shows a flowchart demonstrating how the script generates the Koch curve as one of the simplest examples.

Although the above described alphabet is very simple, it allows us to create quantities of various fractals which are discussed in the following section.

3. Examples

Fractals can be generally divided into 3 different groups according to their geometrical properties. Our L-System generator is able to create all kinds of fractals in the XY plane. Let us imagine a fractal set R with Hausdorff [2] interior dimension D . Dimension of the boundary ∂R will be denoted d . Then:

- a) D is an integer and d is a non-integer, respectively. A planar object with a fractal boundary like the Koch snowflake, where $D = 2$ and $d \sim 1.26$, is an example.
- b) d is an integer and $D < 2$ is a non-integer, resp. Fractal curves like the Minkowski ($D = 1.5$) and the Koch ($D = 1.26$) belong to this group. Also fractal trees satisfy these conditions.
- c) Both d and D are non-integers. A porous material or a natural snowflake are examples.

In the following section, some examples of the script capability are shown.

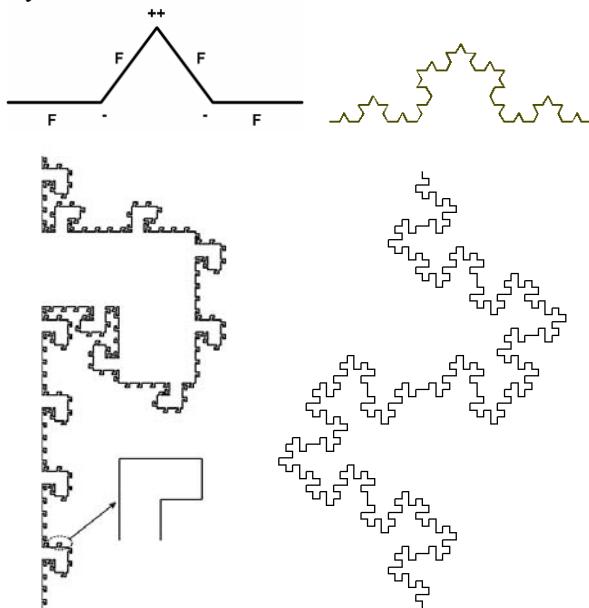


Fig. 2. Generation of Koch Curve (top), Coastline (bottom left) and Minkowski Curve (bottom right), both iteration 3.

3.1 Fractal Curves

- *Koch Curve*

S: F
R: $F \rightarrow F-F+F-F$, $\theta=60^\circ$
see Fig. 2 (at the top) for generating process details.

- *Coastline*
S: F
R: $F \rightarrow FFFF+F+F-F-FF-FF+FF$, $\theta=90^\circ$
see Fig. 2 (three iterations performed).
- *Minkowski Curve*
S: F
R: $F \rightarrow F-F+F+FF-F-F+F$, $\theta=90^\circ$
see Fig. 2 (three iterations performed).
- *Hilbert Curve*
S : L
R1: $L \rightarrow +RF-LFL-FR+$, $\theta=90^\circ$
R2: $R \rightarrow -LF+RFR+FL$, $\theta=90^\circ$
see Fig. 3 for Hilbert Curve, 1 and 4 iterations shown.

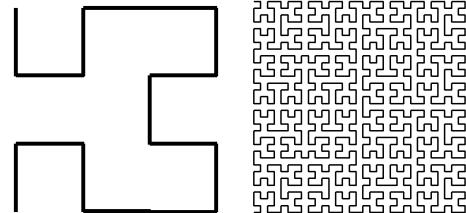


Fig. 3. Hilbert Curve, iteration 1 and 4.

3.2 Planar Closed Sets with a Fractal Boundary

- *Quadratic Koch Island*

S: $F+F+F+F$
R: $F \rightarrow F-F+F+FFF-F-F+F$,
 $\theta=90^\circ$, see Fig. 4 (three iterations performed).

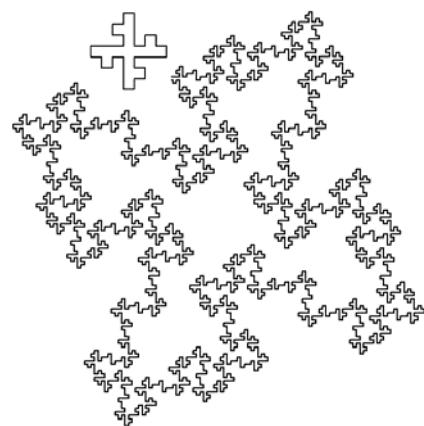


Fig. 4. Quadratic Koch Island, iteration 3. Iteration 1 shown also for the comparison.

3.3 Other Planar Fractals

- *Tegel I*

S: $F-F-F-F$
R: $F \rightarrow FF[-F-F-F]F$, $\theta=90^\circ$
see Fig. 5 (left).

- *Tegel II*

S: F-F-F-F

R: F→FF[-F+F+F]FF, $\theta=90^\circ$

see Fig. 5 (right).

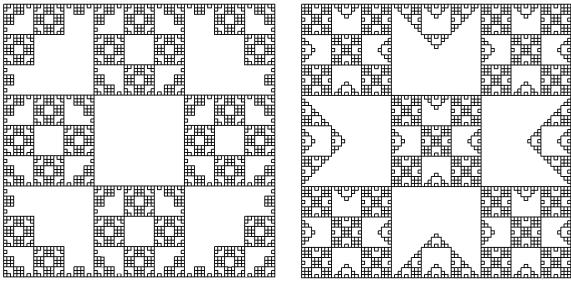


Fig. 5. Tegel I and Tegel II, iteration 4.

- *Snowflake I*

S: [F]+[F]+[F]+[F]+[F]+[F]

R: F→FF[+F][-F]F, $\theta=60^\circ$

see Fig. 6 (left).

- *Snowflake II*,

S: [F]+[F]+[F]+[F]+[F]+[F]

R: F→FF[+F-F][-F+F]F, $\theta=60^\circ$

see Fig. 6 (right).

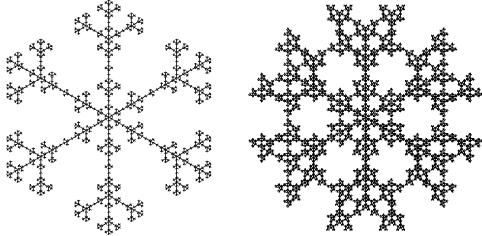


Fig. 6. Snowflake I and Snowflake II, iteration 4.

4. Applications

The main purpose of the presented generator is to study various fractal antennas using numerical EM simulation. Let us consider a fractal patch of the iteration 3 (Fig. 7) defined as follows:

- S : F+F+F+F

R1: F→FF+GGGG-G-GGGG+F

R2: G→GGG

$d = 1, e = 0.1, \theta = \varphi = 90^\circ$

The Matlab script produces the antenna geometry in a .3dt format which can be directly imported to the full wave simulator IE3D. The simulation was carried out with the infinite ground-plane, the air substrate $h = 1$ mm and was fed by a coaxial probe. The antenna's first resonance appeared at 3 GHz, which is approximately 65 % reduction in frequency compared to a square patch of the same outer dimensions. This is addressed by the fact that the fractal geometry forces currents to flow along the patch's diagonal (see Fig. 8a). For verification purposes, modal currents have been calculated by using the cavity model theory [3],

[4] implemented in the Femlab software. Except for the fundamental mode #1, the mode #4 is also shown to check the agreement between 2 different simulation approaches.

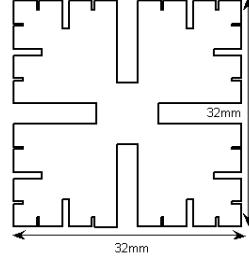


Fig. 7. Fractal patch antenna created by our L-system script.

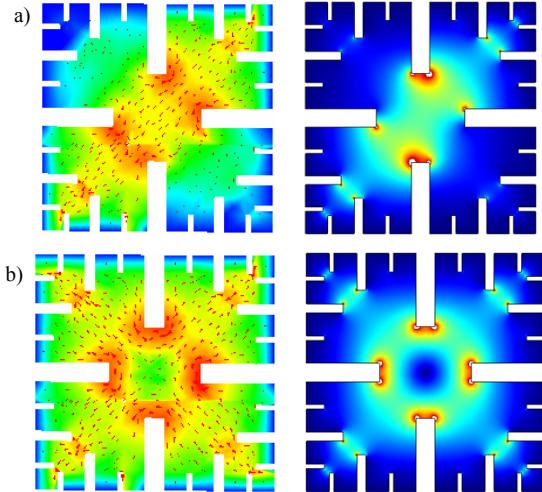


Fig. 8. Mode #1 and #4 current densities calculated by the full-wave model and the cavity one: a) mode #1, full-wave IE3D (left), cavity FEMLAB (right), b) mode #4, full-wave IE3D (left), cavity FEMLAB (right).

Another interesting antenna structure created with the L-System described is a so-called Inverted Koch Square patch [5]. The structure is obtained by the following ruling:

- S: F<F<F<F
- R: F→F-F++F-F.

We use the notation IKS $n\text{-}\alpha\text{-}J$ (n stands for the number of performed iterations, α is so-called indentation angle [5], J denotes that the J -probe is used as the feeding system). The indentation angle α is defined within the operators + and -. The seed is a rectangle, the angle for the operator < is thus $\beta = 90^\circ$. For the analysis, IKS3-85-J is selected (Fig. 8).

First, the modal analysis is employed to predict resonances and surface currents, and the fundamental mode with the lowest resonant frequency is chosen. The modal estimation predicts the resonant frequency to be lower, ~ 30 % of the rectangular patch antenna with the same edge dimensions (31.32mm \times 31.32mm). The comparison of surface currents obtained from full-wave analysis and the modal one is shown in Fig. 9. Arrows clearly show two main current paths, both being in-phase. Thus, the expected radiation pattern should be broadside.

Second, the suitable feeding technique is employed to provide the appropriate matching. We use the so-called J-

probe (see Fig. 8) and adjust the main parts of the probe using the parametric IE3D full-wave simulation. A partial result of the parametric study performed is shown in Fig. 10, the final dimensions of the J-probe are $L_{v1} = 20$ mm, $L_h = 10$ mm, $L_{v2} = 8$ mm, $L_p = 30$ mm resulting in a relative frequency bandwidth of 18 %. Let us note that dimensions of the edges are $0.22 \lambda \times 0.22 \lambda$, thus the antenna is notably smaller than the conventional rectangular patch antenna operating at fundamental “ 0.5λ ” mode.

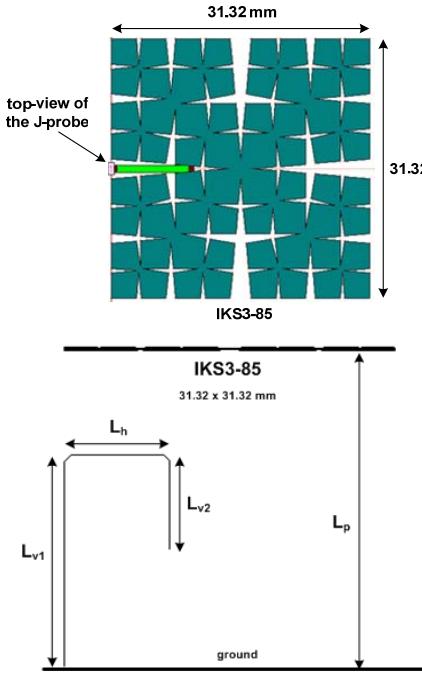


Fig. 8. Layout of the IKS3-85-J antenna, top and side view.

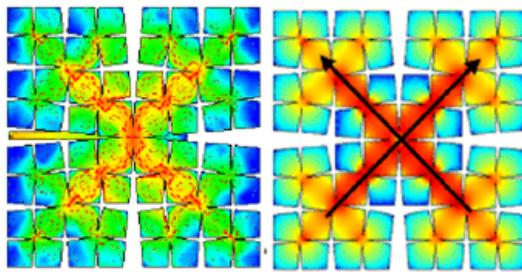


Fig. 9. IKS3-85-J. Surface currents at the fundamental mode – full wave results (left), modal analysis (right).

The antenna exhibits directivity of 4.5 to 6.6 dBi across the working band, the radiation pattern at the central frequency is shown in Fig. 11. A more detailed discussion on radiation properties could be found in [5].

5. Conclusion

This paper presents a widely configurable fractal geometry generator based on an L-System algorithm implemented in a Matlab environment. Various outputs from the generator are shown together with the simplicity of entering the input data. The script is ready to be used with-

in the optimization loop as all the parameters are simple to access and with clear impact on the generated geometry. Future plans are to use the modal analysis together with the described generator to optimize the frequency response of planar fractal antennas. Finally, two examples of microstrip fractal antennas created with the script are given.

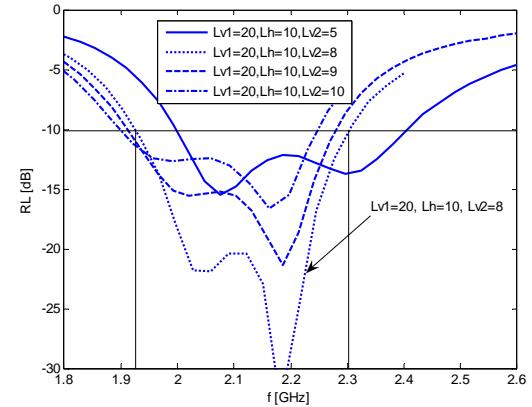


Fig. 10. Return loss of IKS3-85-J antenna with $L_{v1} = 20$ mm, $L_h = 10$ mm, $L_p = 30$ mm, L_{v2} being variable.

—○— $f=2.12069\text{GHz}$, E-theta, phi=0 (deg), PG=-4.1689e-005 dB, AG=6.12289 dB
—○— $f=2.12069\text{GHz}$, E-theta, phi=90 (deg), PG=-6.24075 dB, AG=12.6954 dB
—○— $f=2.12069\text{GHz}$, E-phi, phi=0 (deg), PG=-48.9407 dB, AG=54.6686 dB
—○— $f=2.12069\text{GHz}$, E-phi, phi=90 (deg), PG=-0.4888917 dB, AG=-5.823662 dB

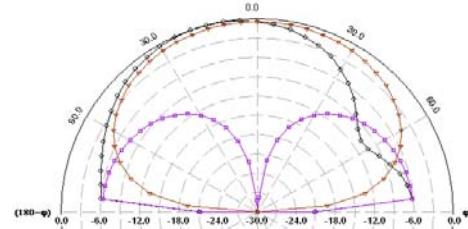


Fig. 11. Two principal far-field cuts @ 2.12GHz for IKS3-85-J, $L_{v1} = 20$ mm, $L_h = 10$ mm, $L_{v2} = 8$ mm, $L_p = 30$ mm.

Acknowledgements

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